Multi-Dimensional Reasoning in Competitive Resource Allocation Games: Evidence from Intra-Team Communication

Ayala Arad* and Stefan Penczynski**

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Abstract

We experimentally investigate behavior and reasoning in various competitive resource allocation games with large strategy spaces. In the experiment, a team of two players plays as one entity against other teams. Team members communicate with one another before choosing a strategy. We analyze their messages using three different classification approaches and find that the vast majority of players think in terms of dimensions or characteristics of strategies rather than in terms of individual elements of the strategy space. Furthermore, the dimensions' metric allows linking the reasoning across the different games. Thus, we suggest that multi-dimensional reasoning is a frequently used decision procedure that connects the behavior observed in various resource allocation games.

Keywords: Blotto games, Bounded Rationality, Communication, Multi-dimensional reasoning, Multi-unit auctions, Text analysis.

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* Corresponding author. Coller School of Management, Tel Aviv University. Adress: Coller School of Management, Tel Aviv University, Ramat Aviv, Tel Aviv, Israel, 69978. Email: aradayal@tauex.tau.ac.il.

** School of Economics and Centre for Behavioural and Experimental Social Science (CBESS), University of East Anglia.

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1. Introduction

In games with large and complex strategy spaces, it may be natural for players to deliberate over the characteristics of strategies rather than the strategies themselves. Arad and Rubinstein (2012a) introduce the concept of *multi-dimensional reasoning*, which posits that players classify strategies along a number of dimensions and make a separate decision for each of them.[1] In an experiment of the Blotto game, they find traces of level-k-style belief iteration in each dimension they identified. Selten et al. (2011) provide experimental evidence for multi-dimensional reasoning in a monopoly context, in which participants focus on achieving a subset of goals rather than on maximizing the monopoly's profit.

To illustrate the concept of multi-dimensional reasoning, consider the strategic decision-making in the allocation of in-house R&D funds over six products. Instead of deliberating over 6-dimensional allocation vectors (6-tuples) in what we call "strategy-wise reasoning", it is easier to make a series of decisions determining the following: the number of products to concentrate on and thus the rough amount of funds available for each product; the identity of the products to concentrate on; and finally the exact amount of funds for each product. Each decision may be based on beliefs about the products' competitive environments. Within each dimension, a decision rule such as level-k reasoning (Nagel, 1995; Stahl and Wilson, 1995) may be used to make decisions. For example, when deciding about the first dimension – the number of products to concentrate on, a belief that the competing company allocates its similar budget equally among the six products ("level-0") could motivate a concentration on only five products ("level-1"). Eventually, such a series of decisions leads to a smaller set of 6-dimensional allocation vectors and sometimes to one specific strategy.

It is established that in various domains, people often use heuristics and simplify problems to obtain approximate solutions (Simon, 1957; Kahneman, 2003; Gigerenzer, 2008). Furthermore, when a problem is more difficult to solve, people more often use heuristics, including broader categorization (Liberman et al., 2002; Broder and Schiffer, 2006; Rieskamp and Hoffrage, 2008). This evidence is consistent with our hypothesis that people categorize strategies along multiple dimensions in complex games and reason within these dimensions.

In order to better understand and predict behavior in games with a complex strategy

[1] A number of economic models incorporate the related idea of categorization. For example, Piccione and Rubinstein (2003), Jehiel (2005), and Arad and Rubinstein (2019) in the case of beliefs; Manzini and Mariotti (2012) in the case of choice sets; and Thaler (1999) in the case of cash flows. Multi-dimensional reasoning involves the simultaneous categorization of strategies along a number of dimensions, where each dimension is a different partition of the strategy space.

space, additional empirical evidence is needed for answering currently open questions: Are the strategy dimensions proposed by Arad and Rubinstein (2012a) in the context of the Blotto game indeed the ones that structure a subject's reasoning? What are the cognitively relevant dimensions in other games? What is the nature of the decision rules within dimensions? Does the consideration of additional dimensions improve the player's success? Are the dimensions identified in one game useful for predicting those in other games? Does the invoked number of dimensions correlate between games? We attempt to answer these questions by looking at team chat transcripts in addition to chosen strategies.

In this paper, we investigate a family of complex strategic interactions, namely simultaneous resource allocation games. We study three classes of these games: Colonel Blotto games, multi-object first-price auctions with budget constraints, and all-pay multi-object auctions with a limited endowment. In all of them, a strategy is a resource allocation in the form of an n-tuple, i.e., a division of a limited budget among a number of tasks, although the payoff functions differ across the games.

In the Colonel Blotto game, a player allocates X troops among n battlefields and wins on the battlefields in which he assigned more troops than his opponent (Borel, 1921). The game has been used to describe the budget allocations of political campaigns across "battlefield" states (see, for example, Brams, 1978), political vote-buying (e.g., Myerson, 1993) and the allocation of defense costs in cyber security (Hausken, 2008; Hui and Chuang, 2011). In multi-object first-price auctions, a player places bids on n objects with different (though known) values and the highest bidder obtains the object. In the all-pay version, all players pay their bids rather than only the winner. Multi-object auctions of various types, such as those for oil leases and spectrum licenses, have naturally attracted the interest of economists (Krishna, 2002).

Experimental behavior in these so-called "multi-battle contests" has been studied separately in the past (see Dechenaux et al., 2015 and Kagel and Levin, 2014 for a review). However, the vast majority of these studies investigate deviations from the game equilibrium or conditions for convergence to equilibrium during the course of repeated play, rather than the reasoning and decision rules used by the players. We provide evidence for a decision procedure that has different implications than the standard solution concepts and which allows us to *connect* the initial reasoning and behavior across different resource allocation games. Our findings of considered dimensions and decision rules enable better predictions of initial behavior in such games. Furthermore, our framework of multi-dimensional reasoning explains systematic deviations from equilibrium in repeated play that were considered anomalies in previous studies (e.g., Chowdhury et al., 2013; Montero et al., 2016; Chowdhury et al., 2021).

In order to shed more light on multi-dimensional reasoning, we go beyond exploring players' chosen strategies and analyze their reasoning process as expressed in team chat transcripts. In the experiment, a team of two players plays as one entity against other similar teams. Teammates are allowed to electronically communicate with each other before choosing a strategy. We use a variation of the communication protocol introduced in Burchardi and Penczynski (2014), in which each team member can suggest a strategy and justify it in a written message. After the simultaneous exchange of the "suggested strategy" and the message, the team members decide individually on the strategy they wish the team to follow. One of the two "final strategies" is randomly implemented as the team's strategy. Thus, participants have an incentive to persuade their teammates using arguments that support their suggested strategy. Since the messages are exchanged simultaneously, the text reveals the individual's reasoning before he is exposed to his teammate's ideas.

The analysis of various forms of communication in experiments for the purpose of better understanding players' motives and reasoning processes has proved to be enlightening in a number of different contexts (Schotter, 2003; Cooper and Kagel, 2005; Schotter and Sopher, 2007; Cason et al., 2012; Penczynski, 2016a, 2017). Here we use intra-team communication as a powerful diagnostic tool to examine multi-dimensional reasoning and decision rules within dimensions. If multi-dimensional reasoning takes place, it shapes the team's conversation and will be detected by this method. Of course, if the conversation does not feature a certain type of reasoning, we cannot infer that it did not take place.

The analysis of the communication between the team members indeed indicates that in all the resource allocation games players classify strategies according to a number of dimensions and perform their strategic deliberation within the space of those dimensions. At the same time, only a small proportion of the messages indicate the standard strategy-wise reasoning. We also find that the main dimensions detected are common to all three games. Thus, analyzing the written messages and the chosen strategies within the framework of multi-dimensional reasoning makes it possible to link the three resource allocation games in terms of the decision-making process. The communication protocol does not change the types of strategies most commonly used and can therefore be viewed as a non-intrusive tool in this respect.[2]

[2] Arad, Grubiak and Penczynski (2022) explore this methodological issue, among others in the Blotto game, and find that individuals' suggested strategies in teams, given this communication protocol, are not significantly different from the strategies played by individuals who are not part of a team.

For each of the three games, the findings contribute to the understanding of the game's dominant reasoning processes and decision rules. We present the first direct evidence – to the best of our knowledge – that multi-dimensional reasoning indeed structures participants' arguments in the well-studied Blotto game and in other games. We also investigate the decision rules that are used to determine the preferred characteristic within a dimension. In addition to "level-k" decision rules of iterated best responses within dimensions, we find evidence of non-belief-based decisions rules, such as aiming to win the majority of battlefields in the Blotto game and aiming to achieve a minimally desired gain in auction games. The results are robust to changes in parameters in all three games. Interestingly, after the exchange of messages, the final strategies continue to reflect multi-dimensional reasoning to a similar extent.

Despite the differences between the three classes of resource allocation games, the findings for all of them provide evidence of multi-dimensional reasoning, a similar consideration of dimensions in the aggregate, and a relatively substantial within-subject correlation of dimensions across the three classes of games. It is also found that considering more dimensions is associated with players' success. Winning strategies feature reasoning in many dimensions and we highlight the most essential ones. In view of the similarities in reasoning across the different allocation games, it is possible that multi-dimensional reasoning is used in additional classes of strategic interactions in order to deal with the complexity arising from large strategy spaces.

The paper proceeds as follows: Section 2 describes the experimental design and specifies multi-dimensional reasoning in our context and the manner in which written messages are classified. Section 3, 4 and 5 discuss the three classes of allocation games in turn, including an analysis of the written messages and of the relationship between performance of a strategy and the reasoning expressed in its accompanying message. Readers that are not interested in one or two of the game classes can skip those sections without any loss of flow. Section 6 explores similarities in the players' reasoning across these games. Section 7 further discusses the paper's method and Section 8 concludes.

2. Experiment

2.1 Hypothesis

The principal aim of our investigation is to provide evidence that is able to support either of two opposed modes of strategic reasoning in complex resource-allocation games: multi-dimensional reasoning and the standard strategy-wise reasoning. In the latter, the fundamental element of consideration is a feasible strategy in the form of a $n \times 1$ allocation vector (n-tuple) and beliefs are concrete, that is, (possibly degenerate) probability distributions over these vectors.

We turn to intra-team communication data since the verbalization of arguments will be influenced by the reasoning process and thus will allow us to discriminate between strategy-wise reasoning and multi-dimensional reasoning. Such discrimination was not possible in Arad and Rubinstein (2012a), which analyzed the strategies chosen by players in the Blotto game and suggested that patterns such as assigning one troop to a front are an outcome of multi-dimensional reasoning.

Hypothesis: *The written arguments behind decisions in complex resource-allocation games will involve elements of dimensions of the strategy space rather than strategies.*

If and when multi-dimensional reasoning can be inferred from the communication, further characteristics of the reasoning will be inferable. They will inform the questions above, such as the nature of the dimensions and the decision rules used within dimensions.

