MONEY UNDER THE MATTRESS: INFLATION AND LENDING OF LAST RESORT*

Gadi Barlevy†, Daniel Bird‡, Daniel Fershtman§, David Weiss¶

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Abstract

This paper examines whether the two key functions of central banks—ensuring price stability and lending during crises—necessarily conflict. We develop a nominal model of bank runs à la Diamond and Dybvig (1983) in which individuals can store the money they withdraw "under the mattress." In this setting, lending of last resort need not be inflationary. Whether it is depends on the interest rates the central bank charges on its loans. Our results suggest that the central bank must not charge a rate that is too low if it wants to ensure price stability, and must charge a high rate if it wants to robustly attain the ex-ante efficient outcome. These rationales for charging high interest rates on loans during a crisis are distinct from the arguments Bagehot originally relied on to advocate for a similar rule.

Keywords: Bagehot rule, bank runs, financial stability, price-level stability
JEL Codes: E31, E50, G01

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†Federal Reserve Bank of Chicago; email: gbarlevy@frbchi.org

‡Eitan Berglas School of Economics, Tel Aviv University; email: dbird@tauex.tau.ac.il

§Eitan Berglas School of Economics, Tel Aviv University and Emory University; email: danielfer@tauex.tau.ac.il

¶CORRESPONDING AUTHOR Eitan Berglas School of Economics, Tel Aviv University; email: davidweiss@tauex.tau.ac.il; phone: +972584833163

1 Introduction

Historically, modern economies have looked to central banks to carry out two main functions: to act as lenders of last resort during credit crunches and to maintain price stability. An important question is whether these two goals conflict with one another: central banks that act as lenders of last resort create money to lend to distressed banks, and such liquidity injections can be inflationary. Mishkin (2001) echoes this concern in his discussion of an international lender of last resort, noting that "[i]njecting reserves, either through open market operations or by lending to the banking sector, causes the money supply to increase, which in turn leads to a higher price level."

Consistent with this view, proposals to bail out distressed financial intermediaries during the Global Financial Crisis that started in 2007 generated widespread concern about impending inflation.[1] Similar concerns were voiced when central banks were forced to deal with financial stability concerns while trying to contain inflation in the wake of the Covid pandemic. Examples include the shift by the Bank of England from selling to buying gilts in September 2022 during the Fiscal Tantrum that threatened pension funds, and the creation of the Bank Term Funding Program by the Federal Reserve in March 2023 that lent to banks after the collapse of Silicon Valley Bank.[2]

Academic research has also suggested that bank bailouts may be inflationary. For example, recent work by Schilling, Fernández-Villaverde and Uhlig (2022) reinforces the idea that financial stability and price stability conflict with each other. While their paper studies a run on a digital currency issued by the central bank rather than a run on banks, and does not consider bailout policies, one of their key results is that a central bank intent on preventing a run and attaining the socially efficient allocation will not be able to maintain a stable price level across all states of the world. More generally, Bordo (2018) describes the modern history of central banks in developed countries as a "varying evolution between monetary stability and financial stability," suggesting that the two goals are inherently incompatible.

[1] For real-time examples of these concerns, see "A Wider U.S. Financial Bailout Could Fuel Inflation," *New York Times*, March 20, 2008, and "Deep Pockets," *The Economist*, October 9, 2008.

[2] For a discussion of the tensions experienced by the Bank of England and the Federal Reserve, see respectively "A Central Bank on the High Wire," *New York Times*, October 13, 2022, and "Fed Walks Tightrope between Inflation and Bank Turmoil," *Wall Street Journal*, March 22, 2023.

The present paper argues that the two goals of central banks do not need to conflict: it is possible for a central bank to bail out banks during a panic without having to deviate from its commitment to price stability. We focus on the case in which loans by the central bank are neither sterilized nor offset by changes in fiscal policy. In this case, loans from the central bank could in principle work against price stability. Nevertheless, we show that even in this case, the central bank can maintain price stability by setting the interest rate on loans to banks appropriately. Key to this result is that depositors who run on banks can save outside of the banking system after they withdraw their money rather than necessarily spend it on goods and bid up prices.

Intuitively, the reason bailouts need not be inflationary is that the households that run on banks during a panic do not necessarily want to spend the money they withdraw. These households are patient and inclined to save but have lost faith in financial institutions. When offered a substitute for bank deposits—be it hoarding cash "under the mattress" or buying a different financial asset like government bonds—they may prefer saving to spending. If the central bank sets the interest rate on its loans in a way that keeps long-run prices in check, it will encourage patient households to hold money, which will in turn keep the short-run price level in check. If it lets long-run prices rise, patient households might opt to spend, driving up the short-run price level as well.

We establish this result using a monetary version of the quintessential Diamond and Dybvig (1983) model of bank runs. We show that the central bank can avoid raising the price level by setting the interest rate on its loans to banks above some threshold. This includes a zero interest rate, which exactly drains out the liquidity the central bank injected. However, we show that positive rates can be equally effective at preventing a higher price level while liquidity is high, and that even negative interest rates can be consistent with price stability in the short run. At the same time, we show that there is a limit on how low interest rates can go without triggering a rise in the price level.

While bailouts allow banks to avoid inefficient liquidation in the event of a run, households may still choose to run if they don't believe the central bank will extend loans to distressed banks. This raises the question of whether beyond maintaining price stability in the event of a run, a bailout policy can attain the ex-ante efficient outcome should a run occur. We show that this is possible if the central bank charges a sufficiently high interest rate on loans. If the rate it charges is too low, the value of bank equity will be high and households that wait will receive more than the ex-ante efficient consumption level while those that run will receive less than the ex-ante efficient consumption level. To ensure that all patient households receive the ex-ante efficient level of consumption, regardless of whether they run or not, the central bank must charge a high interest rate to prevent banks from profiting off bailout loans. However, if households consume the same ex-ante efficient amount whether they withdraw early or not, there is nothing to discourage them from running. Ensuring the ex-ante efficient outcome in the event of a run thus allows a run to occur in equilibrium.

Our results that central banks should avoid charging low interest rates to maintain price stability and should charge high rates to ensure ex-ante efficiency accord with the well-known Bagehot (1873) dictum that central banks should lend to banks at high interest rates during a crisis. However, our reasons are different than those originally cited by Bagehot for setting high rates, and our results come with the caveat that high interest rates may also validate the decisions of agents to run on banks.