2.2 Experimental design

The experiment was carried out in the Interactive Decision Making Lab at Tel Aviv University and the Experimental Economics Lab at Ben-Gurion University. The experiment was programmed in z-Tree (Fischbacher, 2007). It consisted of 16 sessions with a total of 249 subjects, about 50% of whom were women. The participants were Tel Aviv University and Ben-Gurion University students in various fields of study. Recruitment of participants was done via ORSEE (Greiner, 2015).

Subjects played four games in the experiment, two games of two game classes. They were rematched into different teams in each game and played against other teams. Anonymity within a team and between teams was maintained both during and following the experiment. Payoffs in the games were stated in terms of points. Points accumulated during the session were converted to cash at the end of the experiment according to a fixed exchange rate: 5 points = NIS 1. Participants received NIS 35 (ca. USD 10) for participation and in addition were rewarded according to their performance, with the team's payoff being divided equally among its members. Sessions lasted about an hour.

We used a variation of the communication protocol introduced in Burchardi and Penczynski (2014), in which two anonymous participants play as a team and communicate electronically in the following manner: each member suggests a strategy and justifies it in a written message. After the simultaneous exchange of the suggested strategy and the message, the team members decide individually on the strategy they wish the team to take. One of the two final strategies is randomly implemented as the team's strategy.

In order to have the team partner's previous messages not influence subsequent suggested strategies, the experiment involved sequentially suggesting strategies for four games first, before the two teammates received each other's suggestions and made their final decisions (van Elten and Penczynski, 2020). Further, the different game class of games 3 and 4 was introduced only after game 2's suggested decisions had been taken in order to prevent the behavior in the first block from being influenced by the description of games in the second block; see Table 1.

Order		**Games**			
		First Block		**Second Block**	
1	General instructions				
2	Practice round of the communication interface				
3	Instructions	1	2		
4	Suggested strategies	1	2		
5	Instructions			3	4
6	Suggested strategies			3	4
7	Final strategies	1	2	3	4
8	Feedback on outcomes & payoffs				

Table 1: Sequence of events in the experiment.

Subjects seem to take seriously the intra-team communication consisting of the suggested decision and the message. As senders in the communication, subjects describe their reasoning in a message in 85% of all decisions. A simple way to detect the impact of the received communication is to compare final strategies to suggested ones. In 23% of all decisions, subjects' final strategies coincide with the suggested strategies of their teammates. In additional 29% of all decisions, the change between subjects' suggested and final strategies is in the direction of their teammates' suggested strategies.[3] Only in 8% of all decisions, this

[3] We measure change as the average relative adjustment over all fronts. In each front, we define the relative adjustment as the change from suggested to final strategy ("adjustment") divided by the difference in suggested strategies between the team members. While this measure is not necessarily a perfect reflection of individual

change is in the opposite direction. In the remaining 40% of decisions, subjects' final and suggested strategies are the same.

Strategic games

Following are the three classes of games that were played by teams in the experiments. The complete translated instructions are shown in Online Appendix C.

1. Colonel Blotto tournament

"You are playing the role of a colonel during wartime and other teams in the experiment are your opponents. Each team is allocated a given number of 'troops' that need to be allocated among a given number of separate 'fronts'.

"You win the battle in a particular front if you assign more troops than your opponent. In the case where you and your opponent both allocate the same number of troops to a particular front, both teams lose the battle.

"Your team will participate in a round-robin tournament, in which your team's deployment of troops will automatically face those of all other teams in the experiment. You cannot choose different deployments against different teams. Your team's total score will be the overall number of fronts you win against all other teams. The winner of the tournament will be the team with the highest score. If there is a tie in the total score, the winner will be determined randomly."

This class of games consisted of two games: "Blotto 6" with 120 troops allocated to 6 fronts and "Blotto 7" with 210 troops allocated to 7 fronts.

2. First-price multi-object auction with budget constraints

"In this game, your team plays against two other teams. You can participate in up to three auctions: A, B, and C. If you win Auction A, you receive a prize of W; if you win Auction B, you receive a prize of X; and if you win Auction C, you receive a prize of Y. (In the second game, there is an additional Auction D that features prize Z.)

"You win a particular auction if your bid is the highest. In the case of a tie, a lottery will determine which of the teams wins the auction. If you win an auction, and only in that

persuasion, its sign indicates whether a change in the final strategy goes in the direction of the partner's suggested strategy or not, also in multi-dimensional spaces.

case, you will pay according to the bid you placed in that auction. You can bid in all three auctions, as long as the sum of your bids is at most M."

This class of games consisted of two games: "Auction 3" with W=X=Y=100 and "Auction 4" with W=X=Y=90 and Z=110. In both games, the budget was M=120.

3. All-pay multi-object auction with a limited endowment

"In this game, your team plays against two teams. You can participate in up to three auctions: A, B, and C. If you win Auction A, you receive a prize of W; if you win Auction B, you receive a prize of X; and if you win Auction C, you receive a prize of Y. (In the second game, there is an additional auction D that features prize Z.)

"You win a particular auction if your bid is the highest. In the case of a tie, a lottery will determine which of the teams wins the auction. You receive an endowment of M, which you may use for bidding in the auctions. In each auction, you pay your bid even if you do not win the auction. Unused points will remain in the possession of the team at the end of the game, and will be added to the team's winnings."

This class of games consisted of two games: "All-pay 3" with W=X=Y=90 and "All-pay 4" with W=X=Y=90 and Z=80. In both games, the endowment was M=60. In contrast to the limited budgets in the previous class of games, unused endowments contributed to the participants' payoff.

Each session consisted of only two classes of games, with the two parameterizations of each particular class played sequentially. In order to avoid confusion, we never included the first-price and the all-pay multi-object auctions in the same session, because they share a very similar language and important details might be overlooked. The Blotto game, in contrast, features very different language and was played in all sessions. The choice of classes of games and their order was altered between sessions. Table 2 describes the four types of sessions run in the experiment and the number of subjects in each type.[4] We had 10 sessions of 18 participants and 6 sessions of 12 participants.

[4] In three cases, research assistants filled in for subjects in order to complete a session. Subjects were not aware of that. We instructed the RAs to suggest the simplest strategy of an equal split and to send an empty message. Note that the suggested decisions in all four games were taken individually before any suggestion of the team partner was seen. Thus, the RA's basic strategy suggestion could only affect the final decisions of their teammates. The three RA's data are excluded from the analysis.

n=249	**First Block (games 1+2)**	**Second Block (games 3+4)**
54	Blotto 6, Blotto 7	Auction 3, Auction 4
69	Blotto 6, Blotto 7	All-pay 3, All-pay 4
54	All-pay 3, All-pay 4	Blotto 6, Blotto 7
72	Auction 3, Auction 4	Blotto 6, Blotto 7

Table 2: The number of subjects in each of the four types of experimental sessions.

Note that each subject played two classes of games that, by design, differ in various aspects. In the Blotto game, the story is described in terms of a strategic war game, resources are troops that are allocated to 6 or 7 fronts, and the score is determined only by the number of battles won (i.e., conditional on winning the front, the assignment size has no effect on the score). In the multi-object auctions with budget constraints, the resource is money, there are 3 or 4 possible objects to bid on (i.e. 3 or 4 auctions), and the size of the bid in case of winning is relevant to the payoff, because the player obtains the value of the object minus the bid. In the all-pay multi-object auctions with limited endowment, the resource is money and there are 3 or 4 objects, as in the previous game, but unused resources contribute to the player's payoff. Another notable difference is that the Blotto game is defined as a round-robin tournament[5] whereas both variations of the multi-object auctions are 3-player games. Thus, the differences between the three classes of games are in the framing of the story, in the number of "fronts", in the budget, and in the score function, although all of them feature a winner-takes-all success function. This design allows us to examine more cleanly, i.e., without triggering common reasoning by using similar stories or parameters, whether there are similarities between these games in the reasoning process. We find it interesting to explore the similarity in reasoning between the games using both between-subject and within-subject analysis (see Section 6).

Comment on the Nash equilibria of the games

The six games in the present study do not feature pure strategy equilibria. Their rich structure induces mixed-strategy equilibria that are often difficult to calculate, even for game theorists.[6]

[5] One of the reasons for our choice of a tournament structure is to make the Blotto game comparable to the game studied in Arad and Rubinstein (2012a).

[6] In the equilibrium of a two-player zero-sum Blotto game, the marginal distribution over each front is symmetric around the number of troops divided by the number of fronts (Roberson, 2006; Hart, 2008). However, we are not aware of a characterization of equilibrium for the case of a round-robin tournament of the Blotto game, which we study here. Kvasov (2007) and Roberson and Kvasov (2012) characterize equilibrium in a two-player multi-object

Furthermore, we examine reasoning and behavior in *one-shot* resource allocation games rather than in repeated play of these games, which leaves no room for the lengthy equilibration process that might occur in repeated interactions (Camerer et al., 2003; Holt and Roth, 2004). Therefore, in the analysis of the data we focus on subjects' inferred reasoning without referring to equilibrium behavior.

2.3 Classification of communication transcripts

In our main analysis, we use a manual classification method in order to classify the written messages along the lines of multi-dimensional reasoning. We refer to this method as the *guided classification*. As a complementary analysis, we use a *free-form classification* in which a research assistant categorizes the decision rules in the messages without any guidance. In addition, we use a machine-learning technique to classify the messages and refer to the method as the *computer classification*. We describe the details of the latter two methods in Online Appendix B1 and B2 and summarize their results in Section 7.

In the guided classification, for each game, two research assistants independently read the messages and classified them into two categories: multi-dimensional reasoning (i.e., thinking about dimensions of strategies) and strategy-wise or other reasoning (e.g., responding to a distribution of strategies chosen by the other players or choosing a strategy randomly).

If a message was classified as reflecting multi-dimensional reasoning, the research assistant further classified the message according to two additional criteria: the *strategy dimensions* (i.e. features) mentioned in the text and the *dimensional decision rule* used within each of the dimensions. A list of possible dimensions and decision rules was composed by the authors after reading the messages in a pilot study. The list was given to the research assistants, but they were allowed to add new categories in both the dimensions and the dimensional decision rules. After deciding on their classifications independently and submitting them, the research assistants met to reconcile any disagreements between them and provided a joint classification for the cases on which they had disagreed. They were allowed to keep their two classifications in case of unresolved disagreement. Initially, the RAs agreed on 84% of the dimension classifications and on 61% of the more demanding decision rule classifications. While there were no remaining disagreements on the dimensions after the reconciliation, 35

all-pay auction. However, we do not know the equilibrium of a three-player game, which is the version played in the experiment. Finally, to the best of our knowledge, the equilibrium of the first-price multi-object auction with budget constraints studied here has not been characterized, not even for a two-player game.

out of 1655 decision rule classifications remained without agreement. Here, we only report on the agreed classifications. The full classification instructions appear in Online Appendix D.