While we focus on central bank loans, the same logic applies to fiscal bailouts financed by debt. A fiscal authority can lend to banks to allow them to pay their depositors. Patient households that withdraw cash can then use it to buy newly issued debt. If households return the cash they withdraw to the Treasury by buying public debt, no new money ends up circulating. The Troubled Asset Relief Program (TARP) during the Global Financial Crisis is an example of such a program: the government purchased preferred shares from banks at a predetermined rate of return, financing these purchases with bonds. A fiscal bailout can thus replicate lending of last resort: the return on preferred shares cannot be too low to maintain price stability, just as is true for the interest rate on bailout loans.

A fiscal bailout of this type is akin to a sterilized monetary intervention. Suppose that a central bank creates money to lend to banks during a run, but simultaneously sells government bonds to the public to drain the liquidity it adds. The end result is identical to a fiscal bailout: patient households can be satisfied without the central bank needing to increase the money supply. We further show that fiscal interventions that rebate to households any profits the central bank earns on its loans will keep the long run price level unchanged, inducing patient households who withdraw early to store money rather than spend it. Thus, unsterilized interventions, sterilized interventions, and interventions in which central bank profits are offset by fiscal policy can all be used to ensure that avoiding inefficient liquidation does not raise the short-run price level.

We proceed as follows. The remainder of this section discusses the related literature. Section 2 lays out the basic model and characterizes the efficient allocation. Section 3 discusses implementation of the efficient allocation in a nominal setting, including the potential for a bank-run equilibrium that features deflation and a fire sale. Section 4 discusses central bank interventions that can avoid liquidation during a bank run and emphasizes the importance of the interest rate that the central bank charges banks in determining both the price level and welfare. Section 5 interprets our findings. Section 6 concludes. All proofs are contained in Appendix A.

1.1 Related Literature

This paper contributes to a growing literature that incorporates money into the Diamond and Dybvig (1983) framework. While some results in this literature are related to ours, we offer new insights into the tradeoff between financial stability and inflation.

One branch of this literature uses models in which any money created by the central bank is drained early on, and it is up to banks to create money in response to higher than expected demand for liquidity, e.g., if there is a run on banks or a larger number of impatient households than expected. Examples include Skeie (2008), Allen, Carletti and Gale (2014), and Schilling, Fernández-Villaverde and Uhlig (2022).

Skeie (2008) considers a model in which households that withdraw money must use it to buy goods from banks. Banks thus never run out of money, even in a run: any money households withdraw simply returns to the bank as spending on goods. While runs are possible, there is no need for the central bank to bail out banks if a run occurs.

Schilling, Fernández-Villaverde and Uhlig (2022) apply Skeie's model to study the implications of issuing a central bank digital currency. They argue that it is not possible to achieve efficiency, avoid runs on the currency, and maintain price stability across all states of the world. Since households that run on the digital currency must buy goods, spending on goods increases during a run. If the central bank succeeds in preventing inefficient liquidation, the amount of goods sold remains fixed and the short-run price level rises. By contrast, households that run (on banks) in our model can store cash rather than be forced to spend it on goods. The price level therefore does not have to rise if inefficient liquidation is averted.

Allen, Carletti and Gale (2014) consider a model in which the fraction of impatient consumers and the return on long-run investments are both random. The socially efficient outcome requires patient households to consume more when the mass of impatient households is low or when the realized return on long-run investments is high. Allowing households to trade money can help to attain the socially efficient outcome. Allen, Carletti and Gale show that one can implement the socially efficient outcome without creating inflation. While their results are similar to ours, the questions they study differ substantially. Efficiency in our model involves preventing financially distressed banks from liquidating long-run investments, a feature absent from their model. Moreover, while money in their model allows households to trade, it is otherwise "super-neutral" and has no effect on outcomes. In our model, money is decidedly not neutral given that more of it can help to prevent costly liquidation.

A separate branch of the literature assumes that banks cannot create money and must instead acquire money created by the central bank. Examples of such models include Antinolfi, Huybens and Keister (2001), Carapella (2012), Robatto (2019), Andolfatto, Berentsen and Martin (2020), and Altermatt, van Buggenum and Voellmy (2022). Our model is closer to these papers: banks in our model also hold a mix of money and long-run real investments, providing money to impatient households that need liquidity and the proceeds from real investments to patient households. While some of our results mirror those in the aforementioned papers, others are novel to our model.

Antinolfi, Huybens and Keister (2001) consider an infinite-horizon setting in which a random fraction of households need liquidity each period. Banks choose how much liquidity to hold before knowing how many households need it and thus can run short on liquidity. Antinolfi, Huybens and Keister show that the central bank can achieve the first-best outcome without generating inflation by extending zero interest rate loans to banks when demand for liquidity is high. However, such loans admit additional equilibria with inflation. They go on to study the case where the central bank charges a fixed zero *real* interest rate on its loans, raising the nominal rate if there is inflation and then spending any profits it earns on loans to buy goods. Under this rule, the unique equilibrium features no inflation and the equilibrium nominal rate is zero. In our model, loans to banks prevent inefficient liquidation rather than maintain the consumption of households that withdraw early. Multiplicity of equilibria is not an issue in the same way as in their model. Finally, we assume that neither the central bank nor the government acts to offset any profits that the central bank earns on its loans. Changes in the nominal interest rate on loans in our model affect the resources that back the government's liabilities, which in turn affect prices.

Carapella (2012) and Robatto (2019) develop models in which banks experience financial distress that is similar to bank runs in our model. They show that distress leads to deflation, as is true in our model. Robatto (2019) shows that a policy of injecting money during a crisis and rebating central bank profits to households can avoid financial distress without raising either the short-run or long-run price levels above their initial values. Since central bank profits are not offset in our setting, bailouts in our model do affect the long-run level, and the key mechanism we study is how such changes affect the short-run price level.

Andolfatto, Berentsen and Martin (2020) develop a model in which households can trade money and long-run assets directly or deposit money in banks that invest in long-run assets. They compare asset trading and banking and study the effects of inflation on allocations in each of these two cases (as well as when both coexist). While they devote a section to bank runs, they only consider runs on a zero-measure set of banks that leave the aggregate price level unchanged. By contrast, we consider systemic bank runs that can affect the aggregate price level. We also consider the effect of changing the interest rate on loans to banks, while they only discuss zero interest rate loans.

Altermatt, van Buggenum and Voellmy (2022) also consider an economy in which impatient households want money while patient ones seek returns on long-run investments. In their model, a run does not affect the price level of goods but does affect the fraction of entrepreneurs that produce. Likewise, emergency liquidity provision has no effect on prices. In our model, a run can lead to deflation, and emergency liquidity can produce inflation if the central bank charges a low interest rate on its loans.

Additional work by Martin (2006) and Keister (2016) considers bailout policies without incorporating money. These papers thus have nothing to say about the implications of bailout policy for price stability.

Finally, our work is related to a large literature on lending of last resort and the Bagehot rule. A nice overview is provided in Bordo (2014, 2018). We complement this literature by providing new justifications for setting a high interest rate on loans to banks during a crisis that are distinct from those cited by Bagehot.

2 Model

We study a monetary version of Diamond and Dybvig's (1983) canonical model of bank runs that allows us to analyze price stability. The model features three periods: $t = 0, 1, 2$, and three types of agents: households, banks, and a government.

Households are risk-averse and ex-ante symmetric. The mass of households is normalized to 1. In period 0, each household is endowed with 1 unit of a good and M units of cash (issued to it by the government). With probability λ a household is impatient and values consumption only in period 1, and with probability $1 - \lambda$ it is patient and values consumption only in period 2. Utility from consumption in the relevant period is given by a utility function $u(\cdot)$ that is strictly increasing, twice differentiable, and features a relative risk aversion coefficient that is strictly greater than unity; that is, for all $c > 0$, $-cu''(c) > u'(c)$. Households privately learn their type at the beginning of period 1. In terms of technology, households can store goods and money across periods but cannot undertake productive investments on their own.

We assume that the mass of banks is also equal to 1. Banks offer households two key financial services. First, we assume that banks can access a technology that converts 1 unit of goods invested in period 0 into $1 + R$ units of goods in period 2 for some $R > 0$. Banks can thus invest on behalf of households.[3] Banks can liquidate these long-run investments in period 1. If so, they receive $1 - \kappa$ units of goods per unit invested for some $\kappa \in (0, 1)$. Since $\kappa < 1$, liquidating investments can provide banks with resources in period 1 should they need them.

Second, banks can pool resources and insure households against the risk of being impatient. Households are better off if they turn out to be patient given that investments

[3] We could allow households to have access to the same technology, but in line with Jacklin (1987), we would need to prevent households from trading equity shares in their investments in order to attain the first-best allocation. We find it easier to assume that only banks can make investments and households enter into exclusive contracts with banks that cannot be sold to others.

only pay off in the long run. Risk-averse households are therefore willing to give up some of the returns on investments they consume as patient households in period 2 in exchange for higher consumption as impatient households in period 1. Pooling resources allows banks to insure risk-averse households and achieve this outcome.

In principle, banks can provide for impatient households either by storing goods to supply to households or by holding money to give to households so that they can buy goods elsewhere. We assume banks do not have the capacity to physically store goods. They can only hold money or buy goods to be deployed off site in long-run investments. In practice, banks transfer money rather than engage in storage or productive investment. However, our main purpose isn't empirical plausibility but to ensure that banks choose between money and long-run investments similarly to the models of Antinolfi, Huybens and Keister (2001), Robatto (2019), and Andolfatto, Berentsen and Martin (2020). We discuss the role of the assumption that banks cannot store goods in Section 5.3.

Finally, the government issues and reclaims money. With a finite horizon, households need a reason to hold cash until period 2. We assume that in period 2 the government is endowed with T goods, which it commits to sell for cash at the end of period 2. Money thus serves as a claim to buy goods from the government in period 2.[4] As will become clear below, we need to impose restrictions on the values T can assume.

2.1 The Efficient Real Allocation

We begin by characterizing the social planner's problem. The planner allocates goods to maximize the ex-ante welfare of the representative household. Denote the consumption of impatient and patient households by c_1 and c_2, respectively. The social planner's objective is to maximize

$$\lambda u(c_1) + (1 - \lambda) u(c_2), \tag{1}$$

[4]Our approach is similar to the fiscal theory of the price level in which households must pay a real tax obligation of T in period 2 using cash. See Cochrane (2023) for an extensive treatment. Diamond and Rajan (2006) and Robatto (2019) also use the fiscal theory to model money and banks in a finite-horizon environment. We find it easier to assume that the government sells goods rather than collects taxes paid with cash in order to avoid the question of what households do if they cannot afford their tax liability.

subject to the budget constraint

$$(1-\lambda)c_2 \leq (1-\lambda c_1)(1+R) + T \tag{2}$$

and the feasibility constraint

$$\lambda c_1 \leq 1. \tag{3}$$

When $T = 0$, as in Diamond and Dybvig (1983), the solution to the planner's problem is interior; i.e., $\lambda c_1 < 1$ and banks undertake some investment. When $T > 0$, we must restrict T to ensure an interior solution, as described in the next lemma.

Lemma 1. *There exists a cutoff $T^* > \frac{1-\lambda}{\lambda}$ such that if $T < T^*$, then the solution to the planner's problem, (c_1^*, c_2^*), is unique, interior, and characterized by*

$$u'(c_1^*) = (1+R)u'(c_2^*). \tag{4}$$

Moreover, $c_1^* > 1$ and $1 < \frac{c_2^*}{c_1^*} < 1 + R$.

For large values of T, the planner gives all of the goods in period 0 to impatient households and lets patient households consume the T goods that impatient households cannot access. To see why the bound on T exceeds $\frac{1-\lambda}{\lambda}$, suppose that $T = \frac{1-\lambda}{\lambda}$. If we assign all of the goods in period 0 to impatient households, they would consume the same amount as patient households. Since $u(\cdot)$ is concave and $R > 0$, such an allocation is suboptimal. In what follows, we assume $T \leq T^*$.

3 Decentralizing the Efficient Allocation

There are various ways to decentralize trade in this economy in order to attain the social optimum. Given our interest in studying the connection between bank bailouts and inflation, we focus on a decentralization that allows for bank runs.

Using a model that incorporates money into the Diamond and Dybvig framework, Skeie (2008) considers a decentralization in which banks buy all goods from households in period 0, storing some and investing the rest. Skeie further assumes that households have to spend all the cash that they withdraw and cannot store it. Every dollar households withdraw from banks thus returns to banks as payment for goods, and so banks never run out of cash and never need to be bailed out.

We consider a different decentralization in which banks can run out of cash. First, we allow households to store the cash they withdraw "under the mattress" rather than spend it. Second, since we assume banks cannot physically store goods, households that withdraw cash and do spend it on goods in period 1 buy them from patient households that stored goods in period 0, not from banks. Money withdrawn and spent in period 1 is thus held by patient households and does not flow back to banks.