In contrast to the guided classification, in the free-form classification the research assistant was not aware of previous publications, was not introduced to the concept of multi-dimensional reasoning and was not provided with a list of categories, but rather was asked to identify autonomously the main categories of decision rules that subjects use in their messages and to classify them accordingly. It turns out that the RA's categories are analogous to the categories of dimensions and that similar considerations appear in the two classifications with similar proportions. Thus, the free-form classification provides further support for the insights obtained by the guided classification.

The robustness and replicability of guided classifications of the type described above are examined with a large set of human classifiers recruited on MTurk in Eich and Penczynski (2017). Furthermore, the results of such classifications are shown to be replicable with machine-learning techniques (Penczynski, 2019). In our experiment as well, the results of the computer classification of the messages indicate similar patterns to those found in the guided classification.[7]

Dimensions

Table 3 presents the exhaustive list of dimensions for all the six games in this study. Note that we use the general term "front" both in the Blotto game and when we refer to an individual auction in the multi-object auctions. Furthermore, a "reinforced" or "disregarded" front relates to a front with a high or a low assignment, respectively, relative to a uniform allocation of the resources. An illustration of each dimension and classification examples will be given separately for each class of games in Sections 3–5. Note that the dimensions of reasoning are distinct from the vector components of the games' strategies. We do not observe these components to be discussed separately in the participants' discussions.

The differences between and within classes of games induce slight differences between dimensions and in their salience. Three of them are worth mentioning.

[7] In a procedure called cross-validation, the algorithm uses a large portion of the manually classified messages as a training set and classifies the rest of the messages with the trained model. Repeating this exercise using different portions of the classified messages as training sets, the computer eventually classifies all messages on the basis of a model that was trained on other messages. For each message and each dimension, the algorithm provides the probability that the dimension is mentioned in the text. If the probability is higher than 0.5, then the final binary classification is "positive" for that message, i.e. the dimension is identified in the message.

	Dimension
D1	Number of reinforced fronts
D1A	Asymmetric assignments to reinforced fronts
D2L	Type of assignment to disregarded fronts
D2H	Type of assignment to reinforced fronts
D3	Considerations of the identity of fronts (assignment order)
D4	How much of the budget to use

Table 3: The dimensions considered by subjects in all six games in the experiment.

First, note that dimension D4 is relevant only in the all-pay auctions. Not using all resources is theoretically unreasonable in the Blotto games because no advantage is derived from withholding troops. Such choices are moreover very rare in first-price auctions, probably because a player does not benefit from an unused budget. The avoidance of a wastefully high bid when a much lower bid would have sufficed is only a secondary consideration.

Second, referring to dimension D3 is almost a necessity in the multi-object auctions in which the items' values are not the same, i.e., Auction 4 and All-pay 4, because a player needs to specify how the auction with a unique item value is treated. In the rest of the games, discussing the identity of the fronts to focus on is a response to the order of the fronts and not to a payoff-relevant aspect.

Third, in the Blotto games, messages discussing the type of assignment to reinforced fronts included mainly arguments about the unit digit and hence D2H was described in the instructions for the classifiers as "the specific assignment to reinforced fronts: the unit digit in such fronts". By contrast, in the first-price auctions the arguments focused on the rough magnitude of the bid in a particular auction. Accordingly, D2H was described for the classifiers as "values in reinforced (high-bid) auctions: considerations of the specific bid or an approximate value of the bid." This difference could be a result of both the different score functions (and particularly the high values of fronts in the auctions compared to the Blotto game) and the difference in the number of fronts in the games.

We intentionally chose to study games that differ in these aspects, in order to examine whether multi-dimensional reasoning is a common decision procedure in a wide class of substantially different games. These differences do not reduce the applicability of multi-dimensional reasoning as research may identify the games' properties that induce such differences and quantify them.

Dimensional decision rules

Following the analysis of Arad and Rubinstein (2012a), the concept of level-k decision rules for the full strategy space provides a set of candidate decision rules that we apply within the dimensional space. The classification exercise gave rise to a further common decision rule, which we summarized as a "reasonable argument without a specific belief" and labelled R. Table 4 reports the type of decision rules that were detected in the written messages.

	Decision rule
L0	Intuitive or random
L1	Response to a specified (categorical) belief
L2	Response to a belief that others are L1 (or L1 & L0)
R	Reasonable argument without a specified belief (e.g., an attempt to guarantee a particular minimal score)
N	No explanation

Table 4: Common decision rules (within dimensions) in classified messages.

In order to see the general classification of decision rules at work, consider the following message written by subject #7 suggesting the strategy (0,38,41,41,0,0) in Blotto 6: "*In my opinion, we should focus on three fronts so that we will have a chance to win half of the fronts. The 41 is because usually people assign round numbers, such that if someone goes for the same method, he will probably assign 40. This way we will win in at least two fronts and in one we will have a high chance*" (direct translation from Hebrew).

When subject #7 considers the unit digit in the reinforced fronts (D2H), the subject responds to the belief that others are intuitively allocating troops in multiples of ten in this dimension. Accordingly, the dimensional decision rule of the subject in D2H is classified as L1, i.e., a response to a belief that the opponent is not strategic in this dimension but is attracted to salient choices (Crawford and Iriberri, 2007a, 2007b; Arad and Rubinstein, 2012b).

A useful example for the decision rule R is subject #7's consideration of the number of reinforcements, D1. Here, the subject chooses to reinforce three fronts assuming that this would guarantee a minimal score, which is a belief-independent but reasonable argument. A possible underlying belief that others reinforce four or five fronts or allocate troops equally among the fronts is not mentioned in the message. Similarly, the following message of subject #4 contains a reasonable argument, which is not based on a concrete belief of others' choices within D1: "*No point in allocating the troops equally to six fronts because we will lose for sure. I prefer betting on two strong fronts and hope that others won't reinforce these particular fronts.*" Thus, the R classification includes all decision rules that do not reflect a response to a particular categorical belief but do reflect some strategic reasoning and hence are not purely intuitive. Of course, due to the absence of an *articulated* belief, R can alternatively be thought of as a reasonable argument on the basis of an implicit, ambiguous belief that is not articulated.[8]

Once the decision rules have identified desirable dimensional values and possibly a set of candidate strategies, the player will need to single out one strategy to implement. We think that in the Blotto games and the Auctions the budget constraint becomes relevant at this point, connects the separate dimensional considerations and is used to find one feasible strategy with the desired characteristics. In the All-pay auctions, either the budget constraint or a chosen smaller budget (D4) might play this role.

In each of the three Results sections 3–5, we start with an analysis of the messages and then turn to explore the relationship between the performance of a suggested strategy and the multi-dimensional reasoning expressed in its accompanying message. We report the characteristics of the suggested and final strategies in the Online Appendix.

3. Results: Blotto games

The total number of participants in the two Blotto games was 249. The number of classified messages was 211 in Blotto 6 and 212 in Blotto 7. The remaining participants provided either blank messages or messages that lacked information on the strategy choice, such as "*Trust me, I know what I am doing.*"

3.1 Dimensions and decision rules in Blotto games

Consider again the message written by subject #7 suggesting the strategy (0,38,41,41,0,0) in Blotto 6: "*In my opinion, we should focus on three fronts so that we will have a chance to win half of the fronts. The 41 is because usually people assign round numbers, such that if someone goes for the same method, he will probably assign 40. This way we will win in at least two fronts and in one we will have a high chance*" (direct translation from Hebrew). It suggests that the subject had in mind two dimensions of a strategy: the number of reinforced fronts and the unit digit in reinforced fronts. His message was classified as reflecting multi-dimensional reasoning and, in particular, D1 and D2H.

[8] Note that while the classification of the decision rule in D1 (number of reinforced fronts) is R, the decision rule within D3 (which fronts to focus on) was coded as level-0, as the subject referred to this choice as a random bet.

By contrast, consider now the following message, written by subject #181 suggesting the strategy (19,21,18,22,17,23): "*Just a technique ... because everyone will either do 20 20 20 20… or this technique ... so I think it's good - 19 21 18 22 17 23.*" Such a message is classified as "strategy-wise or other reasoning" rather than multi-dimensional reasoning.

For all dimensions in the Blotto games, Table 5 provides an illustrating strategy and summarizes the proportion of messages that include each corresponding dimension. Figure 1 depicts the distribution of the number of dimensions per message. Messages with zero dimensions correspond to the category of "strategy-wise and other reasoning" that is not multi-dimensional.

	Dimension	**Illustration in Blotto 6**	**% Blotto 6 (n=211)**	**% Blotto 7 (n=212)**
D1	*Number of reinforced (or disregarded) fronts*	(30,30,30,30,0,0): four reinforcements	86%	87%
D1A	*Asymmetric assignments to reinforced (or disregarded) fronts*	(35,35,25,25,0,0): four asymmetric reinforcements	7%	8%
D2L	*Unit digit in disregarded fronts*	(30,30,30,28,1,1): two disregarded fronts with unit digit 1	22%	26%
D2H	*Unit digit in reinforced fronts*	(31,31,31,27,0,0): three reinforced fronts with unit digit 1	21%	18%
D3	*Identity/location of reinforced fronts (order)*	(0,0,30,30,30,30): four reinforced fronts at the right	39%	38%

Table 5: Frequency of dimensions in classified messages in the Blotto games.

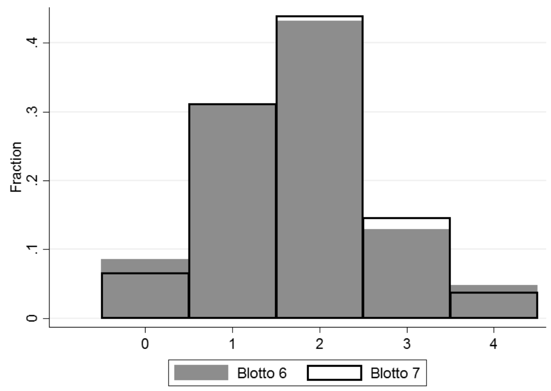

Figure 1: Number of dimensions per message in classified messages in the Blotto games (n=211, n=212).

The main findings from Table 5 are as follows. D1 is very common, mentioned by the vast majority of subjects, D1A is very rarely mentioned, and D2L, D2H, and D3 are somewhere in between, although D3 is more commonly mentioned. Although the proportion of D2L and D2H is similar, it turns out that a considerable group of subjects mention D2L but do not consider D2H and vice versa. In fact, only 5% of the subjects mention both dimensions.

Figure 1 shows that about 60% of the subjects mention at least two dimensions in their message, which provides further support for multi-dimensional reasoning in the Blotto game. While less than 10% of the messages reflect strategy-wise or other reasoning, considering empty messages as the result of such reasoning as well establishes an upper bound of reasoning that differs from multi-dimensional reasoning. Taking this conservative approach, we find that 22% and 20% of the subjects in the Blotto 6 and Blotto 7, respectively, do not mention dimensions explicitly. In other words, at least 78%-80% of the subjects think in terms of dimensions.

Table 6 illustrates dimensional decision rules in the Blotto games and provides some examples of arguments written by subjects and their respective classification. The complete distribution of decision rules in the different dimensions appear in Table S1 in Online Appendix A. The main findings are that L1 and R are the most frequently used dimensional decision rules. In D2L, D2H, and D3, L1 is the dominant decision rule. However, in D1, R is particularly prominent. Interestingly, a significant portion of decisions in D1 are based on an objective to win a particular number of fronts for which oftentimes a motivation is given.

	Argument consistent with the decision rule
L0	*Which fronts? Let's say 1,3,5, and 7 just because they look nice and symmetric.* (D3)
L1	*I assigned 1 to certain fronts because there is a chance that the other team assigned 0, so we can win them*. (D2L)
L2	*I think others will allocate equally to win more fronts, but I also think that others regard us as likely to do the same.* (D1)
R	- *It is better to win in half of the fronts with certainty than to gamble on the outcome in all*. (D1) - *Assigning a few troops to a front is equivalent to giving up the front entirely.* (D2L)

Table 6: Examples of arguments consistent with the various decision rules in the Blotto games.

3.2 Multi-dimensional reasoning and performance in Blotto games

We calculated the expected score of the suggested strategies when playing against the suggested strategies of all subjects in the experiment. This measure eliminates the random aspect of the score in any session with few opponents.[9] Our focus on the suggested strategies score reflects our interest in the individual reasoning. It allows us to explore how thinking about dimensions – at the stage of the suggested strategies - is related to the performance of the corresponding (suggested) strategy. Table 7 presents the top three scores for each Blotto game.

Rank	**Blotto 6**	**Blotto 7**
1	(2,3,41,41,31,2) [3.61]	(1,1,54,55,53,45,1) [4.07]
2	(3,31,31,31,21,3) [3.56]	(4,1,1,51,51,51,51) [4.06]
3	(2,38,38,2,38,2) [3.47]	(2,51,2,51,51,51,2) [4.05]
	& (1,1,31,31,31,24) [3.47]	

Table 7: Winning suggested strategies and their expected score [in brackets] in the Blotto games. For comparison, the scores for a uniform allocation were 2.67 and 2.96, respectively.

The results show that the best performing strategies reinforce three or four fronts, use the unit digits 1, 2, and 3, and assign a small number of troops to the first front, and often to the last one as well, thus suggesting the use of dimensions D1, D2L, D2H, and D3. These properties are similar to those of the best performing strategies in Arad and Rubinstein's (2012a) tournament with thousands of participants. It is natural to inquire whether considering a larger number of dimensions is associated with the strategy's performance.

Table 8 provides some indications of the relationship between dimensional thinking in classified messages and the suggested strategy's expected score. As the table suggests, we found strong support for the positive correlation between the number of considered dimensions and the score, using a Mann–Whitney test. Note that a high score might be the outcome of the consideration of more dimensions but could also be driven by other factors, such as the player's sophistication. Strategically sophisticated players are likely to choose successful strategies and tend to describe more considerations in their messages. Such sophistication may be reflected, for example, in the dimensional decision rules used by the player.

Of course, mentioning more dimensions is likely to be correlated with mentioning more important dimensions. Thus, for subjects who mentioned at least one dimension, we studied the effect of particular dimensions on the score. In a linear regression explaining the score in Blotto 6, the coefficients for the dummy variable for the dimensions D2L, D2H, and D3 were positive and significant. In Blotto 7, D2L was positive and significant and D1 was found to be

[9] Although large and small tournaments induce different incentives and possibly lead to distinct best-performing strategies (Arad and Rubinstein, 2013), the strategies in Table 7 are also very successful in the smaller session-level tournaments.

negative and significant (see Table S6 in Online Appendix A). Note that merely considering D1 was not necessarily beneficial since almost all subjects considered this dimension. Therefore, in D1, the applied decision rule turns out to be very important for the score. To summarize, while different dimensions may have made different contributions to the score, overall, the use of more dimensions was correlated with a higher score.

# of dimensions	n	Average	Significance	n	Average	Significance
		Blotto 6			**Blotto 7**	
1	65	2.49 (0.65)	(+,**, ***)	66	3.12 (0.53)	(N,**,*)
2	91	2.71 (0.51)	(+, ***)	93	3.23 (0.50)	(*, N)
3	27	2.88 (0.40)	(**)	31	3.33 (0.76)	(N)
4	10	3.28 (0.23)		8	3.41 (0.70)	

Table 8: Average score (standard deviation in parentheses) for each number of dimensions in the Blotto games. For each number of dimensions, the column *Significance* indicates significant score differences when compared to the higher numbers of dimensions in the table consecutively. + indicates p<0.1, * indicates p<0.05, ** indicates p<0.01, *** indicates p<0.001, N indicates p>0.1.

4. Results: First-price multi-object auctions

The total number of participants in the first-price multi-object auctions was 126. The number of classified messages was 101 in Auction 3 and 105 in Auction 4.

4.1 Dimensions and decision rules in first-price multi-object auctions

In order to see multi-dimensional reasoning at work in multi-object auctions, consider the message written by subject #21 suggesting the strategy (0,0,42,78) for Auction 4, in which item D is more valuable than the other items: "*I think we should bid only in two games. Specifically, we should participate in Auction D – because everybody thinks there is no point in participating in it because everybody else will, and hence in practice it will have low bids or no bids at all*" (direct translation). This message was classified as reflecting two dimensions: the dimension of how many auctions to focus on (D1) and the dimension of the identity of those auctions (D3). While the subject did not explain her dimensional decision rule in D1, the message suggests that she practiced two steps of reasoning (L2) in D3. The starting point of her iterative reasoning process was the intuitive rule *to participate in Auction D* because it entails a higher prize and hence the first step of her reasoning was to avoid Auction D and the second step was to participate in the auction after all.

In contrast to subject #21's message, a message suggesting to try winning against a particular strategy assumed to be chosen by others or a distribution of strategies would not be classified as multi-dimensional reasoning. Subject #155's message in Auction 4 illustrates such reasoning: "*If people assign the points uniformly, I think that the best chance of winning is as follows: Auction D - 40, Auction C - 15, Auction B - 33, Auction A - 32.*"

For all dimensions in the first-price multi-object auctions, Table 9 provides an illustrating strategy and summarizes the proportion of messages that included each corresponding dimension. Figure 2 presents the distribution of the number of dimensions per message. Messages with zero dimensions correspond to the category of "strategy-wise or other reasoning" that is not multi-dimensional.

	Dimension	**Illustration in Auction 3**	**Auction 3 (n=101)**	**Auction 4 (n=105)**
D1	*Number of auctions with high bids (or disregarded)*	(60,60,0): two high bids	62%	78%
D1A	*Asymmetric assignments to auctions with high bids*	(70,50,0): two asymmetric high bids	13%	10%
D2L	*Type of assignment to disregarded auctions*	(55,55,10): one disregarded front in the 10s	21%	20%
D2H	*Type of assignment to auctions with high bids*	(53,52,15): two non-round high bids in the 50s	63%	43%
D3	*Considerations of the identity of auctions*	(0,60,60): two high bids in B and C	51%	88%
D4	*How much of the budget to use for bids*	(30,30,0): Using 60 of a budget of 120	5%	4%

Table 9: Frequency of dimensions in classified messages in the auctions.

The main findings from Table 9 are as follows. D1, D2H, and D3 are used by the majority of subjects, where D1 and D3 are naturally more common in Auction 4 due to the asymmetry in the auctions' prizes.[10] The proportions of D3 and D2H are higher than in the

[10] This is in line with Chowdhury et al.'s (2021) results that the effect of value salience (asymmetry in values) on deviations from Nash equilibrium in a multi-battle contest is higher than that of label salience.

Blotto games, while dimension D2L is mentioned by about 20% of the subjects as in the Blotto games. D1A and D4 are rarely mentioned in the auctions.

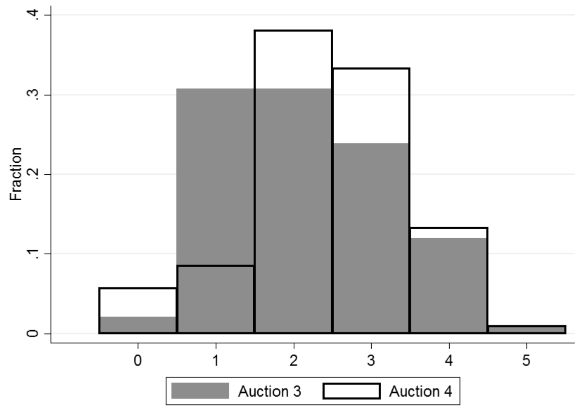

Figure 2: Number of dimensions per message in classified messages in the auctions (n=101, n=105).

Figure 2 shows that 67%–85% of the subjects mention at least two dimensions in their messages and 37%–47% mention at least three dimensions, which provides support for multi-dimensional reasoning in the multi-object auctions. While only a small number of messages reflect strategy-wise or other reasoning, considering empty messages as the result of such reasoning as well, we find that 21% and 20% of the subjects in the Auction 3 and Auction 4, respectively, do not mention dimensions explicitly. In other words, at least 79%-80% of the subjects think in terms of dimensions.

Table 10 illustrates dimensional decision rules in the first-price auctions and provides some examples for arguments written by subjects and their respective classification. The complete distribution of decision rules in the different dimensions appears in Table S2 in Online Appendix A.

	Argument consistent with the decision rule
L0	*I just chose two particular auctions randomly*. (D1)
L1	*I think others will focus on the higher-value auction and hence I focused on the other auctions.* (D3)
L2	*The rest of the people will not participate in Auction D, because they think everybody else will, so I am assigning an average amount there.* (D3)
R	- *No point in assigning too many points on an auction because then we will get a low payoff.* (D2H) - *I think we should assign a lot to one auction so that we will win it, at least.* (D1)

Table 10: Examples of arguments consistent with the various decision rules in the first–price auctions.

The most frequently observed decision rules were L1 and R. In contrast to the findings in the Blotto games, in the auctions L1 was the most frequent rule in D1, not R. However, in dimensions D2L and D2H it was only the second most frequent rule, after the collection of rules classified as R, whereas in the Blotto games L1 was the most frequent rule in these dimensions.