Our discussion proceeds as follows. We first describe how decentralized trade unfolds over time. We then show that there exists a unique equilibrium that implements the efficient allocation, and that in this equilibrium only patient households withdraw early. We then argue that under the equilibrium contract, patient households can unexpectedly coordinate on a continuation equilibrium in which patient households withdraw early and banks are forced to liquidate investments.

3.1 Markets and Equilibrium

Trade between banks and households unfolds as follows.

Period 0: Households choose a bank in which to deposit their cash endowment. In exchange, they are entitled to a share of the bank's equity in period 2, which they can give up in period 1 in exchange for a cash payment of d from the bank.[5] Banks must repay their deposit obligations even if it conflicts with maximizing the value of equity. Since banks cannot store goods and liquidating investment is costly, banks prefer to pay households in period 1 with money rather than goods obtained through liquidation.

Contracts with banks are exclusive and nontransferable; i.e., households cannot sell their claim to a bank's equity to other households. Competition among banks leads

[5] The assumption that households deposit all their cash endowment in equilibrium is without loss of generality. Below we show that the prices in period 0 and period 1 are identical in the equilibrium that implements the efficient allocation. Hence, the ability of banks to store cash between periods 0 and 1 is functionally equivalent to storing goods. It follows that banks have (weakly) more options than households. We further show that the equilibrium features some long-run investment. Households are therefore strictly better off depositing with banks than under autarky.

them to pledge all of their equity in period 2 to households that do not withdraw in period 1. The only relevant parameter of the contract is thus the amount of cash d that households receive if they withdraw early. The value of a bank's equity depends on the bank's investment decisions in period 0 and on how much investment it liquidates in period 1. For ease of exposition, we leave the details of how a bank's equity depends on these choices to Online Appendix B.

After households deposit money in banks, a Walrasian market opens in which goods are traded for cash. At this point, banks hold all the cash and households hold all the goods. Banks must buy goods with cash to undertake long-run investments.

Let P_0 denote the price of goods in terms of money in the Walrasian market. Since households are symmetric, we assume that their trades (and those of banks) are identical. Let m_0 denote a household's cash holdings once the market closes. At the end of period 0, each household holds $s_0 = 1 - \frac{m_0}{P_0}$ units of goods and m_0 units of cash, while banks invest the $z = 1 - s_0$ units of goods they acquired and hold $M - m_0$ units of cash.[6]

Period 1: At the beginning of period 1, households learn their type and simultaneously announce whether they intend to withdraw early. Unlike in the canonical Diamond and Dybvig model, in our model there is no sequential service constraint. All economic outcomes are contingent on the *total* fraction of patient households that withdraw early. Households are paid only after all of them declare their intentions, and the amount each household that withdraws early receives depends on what all households choose. Let $\mu \in [0, 1]$ denote the fraction of patient households that withdraw early.

Impatient households want to consume the s_0 goods they stored in period 0, spend the m_0 units of cash they chose to store in period 0, and withdraw their deposit so that they can buy still more goods. Patient households must decide whether to store their s_0 goods or sell them, whether to withdraw or wait to receive an equity payment, and whether to store any cash in their possession and buy goods in period 2 or to buy goods in period 1 and store them for consumption in period 2.

If households collectively demand more cash than the $M - m_0$ units of cash that banks have at the beginning of period 1, banks would have to liquidate their long-run

[6]Households cannot deposit cash with banks at the end of period 0 as they could at the beginning of period 0, since at the end of period 0 banks have already made investment decisions.

investments to raise cash and meet their obligations. To deal with this scenario, we assume that two Walrasian markets operate sequentially within period 1.

First, there is a "liquidation" market that operates after all households announce whether they intend to withdraw, but before they receive any cash from the banks. In this market, households can buy goods with the m_0 units of cash they stored and sell any of the s_0 goods they stored in period 0, and banks can sell goods obtained from investments that they liquidate. Let $P_1^L(\mu)$ denote the price of goods in the liquidation market if a fraction μ of patient households withdraw early.

After the liquidation market closes, banks pay the households that announced their intention to withdraw early. If banks have enough cash to cover all withdrawals, they pay each of these households d units of cash. Otherwise, they pay out all of their cash holdings pro rata to the households that ask to withdraw early.

Finally, after banks pay the households that withdraw early, a second "regular" market opens. In this market, households can buy goods with money they receive from banks and money they have after the liquidation market, households can sell any goods they have after the liquidation market, and banks can sell goods obtained from investments they liquidate. Let $P_1(\mu)$ denote the price of goods in the regular market.[7]

Since liquidation is costly ($\kappa > 0$) and households can store goods at no cost, banks have no reason to liquidate investments unless they cannot cover their obligations. If banks have sufficient cash to pay households that withdraw, only households will trade in both Walrasian markets. In this case, the price of goods in the two markets must be equal, i.e., $P_1^L(\mu) = P_1(\mu)$: households will sell goods in the market with the higher price but buy goods in the market with the lower price, so unless prices are the same in both markets there will be unmet demand in the market with the lower price. Equilibrium trade volumes in each market are indeterminate. However, households consume the same amount of goods regardless of where they buy, so this indeterminacy has no real consequences. For simplicity, we assume there is no trade in the liquidation market unless banks are short on funds, i.e., if $(\lambda + (1 - \lambda)\mu)\, d > M - m_0$. That is, we assume trade in the liquidation market only occurs when such trade is strictly needed.

[7]The price in the regular market may depend on what transpired in the liquidation market. While the notation $P_1(\mu)$ highlights its dependence on μ, it should not be interpreted as implying the price in the regular market does not depend on outcomes in the liquidation market.

Period 2: At the beginning of period 2, returns are realized on any investments the banks did not liquidate. Banks distribute any goods and cash they have on a pro-rata basis to the households that chose not to withdraw in period 1.[8]

After the banks distribute the returns on investments, there is a final Walrasian market in period 2 in which the government inelastically sells T units of goods for cash. Patient households consume and the model ends.

The timeline of the model is summarized in Figure 1.