Within the R category, a prominent decision rule observed in D2H, which did not appear in the Blotto games, refers to a choice of bid magnitude that maximizes the likelihood of winning conditional on gaining at least a sum X in the case of winning. Subject #93 suggests the strategy (10,10,40,60) and writes in Auction 4: *"I believe that we should bid a lot of money in two auctions, for example in Auction D, so that we will guarantee winning it. In fact, bidding 60 in D leaves us with 50 points or 5 shekels each, which is quite awesome"* (direct translation). Like subject #93 in this example, a considerable number of other subjects described goals that differ from maximizing profit, such as aiming to win a particular number of auctions or achieving a minimal desired profit.[11] Thus, despite the similarities in the considered dimensions, the reasoning within dimensions differs between the auctions and the Blotto games.

4.2 Multi-dimensional reasoning and performance in first-price multi-object auctions

Table 11 presents the top three suggested strategies in Auctions 3 and 4, where the expected score is calculated by matching each player with any pair of competitors from the whole experiment, thereby eliminating the random aspect of the session score among the small set of players in a session.

Rank	**Auction 3**	**Auction 4**
1	(59,0,61) [58.72]	(45,45,23,7) [88.83]
2	(60,0,60) [58.49]	(41,41,37,1) [88.59]
3	(61,0,59) [58.26]	(50,50,10,10) [88.04]

Table 11: Winning suggested strategies and their expected scores [in brackets] in the auctions. For comparison, the scores for a uniform allocation were 39.28 and 63.75, respectively.

[11] Branas-Garza et al. (2011) report various motivations in a traveler's dilemma, including the "aspiration" for a given amount of money, which resembles the goal of achieving a minimal desired profit described here.

In both games, neglecting one auction proves to be beneficial. In Auction 3, neglecting Auction B turns out to be successful, while in Auction 4, "almost neglecting" the auction with the higher prize, Auction D, is the key. Therefore, it is not enough to understand that one needs to decide on which auctions to focus. The optimal choice of focus is sensitive to the details of the game. The fact that some winning strategies use bids that are multiples of 10 suggests that the unit digit factor is less important in the auction games than in the Blotto games.

We now turn to examine whether considering more dimensions is associated with a higher performance of the suggested strategies. Table 12 presents the average score of suggested strategies as a function of the number of dimensions mentioned in the accompanying message.

# of dimensions	**n**	**Average score**	**n**	**Average score**
	Auction 3		**Auction 4**	
1	30	42.03 (9.55)	8	66.27 (11.43)
2	30	44.51 (9.25)	40	62.6 (17.98)
3	24	41.98 (11.19)	35	63.43 (10.4)
4	12	40.40 (12.98)	14	62.28 (19.47)
5	1	39.94	1	70.48

Table 12: Average score (standard deviation in parentheses) for each number of dimensions in the auctions.

As the table suggests, we found no correlation between the number of considered dimensions and performance (Mann–Whitney ranksum tests). Using a linear regression to study the effect of particular dimensions on the scores, we find that none of the dimensions significantly affect the scores; see Table S6 in Online Appendix A. It may well be that in the auctions the particular choices within dimensions are the crucial element in determining the scores. For example, considering D3 is beneficial only if the player eventually neglects Auction B in Auction 3 and almost neglects Auction D in Auction 4.

5. Results: All-pay multi-object auctions

The total number of participants in the multi-object all-pay auctions was 123. The number of classified messages was 108 in All-pay 3 and 109 in All-pay 4.

5.1 Dimensions and decision rules in all-pay multi-object auctions

In order to see the use of multi-dimensional reasoning in the all-pay multi-object auctions, consider the message written by subject #55, who suggested the strategy (29,0,0,31) in All-pay 4: "*I think we should split the points between two auctions. We should bet half the points specifically on auction D, in which the rest of the teams are likely to invest less since its prize is lower. If we split 30-30 between two auctions, we have a good chance of winning. In addition, I suggest not to assign round numbers but rather 31/41 because usually people tend to use round numbers and one point of difference could make us win the auctions"* (direct translation). This message is classified as reflecting the three dimensions of how many auctions to focus on (D1), the identity of those auctions (D3), and the type of assignment to auctions with high-bids (D2H). The dimensional decision rule within D1 is classified as N and the decision rules within D2H and D3 are classified as L1.

The type of reasoning expressed by subject #55 resembles that observed in the messages in the Blotto games and the first-price auctions. By contrast, the message of subject #59 in All-pay 4 suggesting (1,1,1,1) illustrates a slightly different type of thinking, which is unique to the all-pay auctions: "*In order to win an auction, one should assign it all points, but it is possible that another team will assign its points to the same auction and then the winner will be selected randomly. This is an undesired risk for a potential gain of 20-30 points. Therefore, we better keep the points that we have received. It is worth assigning one point to each auction for the case that in one of the auctions the other teams did not assign any point. Such a situation is very likely and it is a shame to miss a chance to win an auction cheaply.*" This message was classified as reflecting D2L, the type of assignment to disregarded fronts (decision rule L1), and, notably, dimension D4, how much of the endowment to use for bids (decision rule R).

For all dimensions in the all-pay multi-object auctions, Table 13 provides an illustrating strategy and summarizes the proportion of messages that included each corresponding dimension. Figure 3 presents the distribution of the number of dimensions per message. Messages with zero dimensions correspond to the category of "strategy-wise or other reasoning" that is not multi-dimensional.

The main findings of Table 13 are as follows. D1 is used by the majority of subjects. Due to the asymmetry in the auctions' prizes, D3 is the most frequently mentioned dimension in All-pay 4. It is the second most popular in All-pay 3. D4 is mentioned by slightly more than 25% of the subjects, and D1A is very rarely mentioned as in the other games.

Figure 3 shows that 57%–67% of the subjects mention at least two dimensions in their messages and 22%–33% mention at least three dimensions, providing support for multi-dimensional reasoning in the multi-object auctions. While only about 10% of messages reflect strategy-wise or other reasoning, considering empty messages as the result of such reasoning as well, we find that in both games 22% of the subjects do not mention dimensions explicitly. In other words, at least 78% of the subjects think in terms of dimensions.

	Dimension	**Illustration in All-pay 3**	**All-pay 3 (n=108)**	**All-pay 4 (n=109)**
D1	*Number of auctions with high bids (or "disregarded")*	(30,30,0): two high bids	64%	56%
D1A	*Asymmetric assignments to auctions with high bids*	(40,20,0): two asymmetric high bids	2%	1%
D2L	*Type of assignment to "disregarded" auctions*	(25,25,10): one disregarded front in the 10s	23%	34%
D2H	*Type of assignment to auctions with high bids*	(23,22,10): two non-round high bids in the 20s	19%	16%
D3	*Considerations of the identity of auctions*	(0,30,30): two high bids in B and C	38%	63%
D4	*How much of the endowment to use for bids*	(20,20,0): Using 40 of an endowment of 60	28%	26%

Table 13: Frequency of dimensions in classified messages in the all-pay auctions.

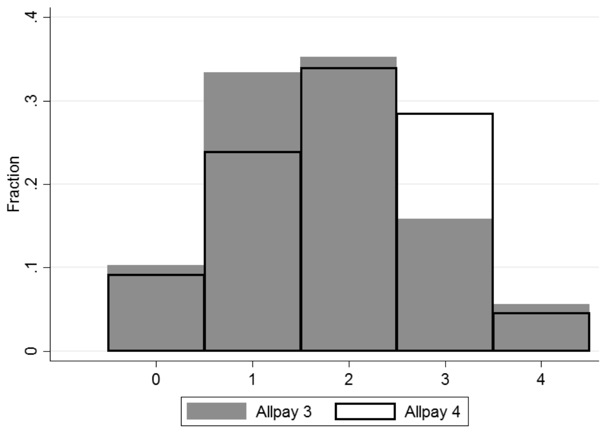

Figure 3: Number of dimensions per message in classified messages in the all-pay auctions (n=108 and n=109).

The dimensional decision rules in the all-pay auctions are similar to those in the Blotto games and the first-price auctions (see Table 6 and 10). The complete distribution of decision rules in the different dimensions appear in Table S3 in Online Appendix A. Overall, the most frequent decision rule in the all-pay auctions is L1, with a similar pattern of usage to that found in the Blotto games, while R is the most prominent decision rule in D1 and D4. We note that, as in the first-price auctions, a considerable group of subjects described goals that differ from maximizing profit.

5.2 Multi-dimensional reasoning and performance in all-pay multi-object auctions

Table 14 presents the top three suggested strategies in All-pay 3 and 4, where the expected score is calculated by matching each player with any pair of competitors from the experiment, thereby eliminating the random aspect of the session score among the small set of players in a session.

Rank	**All-pay 3**	**All-pay 4**
1	(10,5,45) [121.8]	(19,18,17,6) [162.7]
2	(6,48,6) [120.3]	(21,31,7,1) [162.4]
3	(6,20,34) [120.2]	(30,4,4,22) [160.7]

Table 14: Winning suggested strategies and their expected scores [in brackets] in the all-pay auctions. For comparison, the scores for a uniform allocation were 106.0 and 148.4, respectively.

Recall that in these games, any unused endowment is counted as part of the score and a considerable group of subjects indeed kept a portion of their endowment of 60 points. It is noticeable that in both games, the winning strategies used all 60 points in the bids. In All-pay 3 they focused on one auction (B or C) and made small bids in the others. In All-pay 4, they focused on two or three auctions. Some of the winning strategies used bids that were multiples of ten in one of the auctions, but most of their bids were not multiples of ten.

We now turn to examine whether considering more dimensions is correlated with the performance of the suggested strategies. Table 15 presents the average score of suggested strategies as a function of number of dimensions mentioned in the accompanying message.

As the table suggests, the number of considered dimensions generally correlates with performance. We found partial support for that in All-pay 3 and strong support in All-pay 4, using a Mann-Whitney test. For subjects who mentioned at least one dimension, we studied the effect of particular dimensions on the scores. In a linear regression explaining the score in All-pay 3, the coefficients for the dummy variables for the dimensions D1A, D2L, and D3 were positive and significant. In All-pay 4, the coefficient for D2L, D2H, and D3 were significant and positive. In both games, the use of D4 was significant and negative, suggesting that saving a portion of the endowment is not the right tool, given the experimental behavior of the other players; see Table S6 in Online Appendix A. Although consideration of particular aspects of a strategy could relate with a lower strategy score, overall, we found that the use of more dimensions correlates positively with performance, as in the Blotto games.

# *of dimensions*	**n**	**Average score**	**Significance**	**n**	**Average**	**Significance**
		All-pay 3			**All-pay 4**	
1	36	94.93 (15.70)	(N,N,*)	26	108.43	(**, ***,**)
2	38	100.28 (10.17)	(N,$^{+}$)	37	128.91	(N, *)
3	17	99.97 (11.90)	($^{+}$)	31	133.05	($^{+}$)
4	6	108.92 (11.50)		5	152.1	

Table 15: Average score (standard deviation in parentheses) as a function of the number of dimensions in the all-pay auctions. For each number of dimensions, the column *Significance* indicates significant score differences when compared to the higher numbers of dimensions in the table consecutively. $^{+}$ indicates p<0.1, * indicates p<0.05, ** indicates p<0.01, *** indicates p<0.001, N indicates p>0.1.

6. Linking the reasoning process in the six games

In light of the results on the three games, we can come back to our hypothesis regarding multi-dimensional reasoning and state the following result.

Result: *In all games studied here, we find evidence of multi-dimensional reasoning in the messages. Players think in terms of characteristics of strategies and often deliberate over multiple dimensions at the same time.*

Having established this, in this section we investigate the extent to which the dimensions considered in the three classes of games resemble each other. We use the message classification to explore the connection between the reasoning in the different games in two analyses. First, we provide a between-subject analysis of the reasoning in the various games on an aggregate level. This analysis addresses the fundamental question of whether different resource allocation games trigger similar multi-dimensional reasoning. Second, we measure the within-subject correlation of the reasoning between pairs of games that a subject played. As in the previous sections, the analysis is based only on the classified messages.

6.1 The connection between the games on the aggregate level

In order to examine whether the three different classes of games trigger similar reasoning, we consider only the reasoning in the first game played by the subject. This eliminates any possible influence that the reasoning in one game might have on the reasoning in the next. Thus, we focus here on the reasoning in the three games: Blotto 6, Auction 3, and All-pay 3. Note that in each of these games, the different fronts have equal values. Figure 4 presents the distribution of the number of dimensions per message for the three games and Table 16 reports the proportion of subjects' messages reflecting the corresponding dimensions.

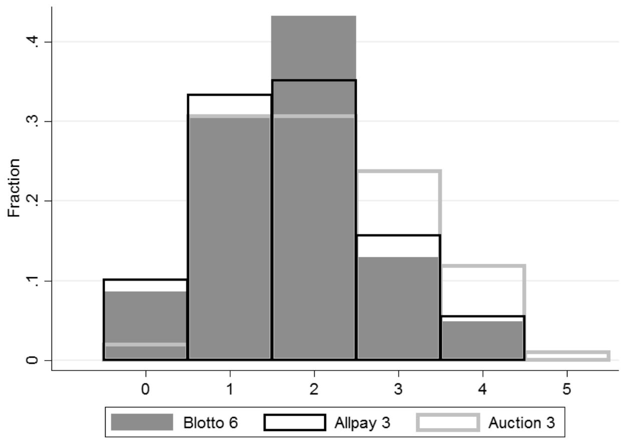

Figure 4: Number of dimensions in classified messages in the first game played.

Figure 4 shows that the distributions of the number of considered dimensions differ between the games, although the proportions are similar. The median number of considered dimensions is 2 in all three games and the average is 1.84 in Blotto 6, 2.31 in Auction 3, and 1.87 in All-pay 3. A ranksum Mann–Whitney test indicates that the numbers of dimensions in Blotto 6 and All-pay 3 are not significantly different ($p=0.87$), whereas the numbers of dimensions in Auction 3 are significantly larger than their counterparts in both Blotto 6 ($p<0.01$) and All-pay 3 ($p<0.05$).

Table 16 reveals that the frequency of usage of the various dimensions follows a similar pattern in all three games: D1 is the most popular dimension, D3 is the second most popular, D2H and D2L follow, and D1A is rarely considered. Overall, the frequencies of those dimensions are similar in all three games. However, as indicated in the table, there are some statistical differences between the games in terms of the proportions of usage of particular dimensions. Two qualitative differences stand out. First, only in All-pay 3 does a considerable group of players consider D4, which is natural since it is the only game in which players can benefit from an unused budget. Second, in Auction 3 the consideration of D2H occurs with a frequency that is almost 40 percentage points higher than in the other games. This finding is naturally reflected in the higher average number of dimensions per message in the game. Thus, in the Blotto games and the all-pay auctions a decision to concentrate only on a number of fronts does not necessarily trigger thinking about the particular assignment, whereas in Auction 3, the type of assignment to an auction with a high bid is frequently discussed.[12]

	Blotto 6 (n=98)	**Auction 3 (n=58)**	**All-pay 3 (n=52)**	**Blotto 6 vs. Auction 3**	**Blotto 6 vs. All-pay 3**	**Auction 3 vs. All-pay 3**
D1	87%	67%	77%	*		
D1A	7%	12%	2%			
D2L	24%	22%	12%			
D2H	22%	60%	23%	***		***
D3	43%	66%	56%	*		
D4	-	3%	17%			*

Table 16: Frequency of dimensions in classified messages in the first game played. Bonferroni-adjusted statistical significance of a proportion test is indicated for the 3 pairs of games in each dimension. + indicates p<0.1/3, * indicates p<0.05/3, ** indicates p<0.01/3, *** indicates p<0.001/3.

To summarize, the aggregate data presented in the above tables indicate that in the first resource allocation game they play, people think of the same dimensions in the different games. In other words, these three games trigger a similar type of reasoning. Yet, each different game has some unique aspects and a slightly different focus due to its particular nature and specific parameters. In the next subsection, we inquire whether an individual player tends to reason in a similar manner in different allocation games.

[12] Recall that in the Blotto games, D2H included only arguments about the unit digit in the assignment to reinforced fronts whereas in the two types of auctions, D2H had a wider definition and included both arguments on the rough magnitude of the bid and arguments on the unit digit.

6.2 The connection between the games on the individual level

Does a participant who thinks about a large number of dimensions in one of the games tend to do so in the other two as well? In comparing the games – both within and across blocks – there is a positive correlation between the number of considered dimensions in each pair of games (0.19–0.58). Unsurprisingly, the within-block correlation tends to be stronger (see Table S4 in Online Appendix A).

We further consider whether players who think of a particular dimension in one game tend to think of that same dimension in one of the other games. Before presenting the results, it is worthwhile noting that correlations of specific actions between not very similar pairs of games have been found to be very small (Georganas, Healy, and Weber, 2015). Rubinstein (2016) reports an average correlation of 0.04. The average correlation he finds between a player's type ("contemplative index"), defined by his behavior in a series of independent games, and a "typical" (contemplative) action in a different game was 0.089.

We start by looking at the dimensions considered in a pair of games of the same class. Table 17 presents the within-block correlations in the various dimensions for the three classes of games.

Dimension	**Blotto (n=197)**	**Auction (n=93)**	**All-pay (n=98)**
D1	0.327***	0.161	0.256^{+}
D1A	0.187*	0.06	-0.015
D2L	0.531***	0.254^{+}	0.573***
D2H	0.543***	0.161	0.218
D3	0.512***	0.053	0.261^{+}
D4	NA	-0.051	0.334**

Table 17: Within-block correlation in the usage of particular dimensions. Significance levels are Bonferroni-adjusted by the number of dimensions per game δ, 5 or 6, respectively. $^{+}$ indicates p<0.1/δ, * indicates p<0.05/δ, ** indicates p<0.01/δ, *** indicates p<0.001/δ.

Overall, the use of a particular dimension is highly correlated within the Blotto games and within the all-pay auctions, whereas the two first-price auctions (except for the shared dimension D2L) seem to trigger the consideration of different dimensions. This is in line with the aggregate-level findings in Auction 4 that D3 and D1 are used more frequently than in Auction 3 and D2H less frequently. The average correlations across all dimensions found in Blotto, Auction and All-pay are 0.42, 0.11 and 0.27, respectively. These numbers are reasonably high given the correlation numbers reported in earlier studies, particularly due to the fact that we consider a set of directly inferred elements of reasoning (i.e. dimensions), rather than a freely chosen typology, and correlations between two single games rather than a more aggregate indicator.

We now explore the connection in terms of dimensions between the games across the two blocks. For some players, these are Blotto and All-pay games and for others Blotto and Auction games, in both possible orders. For the 191 subjects whose messages were classified, table 18 reports the average correlations between all dimensions for each pair of games. Over all game pairs, the average correlation is 0.111, which, again, is a reasonably high correlation given the above considerations. Most game pairs' average is close to this number, with the exception of the pair Blotto 6 and All-pay 3, which shows no correlation.

	Auction 3	***Auction 4***	***All-pay 3***	***All-pay 4***
Blotto 6	0.155	0.108	-0.003	0.118
Blotto 7	0.128	0.140	0.083	0.159

Table 18: Average correlations of the usage of dimensions for all game pairs between blocks.

Regarding individual dimensions, the dimensions D2L, D2H and D3 show the highest contribution to these averages, with correlations of 0.193, 0.147 and 0.119, respectively (see Table S5 in Online Appendix A). The use of D2L is most correlated between Blotto and All-pay (0.300) and the use of D2H is most correlated between Blotto and Auction (0.221). Blotto and Auction show correlations in D1 and D3 (0.143, 0.211). No correlation is found in the use of D1. Taking into account that D1A is rarely used and D1 is frequently used in all the games, the results suggest that indeed a player's reasoning is linked across games by the tendency to consider similar dimensions.[13]

The results presented in this subsection indicate that while different people may focus on different dimensions, an individual's reasoning tends to show similarities between two allocation games. These similarities are unlikely to be due to the sequential order of the game blocks. Between-subjects analyses show no systematic increase of mentioned dimensions when a game is located in the second block. The strength of these similarities exceeds the level of persistence shown for other studied reasoning traits, such as level-k thinking and

[13] Because subjects in our experiment were paid based on their performance in all four games, we cannot rule out the possibility that they used different types of strategies in the different games to diversify or hedge risks. We believe that this should not have affected which dimensions are considered but rather only possibly the decision rules within dimensions (e.g., whether to reinforce 3 or 4 fronts).

contemplativeness. It would appear to make sense that an individual for whom the order of assignments matters in the Blotto games also believes that it matters in the auctions and that individuals who figure out that other players neglect certain fronts in the Blotto games will suspect that a parallel tendency exists in the first-price and all-pay auctions. On the other hand, it is possible that the features of a particular game may lead an individual to consider a certain dimension that he would not consider in another game. For example, a person may be more inclined to consider the identity of reinforced auctions in games with asymmetric values of the auctioned items than in other auction games.

7. Discussion

Our results show that incentivized messages provide rich information about individual reasoning. One limitation of the used methodology is that deliberated arguments and their structure can directly be inferred from observing these arguments in a message (for example, a multi-dimensional belief) but not observing a particular argument does not imply that it was not considered. The considerable detail of many messages suggests that incentives were sufficient for most people to communicate main arguments. The fact that some players did not write any messages could be due to lacking incentives, lacking reasoning or a preference to stay silent.