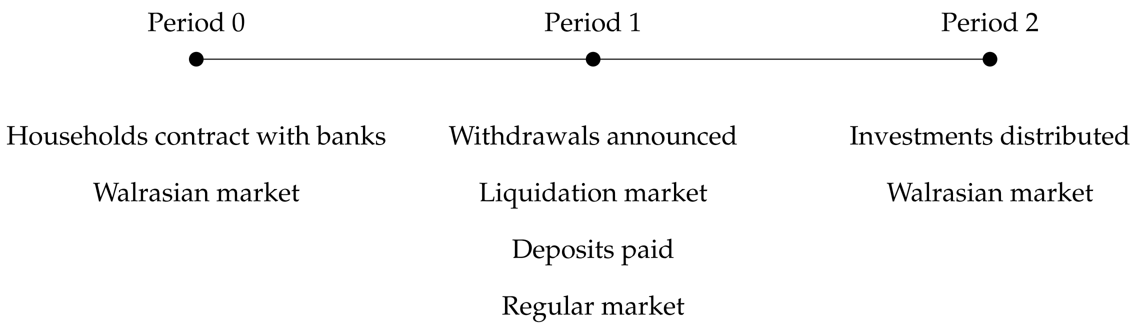

Figure 1: Timeline of Sequential Instantaneous Markets

An equilibrium consists of contract terms offered by banks, a price path, and households and banks' decisions where agents act optimally and markets clear. Formally:

Definition 1 (Equilibrium)**.** *An equilibrium consists of a set of variables that are consistent with market clearing and optimality. Specifically:*

1. *An amount of cash d promised to households that withdraw in period 1.*
2. *A fraction of patient households $\mu \in [0, 1]$ that withdraw early.*
3. *A set of prices P_0, P_1, P_1^L, and P_2.*
4. *Amounts of money m_0 and goods s_0 held at the end of period 0 by each household.*
5. *Amounts of money m_1 and goods s_1 held at the end of period 1 by each patient household.*

[8] If all households withdraw and banks are left with positive equity, we would need to describe what banks do with their equity. However, if a bank has positive equity, an infinitesimal household will prefer to receive all of the equity rather than withdraw, which cannot occur in equilibrium.

In Online Appendix B, we provide a more complete description of the equilibrium that details the contract between households and banks and what agents do in each period. We skip these details here to facilitate the exposition. While we choose not to include the amount of goods z that banks invest in period 0 or the amount of investments $\ell \leq z$ that they liquidate in period 1 in our definition of equilibrium, these objects can be inferred from s_0 and s_1. We refer to these objects below.

3.2 An Efficient Equilibrium

We now look for an equilibrium that implements the social planner's solution. We first look for such an equilibrium where there are no runs, i.e., where only impatient households withdraw in period 1. We then argue that this is the only way to implement the planner's solution. We refer to this as the efficient equilibrium, anticipating that there is an additional continuation equilibrium not associated with the planner's solution in which at least some patient households unexpectedly run on banks in period 1.

Any equilibrium that implements the social planner's solution must satisfy the following four conditions:

First, banks invest the same amount that the planner would choose. This implies that

$$z = 1 - s_0 = 1 - \lambda c_1^*. \tag{5}$$

Second, since banks can only buy goods for investment, the households' money holdings m_0 at the end of period 0 must equal the value of the goods that the banks purchase to invest. That is, in period 0 we have

$$m_0 = P_0(1 - \lambda c_1^*). \tag{6}$$

Third, each impatient household consumes c_1^* in period 1. Since impatient households consume the s_0 goods they previously stored and use the money m_0 left over from period 0 plus the money d they withdraw in period 1 to buy goods, we have

$$s_0 + \frac{m_0 + d}{P_1} = c_1^*. \tag{7}$$

Fourth, no investments should be liquidated, i.e.,

$$\ell = 0. \tag{8}$$

Without a need for liquidation, we know that $P_1^L(\mu) = P_1(\mu)$. For notational ease, we refer to a single price P_1 in the no-run equilibrium without reference to μ.

Next, equilibrium prices must satisfy the following three restrictions:

First, households cannot strictly prefer either money or goods in period 0. In the efficient allocation, households store $\lambda c_1^* \in (0, 1)$ in period 0 and also must hold the cash that they receive from selling goods to banks. Hence, the return on holding money between period 0 and period 1 must equal the return on storing goods, i.e.,

$$P_0 = P_1. \tag{9}$$

Second, for the regular Walrasian market to clear in period 1, patient households must be willing to hold money. For these households to hold money from period 1 for use in period 2, the return on money must at least equal the return on storing goods, i.e.,

$$P_1 \geq P_2. \tag{10}$$

Third, for the Walrasian market to clear in period 2, households must spend all of their money to buy goods from the government. This implies that

$$P_2 = \frac{M}{T}. \tag{11}$$

Finally, we use the above equilibrium conditions to derive one final condition that governs any efficient no-run equilibrium. By Equation (10), if patient households withdraw in period 1, they find it at least weakly optimal to hold their period-1 wealth in cash (as opposed to goods). By Equation (7), we know that their wealth if they withdraw in period 1 is $P_1 c_1^*$. In an efficient no-run equilibrium, this amount cannot buy more than c_2^* goods in period 2; otherwise, patient households could consume more than the socially optimal level, which is infeasible. Hence,

$$\frac{P_1 c_1^*}{P_2} \leq c_2^*. \tag{12}$$

By Lemma 1, $\frac{c_2^*}{c_1^*} < 1 + R$. Equation (12) can therefore be satisfied only if $\frac{P_1}{P_2} < 1 + R$. Combining this inequality with the fact that $P_0 = P_1$ (by Equation (9)), implies that the return on holding cash from period 0 to period 2 is less than the return on investing between periods 0 and 2. Since $\kappa > 0$ and $P_0 = P_1$, investing in a long-run project in period 0 with the intention of liquidating it in period 1 is dominated by holding cash. Banks would thus hold the minimal amount of cash needed to pay depositors in period 1 and use the rest to buy goods to invest. In an efficient no-run equilibrium, the minimal amount of cash banks hold is λd. Hence,

$$M - m_0 = \lambda d. \tag{13}$$

We can use Equations (6)–(9) and (13) to obtain unique values for P_0, P_1, P_2, m_0, and d, which we denote with a star:

$$P_0^* = P_1^* = \frac{M}{1-\lambda} \, , \; P_2^* = \frac{M}{T} \, , \; m_0^* = \frac{1-\lambda c_1^*}{1-\lambda} M \, , \; d^* = \frac{c_1^* - 1}{1-\lambda} M. \tag{14}$$

To solve for s_0^*, we use the fact that it coincides with the social planner's solution:

$$s_0^* = 1 - z = \lambda c_1^*. \tag{15}$$

We can use the value of s_0^* to solve for the amount of money patient households hold at the end of period 1 in the efficient no-run equilibrium:

$$m_1^* = m_0^* + P_1^* s_0^* = \frac{M}{1-\lambda}. \tag{16}$$

Finally, the amount of goods that patient households hold at the end of period 1 is:

$$s_1^* = s_0^* - \frac{m_1^* - m_0^*}{P_1} = 0. \tag{17}$$

In a no-run equilibrium that implements the planner's solution, patient households sell all of the goods they stored in period 0 to impatient households, leaving them with no goods in period 1, i.e., $s_1^* = 0$. Patient households also end up with the entire stock of money at the end of period 1, which they use to buy goods from the government in period 2. Our first result confirms that this allocation is an equilibrium when T assumes a particular range of values.