Having said that, we interpret large differences in the observation of two arguments as meaningful because we do not expect incentives to interact so strongly with the type of arguments that are articulated. For example, in the classified messages in Blotto 6, we find that only 3% of the subjects articulate a standard concrete belief in terms of a strategy or a distribution of strategies; some of them continue with multi-dimensional reasoning, rather than with a concrete best response to their belief. In comparison, beliefs about dimensions of others' strategies are stated much more often, by 52% of the subjects. Overall, we infer from such observation that reasoning about concrete strategies is rare while multi-dimensional reasoning is common. However, we cannot rule out the possibility that concrete beliefs are held by further subjects and are simply not articulated. We can only turn to the fact that many more subjects articulate multi-dimensional beliefs when it might be even easier to articulate concrete beliefs.

We use two additional approaches to classify the written messages in order to provide evidence of the classification's robustness. The first is a free-from classification in which a research assistant is not provided with a list of categories of decision rules and is not introduced to the concept of multi-dimensional reasoning. For the three classes of games, the research assistant categorized the messages in a way that is analogous to the dimensions' categorization, which allows us to compare the proportion of dimensions detected using this approach and the guided classification. The second is a computer classification, which is based on a machine-learning algorithm. We find that both the free-form and the computer classifications are qualitatively similar to the guided classification in terms of the frequency of dimensions. Moreover, most of the messages include more than one dimension and common patterns emerge across the three classes of games, as in the analysis performed using the guided classification. The results reinforce our findings that thinking in terms of a number of dimensions is common in the three classes of games explored in this paper.

We focus on the analysis of text, but we also analyze the suggested and final strategies in light of multi-dimensional reasoning and give details in the appendix. The suggested strategies, which accompany the written messages, seem to reflect the reasoning identified in the messages, even though it is more difficult to infer the reasoning from these strategies. For example, the proportion of people who use unit digits 1–3 in their assignments is generally similar to the proportion of messages that mention dimension D2. Furthermore, mentioning D2 is correlated within-subject with using these unit digits in the suggested strategy. However, such correlations are difficult to quantify because not all considered arguments are necessarily mentioned in the messages and some dimensions are difficult to pin down from the strategies.

Although final decisions are not accompanied by a written message, they turn out to be informative about multi-dimensional reasoning. In terms of reinforcements, the use of unit digits, and the location of assignments, they share qualitative features and appear slightly more sophisticated. In the Blotto games, we find fewer reinforcements of 0, 1, and 2 fronts and in the first-price auctions we find fewer reinforcements of 0 and 1 front. We further find a lower frequency of the unit digit 0 and a higher frequency of the digits 1, 2, and 3 in almost all games. Overall, we find more neglect of the first front, which can be beneficial if one allocates a small assignment to it.

Thus, communication does not diminish multi-dimensional reasoning and increases the sophistication within dimensions, as observed in the final strategies. This finding makes us speculate that multi-dimensional reasoning would also be stable over repetitions of such strategic interactions. In fact, previous studies on a repeated play of multi-battle contests found deviations from equilibrium that may be explained by multi-dimensional reasoning, even in the last rounds. Chowdhury et al. (2013) find that players choose either very low bids or moderate-high bids for a given front (as in D1), more often than predicted by equilibrium. In one version, they find that players use decimals in their assignments – analogue to our D2.

Deviations from equilibrium in Montero et al.'s (2016) could also be described as level-1 reasoning in dimensions D1 and D2. In Chowdhury et al. (2021), deviations from equilibrium are more common in asymmetric variations as players assign more to the salient or the high-value front (as in D3).

8. Conclusion

This study was designed to investigate the reasoning process in competitive resource allocation games and to explore whether there are components of the process that are common to different games in this family of games. We experimentally studied three classes of allocation games: Blotto games, first-price multi-object auctions with budget constraints, and all-pay multi-object auctions with a limited endowment. The analysis of the written communication between team members in the experiment indicates that in all the games studied, players classify the strategies in a number of dimensions and perform their strategic deliberation in the space of those dimensions. Moreover, the main dimensions are common to the three classes of games.

The subjects' written messages suggest that almost all of them reasoned in terms of characteristics of strategies rather than in terms of strategies per se. While a group of subjects appeared to consider only one dimension, 57%–85% mentioned two or more dimensions in their message, depending on the particular game. Overall, the most frequently considered dimensions were how many fronts to focus on (D1) and the identity of those fronts (D3). In games with asymmetric values of items, D3 was naturally more frequent. The less frequently considered dimensions D2L and D2H, which focus on the details of the assignment to a front, distinguished their users as more sophisticated and are often significant determinants of good performance. In some of the games, a consideration of D3 was associated with a high score as well. One of the most prominent dimensional decision rules was L1, i.e., a response to a belief that others choose instinctively within the dimension, e.g., by allocating in multiples of ten. Similarly prominent was R, i.e., a collection of reasonable arguments that are neither instinctive nor explicitly belief-based, e.g., focusing on the majority of fronts under the assumption that it is impossible to win all the fronts and that winning less than half is not enough. Overall, we identify several dimensional decision rules, both within and beyond the level-k types of reasoning.

In Section 6, we found that a player tends to consider similar dimensions in different interactions and that multi-dimensional sophistication is persistent across games. We also showed that in the aggregate some patterns of multi-dimensional reasoning are found in different competitive resource allocation games and do not depend on the details of the interaction. Thus, for example, we imagine that both cyber attackers with limited resources and bidders in multi-object auctions focus on some fronts while nonetheless allocating a small amount of resources to other, almost neglected fronts.

Our study suggests embracing predictions that make use of the concept of dimensions. To illustrate the potential benefits, note that despite the heterogeneity in reasoning and behavior in our experiment, we obtained some general insights into multi-dimensional reasoning and its role in generating behavioral patterns (see the Appendix for the distribution of chosen strategies): in resources allocation games, (1) almost all individuals reinforce some fronts and there is a strong tendency to reinforce at least half of the fronts; (2) about 20%-50% use 1-2-3 as a unit digit in their assignments, at least part of the time, while a considerable proportion use only the unit digit 0 in their assignments; (3) the side fronts tend to be "weaker" on average and are more often neglected. Such a prediction narrows the range of probable outcomes and may be useful in planning a wise strategy in similar situations.

The finding that multi-dimensional reasoning is stable after communication makes us conjecture that in repetitions of such games, players organize the information on other players' past behavior in the space of dimensions, form dimensional beliefs, and respond to those beliefs as they did in this study. Throughout the interaction, beliefs may become more precise and behavior within dimensions may converge to a stable one. Thus, the evidence that players consider similar dimensions in different games and that the patterns of multi-dimensional reasoning are recurrent encourages a discussion of an equilibrium concept that is based on such reasoning. Arad and Rubinstein (2019) formalize an equilibrium notion in which strategy characteristics rather than strategies themselves are the fundamental domain of choice. They thus offer one way to think about stability in repeated interactions with multi-dimensional reasoning. Finally, while our paper demonstrates the prevalence of multi-dimensional reasoning solely in resource allocation games, we believe that thinking in terms of dimensions may arise in other strategic interactions in which the strategy space is large and rich, although the relevant dimensions may differ from one context to another. Investigating the factors that trigger multi-dimensional reasoning and identifying additional classes of games in which dimensions are important are natural avenues for future research.

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Appendix: Dimensions reflected in suggested and final strategies

1. Strategies in Blotto games

In this section, the introduced dimensions in Blotto 6 and 7 are analyzed solely on the basis of the data of the suggested strategies and the final strategies. Table A1 presents the proportion of strategies that corresponds to each number of reinforced fronts (D1).

# of reinforced fronts	**Blotto 6 (n=249)**		**Blotto 7 (n=249)**	
	suggested	**final**	**suggested**	**final**
0	5%	3%	4%	2%
1	7%	2%	4%	2%
2	25%	23%	10%	10%
3	31%	35%	26%	25%
4	27%	33%	31%	39%
5	5%	4%	20%	18%
6			6%	3%

Table A1: Fraction of strategies corresponding to number of reinforced fronts in the Blotto games. Reinforcement is defined as allocating more than 20 or 30 troops, respectively.

Table A1 indicates that strategies with one or no reinforced fronts are very rare. Hence, the proportions in the table are consistent with the findings that almost all subjects considered D1 in their messages. The reinforcement of 0, 1, or 2 fronts, which is not successful, is slightly less frequent in the final strategies than in the suggested strategies.[14]

Table A2, which summarizes the use of unit digits in the strategies (D2L and D2H), shows a moderate use of unit digits 1, 2, and 3. Focusing on the suggested strategies, we found that the unit digit 0 is more frequently used in disregarded fronts than in reinforced fronts. In Blotto 6, 52% of the assignments to the reinforced fronts (n=829) and 65% of the assignments to neglected fronts (n=665) have the unit digit 0. In Blotto 7, 37% of the assignments to the reinforced fronts (n=1115) and 68% of the assignments to the neglected fronts (n=628) entail the unit digit 0.

[14] The final strategies give an idea of which strategies teammates found persuasive in their communication. A deeper analysis of persuasion in the spirit of Penczynski (2016b) goes beyond the main objective of our study.

Strategies	Blotto 6 (n=249)		Blotto 7 (n=249)	
	suggested	**final**	**suggested**	**final**
All assignments have the unit digit 0	43%	33%	22%	18%
Some assignments have the unit digit 1,2, or 3	40%	54%	52%	60%
The rest of the strategies	18%	13%	26%	22%

Table A2: Type of troops assignment in terms of the unit digit in strategies in the Blotto games.

A comparison between unit digit patterns in the suggested strategies and the final strategies suggests that subjects' sophistication increased as a result of communication. Table A2 shows that the incidence of the unit digit 0 decreases overall. The data further shows that this is true for both reinforced and disregarded fronts in Blotto 6 and Blotto 7.

As for dimension D3, Figures A1 and A2 suggest that, in both games, fewer resources are assigned to the first and last fronts than to the intermediate ones. Furthermore, 55–63% of the subjects assigned few or no troops to Front 1 in their suggested strategy. The final strategies are very similar to the suggested strategies in this aspect, with slightly fewer resources assigned to Front 1 and 2 in the final strategies and more to the intermediate fronts.

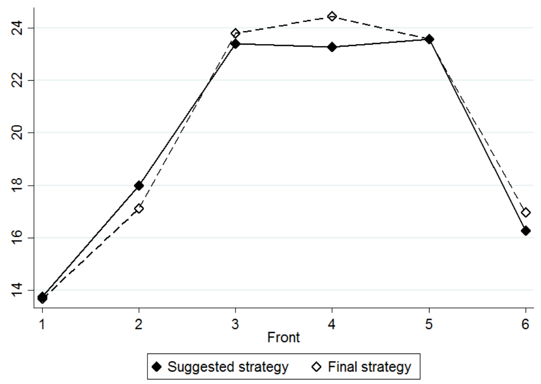


Figure A1: Average assignment to each front in Blotto 6.

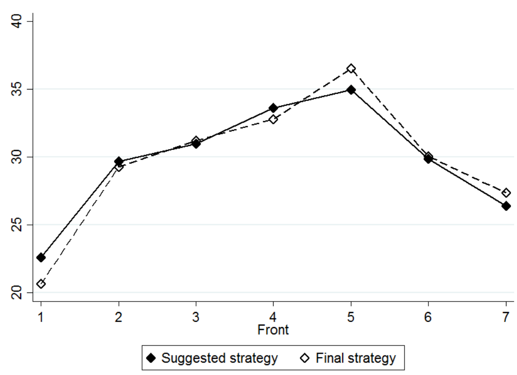


Figure A2: Average assignment to each front in Blotto 7.

We note that the patterns in Blotto 6, reported in Table A1, Table A2 and Figure A1, are similar to those reported in Arad and Rubinstein (2012a), where subjects played the game individually. The finding that the type of strategies suggested here are similar to those played when there is no possibility for communication, suggests that the reasoning is not significantly affected by the communication method used in the current paper.

2. Strategies in first-price multi-object auctions

In this section, the dimensions in Auction 3 and 4 are analyzed solely on the basis of the suggested strategies and the final strategies.

First, we find that 92% of the subjects use all 120 points in the two auction games in their suggested strategies. The average spending is about 118 points, suggesting that dimension D4 regarding the budget use is hardly relevant to strategy choice in the auction games. This is consistent with the rare appearance of this notion in the messages. The final strategies show a similar pattern with 94%–98% of subjects using all 120 points.

Turning to dimension D1, Table A3 presents the proportion of strategies that correspond to each number of high-bid auctions. The numbers are consistent with the findings that the vast majority of subjects consider D1. In Auction 3, the majority decides to have two reinforcements and neglect one auction. The second most popular choice is to neglect two auctions. In Auction 4, the majority of subjects neglect two auctions and the second most popular choice is to neglect only one. In the final strategies, the reinforcement of one or zero auctions is plausibly observed in slightly fewer occasions than in the suggested strategies.

High bids in	**Auction 3 (n=123)**		**Auction 4 (n=126)**	
	suggested	**final**	**suggested**	**final**
0 auctions	11%	5%	4%	1%
1 auction	33%	27%	14%	12%
2 auctions	56%	68%	57%	63%
3 auctions			25%	26%

Table A3: Number of high bids in the auctions. High bid is defined as a bid higher than 120/3 or 120/4, respectively.[15]

Table A4 summarizes the use of different unit digits in the strategies, which corresponds to an aspect of dimensions D2L and D2H (but does not capture these dimensions entirely). The table shows that most of the subjects do not consider the unit digit aspect of their bids and choose all their bids as multiples of tens or zero in their suggested strategies. We can further distinguish between high-bid auctions and neglected auctions in our analysis. Aggregating over all subjects and all auctions, we find that the unit digit of 0 is more frequent

[15] The strategies of three subjects in Auction 3 were eliminated since they used more than 120 points for bids.

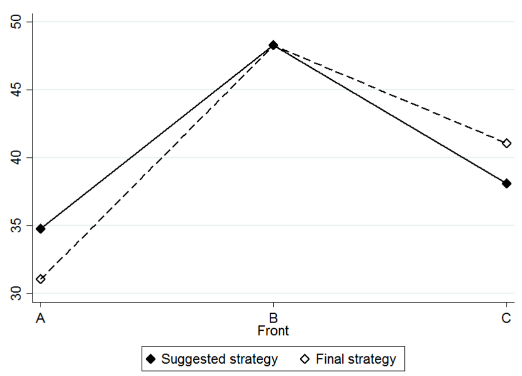
in neglected auctions than in reinforced ones. In Auction 3, 68% of the bids in high-bid auctions (n=232) and 68% in neglected auctions (n=146) have the unit digit of 0. In Auction 4, 64% of the bids in high-bid auctions (n=288) and 76% in neglected auctions (n=216) have the unit digit of 0. The above pattern in the first-price auctions is similar to that observed in the Blotto games, albeit weaker.

Strategies	**Auction 3 (n=126)**		**Auction 4 (n=126)**	
	suggested	**final**	**suggested**	**final**
All bids have the unit digit 0	58%	49%	54%	43%
Some bids have the unit digit 1,2, or 3	21%	30%	25%	36%
The rest of the strategies	21%	21%	21%	21%

Table A4: Type of bids in terms of the unit digit in strategies in the auctions.

Table A4 also shows results for the final strategies and indicates that, as in the Blotto games, the sophistication with respect to the unit digit increases as a result of communication.

As for dimension D3, Figures A3 and A4 show the average bids for each front and suggest that the first and last auctions are less frequently reinforced than the intermediate ones in both Auctions 3 and Auction 4. The pattern of focusing on the intermediate auctions is similar to the observation in the Blotto games. The differences between the average bids in the final strategies and those in the suggested strategies are very small in Auction 4. In Auction 3, however, we find that in the final strategies, the average assignment to Front A is smaller and the assignment to Front C is larger, compared to the suggested strategies.



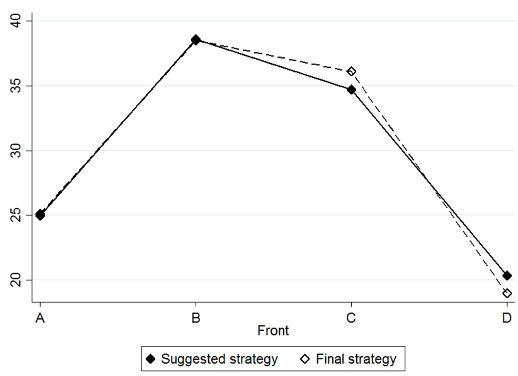


Figure A3: Average bids for each front in Auction 3. Figure A4: Average bids for each front in Auction 4.

3. Strategies in all-pay multi-object auctions

In this section, the dimensions are analyzed solely on the basis of the suggested strategies and the final strategies.

In the suggested strategies, 74% of the subjects used all 60 points in All-pay 3, and 72% did so in All-pay 4. In each game, 9 subjects suggested not bidding at all and the average spending was about 50 points. These findings are consistent with the proportion of D4 in the messages and confirm that this dimension was much more common in the all-pay auctions than in the first-price auctions. In the final strategies, the use of all 60 points went down to 63% in All-pay 3 and 68% in All-pay 4.

Looking at dimension D1, Table A5 presents the proportion of strategies that correspond to each number of reinforced fronts. The numbers are consistent with the finding that the vast majority of subjects considered D1. In All-pay 3 the majority decided to have one reinforced front, and in All-pay 4 the majority focused on one or two fronts.

High-bids in	**All-pay 3 (n=123)**		**All-pay 4 (n=123)**	
	suggested	**final**	**suggested**	**final**
0 auctions	16%	20%	15%	14%
1 auction	59%	59%	41%	36%
2 auctions	24%	21%	36%	43%
3 auctions			8%	7%

Table A5: Number of high bids in the all-pay auctions. High bid is defined as a bid higher than B/3 or B/4, respectively, where B is the used endowment.

Table A6 summarizes the use of various unit digits in the strategies, which corresponds to an aspect of D2L and D2H (but does not capture these dimensions entirely). The table provides information on the individual level and suggests that 41%–47% of the subjects did not consider the unit digit aspect of their bids and chose all their bids to be multiples of tens or zero in their suggested strategies. We distinguish between high-bid auctions and neglected auctions. Aggregating over all subjects and all auctions, we find that the unit digit 0 is used more frequently in the neglected fronts. In All-pay 3, 53% of the bids in reinforced auctions (n=206) and 61% in neglected auctions (n=163) have the unit digit 0. In All-pay 4, 43% of the bids in high-bid auctions (n=247) and 65% in neglected auctions (n=245) have the unit digit 0.

Thus, the above pattern in the all-pay auctions is similar to that observed in the first-price auctions and the Blotto games.

In All-pay 4, the unit digit patterns suggest the higher sophistication of the final strategies compared to the suggested strategies, while in All-pay 3 no such tendency is observed.

Strategies	**All-pay 3 (n=123)**		**All-pay 4 (n=123)**	
	suggested	**final**	**suggested**	**final**
All bids have the unit digit 0	47%	47%	41%	36%
Some bids have the unit digit 1,2, or 3	33%	33%	30%	42%
The rest of the strategies	20%	20%	28%	22%

Table A6: Type of bids in terms of unit digit in strategies in the all-pay auctions.

As for dimension D3, Figures A5 and A6 show the average bid in the different auctions and suggest that the first auction is less frequently reinforced in both games, as is the low-stakes Auction D in All-pay 4. The pattern of focusing on the intermediate auctions is similar to that observed in the other games. In All-pay 3, the final strategies assign fewer resources to fronts A and C, compared to the suggested strategies. In All-pay 4, smaller bids are found in fronts A and B, compared to the suggested strategies.

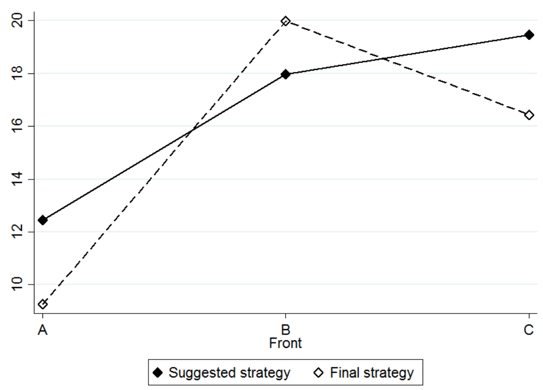


Figure A5: Average bids in each auction in All-pay 3.

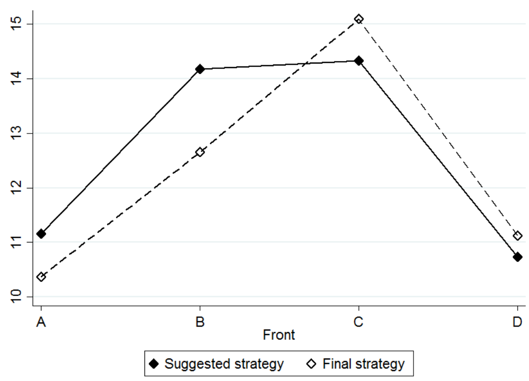


Figure A6: Average bids in each auction in All-pay 4.