Proposition 1. *There exists $T^{**} \in (1-\lambda, T^*]$ such that if $T \in [1-\lambda, T^{**}]$, there exists a unique efficient equilibrium that is defined by* (14)–(17)*.*

A no-run equilibrium that implements the first-best allocation exists if T is neither too small nor too large. Intuitively, money is a claim to the T goods sold in period 2. When T is small, the value of money is too low to sustain trade in period 1. Since households can always store goods and consume them in the next period, the money they receive for selling goods in period 1 must compensate them for the goods they give up in that period. This requires that $\frac{P_1}{P_2} \geq 1$; i.e., households must be able to buy at least one good in period 2 for each good they give up in period 1. Given $P_1^* = \frac{M}{1-\lambda}$ and $P_2^* = \frac{M}{T}$, this is only possible if $T \geq 1-\lambda$. For smaller values of T, a no-run equilibrium still exists, but it does not implement the efficient allocation.

Conversely, when T is large, the value of cash is high and patient households prefer withdrawing in period 1 to waiting. A no-run equilibrium consistent with the planner's solution is thus unsustainable.

Finally, we show that there cannot be an efficient equilibrium in which patient households withdraw early. Efficiency requires that banks avoid liquidation. Banks must therefore have some positive equity in period 2. When equity is positive, a household's utility from waiting tends to ∞ as $\mu \to 1$: the equity is divided over a vanishing mass of households. It therefore cannot be an equilibrium for all patient households to run. The only possible equilibrium where patient households run is one in which they are indifferent between withdrawing in period 1 and waiting for period 2. But as we show in the proof, patient households cannot be indifferent between withdrawing and waiting in equilibrium.

We use the notation P_1^* and P_2^* to denote equilibrium prices for the efficient equilibrium. These prices serve as benchmarks to compare equilibrium prices if there is a run and, subsequently, if the central bank provides loans to bail out banks during a run. In what follows, we assume that $1 - \lambda \leq T \leq T^{**}$.

3.3 A Bank-Run Continuation Equilibrium

We now show that the equilibrium contract in Proposition 1 that supports the efficient allocation can allow for a continuation equilibrium in which some (or all) of the patient households choose to withdraw in period 1, contrary to expectations in period 0. We refer to any situation in which $\mu > 0$ as a run.

In the efficient equilibrium, only impatient households withdraw early and banks hold just enough cash to meet their demand. A run would require banks to liquidate investments to pay depositors, since with $\kappa < 1$ liquidation will generate cash. The reason that there may be a continuation equilibrium with $\mu > 0$ is that withdrawing early is a strategic complement: if households withdraw and force banks to liquidate, the value of equity decreases for other households. In Diamond and Dybvig (1983), this complementarity implies that $\mu = 1$ is always a continuation equilibrium. If all other households run, no equity remains, and so a household is better off withdrawing early and receiving something. In our model, we need an additional condition on the cost of liquidation κ to ensure that $\mu = 1$ is a continuation equilibrium.

Proposition 2. *For every $\mu > 0$, there exists $\overline{\kappa}(\mu) \in [0, 1)$ such that banks have 0 equity if a fraction μ of patient households withdraw in period 1 and $\kappa \geq \overline{\kappa}(\mu)$. Hence, $\mu = 1$ is an equilibrium if $\kappa \in (\overline{\kappa}(1), 1)$.*

To understand the role of κ, recall that households hold positive amounts of goods ($s_0^* > 0$) and cash ($m_0^* > 0$) at the beginning of period 1. They thus do not rely on the cash they withdraw (d^*) to pay for their entire consumption, in contrast to Diamond and Dybvig (1983). While it is still true that in our model $c_1^* > 1$, the consumption value of the cash amount d^* promised to households in period 1 may be less than c_1^*. In such cases, obtaining one good in period 1 for each good invested in period 0 may suffice to cover the banks' obligations even if all patient households withdraw. Banks run out of equity only if they receive less than one good per unit of goods invested, as is the case when $\kappa > 0$. Of course, if the equilibrium real value of d^* is high, then $\mu = 1$ leaves banks with zero equity, just as in Diamond and Dybvig's model.[9]

Proposition 2 concerns the existence of a continuation equilibrium in which all patient households run. This can only be an equilibrium if banks are left with zero equity when all households run. As we noted above, with positive equity, utility for a household form waiting tends to ∞ as $\mu \to 1$, so that waiting is better than withdrawing.

If bank equity is positive when $\mu = 1$, there might still be a continuation equilibrium in which $0 < \mu < 1$. In such an equilibrium, patient households are indifferent between d^* units of cash in period 1 and a share of the bank's nonliquidated investments in period 2. While such a run does not cause banks to fail, it still results in inefficient liquidation. The logic behind bailouts does not hinge on bank failures but on the way inefficient liquidation cuts into what patient households can consume.

More generally, even if there is no continuation equilibrium with $\mu > 0$, we could still ask what would happen off the equilibrium path if a fraction $\mu > 0$ of patient households were to withdraw early.

As with any well-specified dynamic game, we can determine prices $P_1(\mu)$ and $P_2(\mu)$ for any value of μ. We will refer to these as equilibrium prices even for values of μ that are not themselves equilibria. We reserve the term *continuation equilibrium* to refer to values of μ that are consistent with patient households optimally choosing whether to withdraw, as well as to the equilibrium prices given such μ. We will eventually return to the question of which values of $\mu > 0$ are in fact continuation equilibria. In particular, we ask whether bailouts work by avoiding liquidation when runs occur or by preventing runs from even occurring.

Bank Runs, Fire Sales, and Deflation

While our primary interest is whether avoiding liquidation interferes with price stability, we briefly discuss what happens absent any intervention if a positive fraction $\mu > 0$

[9] As a numerical example, suppose that $\lambda = \frac{1}{2}, R = 1, T = \frac{3}{4}$, and households have CRRA utility $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. We can show that $\overline{\kappa}(1) = 0$ iff $\gamma > \frac{\log(2)}{\log(27/14)} > 1$. That is, if liquidation costs are sufficiently small and γ is not too large, banks remain solvent even if there is a full run.

of patient households runs. In such cases, banks need to liquidate investments in the liquidation market to cover their obligations.[10]

In principle, households can also sell some of the s_0^* goods they stored in period 0 in the liquidation market. Unlike banks, households can wait for the regular market in period 1 to sell stored goods. This ensures that $P_1(\mu) \geq P_1^L(\mu)$. Otherwise, no goods would be sold in the regular market at the end of period 1, which cannot be an equilibrium given that impatient households try to buy goods with the cash that they withdraw from banks after the liquidation market closes. Formally, we have

Lemma 2. *$P_1^L(\mu) \leq P_1(\mu)$ for any $\mu > 0$. That is, the equilibrium price of goods in the liquidation market cannot exceed the equilibrium price of goods in the Walrasian market.*

The inequality in Lemma 2 can be strict, i.e., there may be an equilibrium in which $P_1^L(\mu) < P_1(\mu)$. In that case, patient households wait to sell goods in the regular market. Banks will want to wait, but they need to sell goods to raise cash to pay depositors. Banks end up liquidating their investments in what is effectively a fire sale. Impatient households prefer to buy goods in the liquidation market, but we assume they cannot spend cash they have yet to receive from banks. Given the price increase between the two markets, patient households have an incentive to buy goods in the liquidation market to sell to impatient households at a higher price in the regular market. Once impatient households withdraw money, they use this cash to buy more goods in the regular market. Such fire sales are possible if d^* is large, which pushes up $P_1(\mu)$, and m_0^* is small, which pushes down $P_1^L(\mu)$. By (14), these conditions will hold if c_1^* is large.

Although banks sell their investments in the liquidation market, a run can affect the price of goods in the regular market in period 1. The next proposition shows that a run generally depresses the price of goods in the regular market.

Proposition 3. *The equilibrium price $P_1(\mu) \leq P_1^*$ for any $\mu > 0$. More precisely:*

1. *If $T \in (1-\lambda, T^{**}]$, the prices of goods satisfy $\frac{M}{T} = P_2(\mu) \leq P_1(\mu) < P_1^* = \frac{M}{1-\lambda}$.*
2. *If $T = 1-\lambda$, then $P_1(\mu) = P_2(\mu) = \frac{M}{T}$ for all μ.*

[10] ℓ and $P_1^L(\mu)$ may not be uniquely determined for a given μ. Intuitively, if $P_1^L(\mu)$ is low, banks need to liquidate more to cover their obligations, and the higher ℓ lowers the price $P_1^L(\mu)$. If $P_1^L(\mu)$ is high, banks need to liquidate less, and the lower ℓ supports a higher price $P_1^L(\mu)$.

Hence, the price of goods in period 1 is weakly lower if there is a run than if there is no run.

Proposition 3 shows that unless $T = 1 - \lambda$, a bank run coincides with a drop in the price level. This finding is similar to Carapella (2012) and Robatto (2019), who show that financial distress is associated with a lower price level. It is also consistent with empirical evidence on inflation during financial crises. However, the reason the price level is lower during a run in our model is a bit different. In our model, a bank run is associated with liquidation: more goods are up for sale in period 1 during a run than when there is no run, even though the economy as a whole is weaker.[11] At the same time, the amount of money spent on goods is either unchanged or decreases (if banks collapse). While more households hold cash during a run, in equilibrium patient households that run on the bank store money rather than spend it. This is because equilibrium requires that $P_1(\mu) \geq P_2(\mu)$, and so there is no reason for households to buy goods in period 1. Since the same or even a smaller amount of money will chase after more goods during a run, the price level falls. In Carapella (2012), causality runs in the opposite direction: a decrease in the price level causes financial distress because banks earn less on the goods they sell. In Robatto (2019), financial intermediaries redirect money to households with a higher propensity to spend. Distress for intermediaries interferes with this and reduces the amount of money spent, as in our model. However, there is no liquidation in Robatto's model as there is in ours.

4 Intervention

So far, we have shown that the efficient equilibrium in our economy is susceptible to bank runs that can force banks to liquidate investments inefficiently. It is well understood that there are various ways to either avoid a run or avoid liquidation if a run occurs. Diamond and Dybvig (1983) show that suspension of convertibility and deposit insurance can prevent runs. Keister (2016) studies fiscal bailouts using real resources, while Andolfatto, Berentsen and Martin (2020) discuss how central banks can lend money to banks during a run. In this section, we also analyze lending by the

[11] If households that withdraw receive (close to) d units of cash, the lower price level in period 1 enables impatient households to consume more than c_1^* during a run. However, the cost of liquidation implies that aggregate consumption of all households must decrease.

central bank during a run. Our contribution is not in showing that bailouts can prevent inefficient liquidation but in analyzing how the terms on loans to banks matter for price stability and allocative efficiency.

If a fraction $\mu > 0$ of patient households withdraw in period 1, banks need to borrow $\mu(1-\lambda)d^*$ units of cash to avoid liquidation. We assume that after μ is revealed in period 1, the central bank offers to lend banks exactly this amount. Banks that accept these loans are charged a nominal interest rate $i(\mu)$. For notational ease, we mostly suppress the dependence of the interest rate on μ and refer to it as i. Banks must repay the central bank $(1+i)\mu(1-\lambda)d^*$ in cash in period 2. We assume that debts to the central bank are senior to all other obligations, i.e., commercial banks must pay the central bank before distributing their equity to households in period 2.

Banks agree to borrow from the central bank only if the interest rate i is not too high. First, borrowing cannot be costlier than liquidation, i.e., the cost of borrowing a unit of cash, $1+i$, cannot exceed the cost of raising one unit of cash by liquidation, $\frac{P_2}{P_1}\frac{1+R}{1-\kappa}$. Second, banks must be able to repay the central bank. The amount they need to repay is $(1+i)\mu(1-\lambda)d^*$, and the most they can repay is $P_2(1+R)z$ where z denotes the amount of investments held by banks. Hence, $1+i$ must also be below $\frac{P_2(1+R)z}{\mu(1-\lambda)d^*}$. Let $1+\bar{\imath}(\mu)$ denote the lesser of these two rates.[12] We henceforth focus on $i \leq \bar{\imath}(\mu)$. We further restrict attention to loans that require some repayment, i.e., where $i > -1$.

To ensure banks can raise the cash they need to repay central banks, we modify the structure of trade in period 2 to allow for sequential markets, similarly to how we model trade in period 1. Specifically, we assume there is a preliminary "loan repayment" market at the beginning of period 2 after investments mature but before banks pay equity holders. In this market, banks sell the goods from matured investments to raise the cash they need to repay the central bank. Patient households buy these goods with their m_1 cash holdings. Let $P_2^L(\mu)$ denote the price of goods in this market as a function of μ. After this market closes, banks pay back their loans to the central bank and distribute any remaining equity to remaining equity holders on a pro-rata basis.

After banks pay equity holders, a second "regular" market opens. In this market,

[12]Below we show that the prices P_1 and P_2 depend on i. This means that the upper bound $\bar{\imath}(\mu)$ is defined only implicitly. Either the affordability constraint or the cost constraint can be the one that pins down $\bar{\imath}(\mu)$. As $\kappa \to 1$, liquidation becomes infinitely costly and affordability becomes the relevant constraint. As $\mu \to 0$, affordability becomes moot and cost is the relevant constraint.

the government sells its endowment of T goods and patient households can buy goods with their remaining money. Let $P_2(\mu)$ denote the price of goods in the regular market. The equilibrium price path is given by the tuple $\{P_0, P_1^L(\mu), P_1(\mu), P_2^L(\mu), P_2(\mu)\}$.

As in period 1, the presence of two markets can lead to indeterminacy in trade volumes. Households that use their money to buy goods in the loan repayment market are able to buy fewer goods from the government in the regular market. Any cash banks raise in the loan repayment market that isn't used to repay the central bank returns to equity holders as cash, and so the amount of money that is used to buy goods from the government in the regular market is the same even if banks sell more than they need. Below, we show that equilibrium prices in the loan repayment market are the same as in the regular market, and so such trade has no real consequences. To simplify matters, we assume that the only trade that takes place in the loan repayment market is that banks sell the $(1+i)\mu(1-\lambda)d^*$ worth of goods they need to repay their debt.

4.1 Price Dynamics

We now consider whether avoiding inefficient liquidation requires the central bank to sacrifice price stability. While there is no explicit reason in our model for the central bank to care about price stability, there are well-known arguments in favor of price stability that our setup ignores. One example is heterogeneity in price stickiness across producers. In such a case, a failure to maintain price stability can lead to a misallocation of goods in the economy. We abstract from reasons for the central bank to care about price stability and only ask whether intervention is consistent with price stability.

To determine whether there is a tradeoff between avoiding liquidation and maintaining price stability, we need to solve for the equilibrium prices of goods when $\mu > 0$ and banks borrow from the central bank. We begin with the following observation.

Lemma 3. *In any equilibrium in which banks borrow from the central bank, the prices of goods in period 2 are given by* $P_2^L(\mu) = P_2(\mu) = \frac{M - i\mu(1-\lambda)d^*}{T}$.

In contrast to period 1, the prices in the two markets in period 2 are always equal. The difference is that in period 1, households buying in the liquidation market can access only part of the money they are entitled to. Even if the price of goods in that market is low, they cannot shift all of their demand in order to buy at the lower price. This is not the case here: patient households start period 2 with all of the cash in the economy. We can thus refer to a single price in period 2. Since loans allow banks to avoid liquidation, we know that there is a single price in period 1. We therefore refer to the effect of intervention only on the two prices $P_1(\mu)$ and $P_2(\mu)$.

The price $P_2(\mu)$ is equal to the ratio of the cash households bring into the regular market and the goods the government sells. A higher interest rate on loans i drains more money from the private sector, leaving patient households with less cash to spend on the T goods sold by the government. $P_2(\mu)$ is therefore decreasing in i. Raising the interest rate i shifts how the government collects the cash that circulates: it will receive more as profits from the central bank rather than as revenue from selling goods.

Antinolfi, Huybens and Keister (2001), Robatto (2019), and Andolfatto, Berentsen and Martin (2020) focus mainly on the case where $i = 0$. At this interest rate, the central bank removes from circulation the amount it lent out. This is a necessary and sufficient condition to leave the price level unchanged in the long run, i.e., to ensure $P_2(\mu) = P_2^*$.

However, policymakers and academics have raised concerns about how liquidity injections affect the short-run price-level separately from the long-run effects after loans are repaid. As we show below, ensuring that $P_2(\mu)$ is unchanged is not equivalent to ensuring that $P_1(\mu)$ is unchanged.[13]

The price of goods in period 1 depends on what patient households that run on banks do with the money that they withdraw. Do they buy goods and store them, bidding up the price of goods? Or do they store money so that demand for goods is the same as when there is no run? Patient households buy goods when they are cheapest. Whether $P_2(\mu)$ exceeds $P_1(\mu)$ in turn depends on the interest rate i that the central bank charges banks. For $P_1(\mu)$ to equal P_1^*, the price level in period 2 cannot exceed P_1^*. Since $P_2(\mu)$ is decreasing in i, this requires that the interest rate on loans be sufficiently high.

Formally, we know from Lemma 3 that $P_2(\mu) = \frac{M - i\mu(1-\lambda)d^*}{T}$. Given that $d^* = \frac{c_1^* - 1}{1-\lambda} M$

[13] Relatedly, Berentsen and Waller (2011) show how monetary policy interventions that will be undone after one period can be non-neutral in the short run even though they are neutral in the long run. We are similarly interested in the short-run impact of monetary interventions on prices in period 1, which may be different from their long-run impact on prices in period 2.

from equation (14), $P_2(\mu)$ will be at least as high as $P_1^* = \frac{M}{1-\lambda}$ if

$$i \geq \frac{1-\lambda-T}{\mu(1-\lambda)(c_1^*-1)} \equiv i^\dagger(\mu). \tag{18}$$

Since $T \geq 1-\lambda$, this critical interest rate is (weakly) negative. Our next result shows that the short-run price level $P_1(\mu)$ is equal to P_1^* if $i \geq i^\dagger(\mu)$ and exceeds P_1^* if $i < i^\dagger(\mu)$.

Proposition 4. *Suppose that $\mu > 0$ and banks borrow $\mu(1-\lambda)d^*$ at interest rate i from the central bank to avoid liquidation.*

1. *If $i \geq i^\dagger(\mu)$, then $P_1(\mu) = P_1^* = \frac{M}{1-\lambda}$ and $P_2(\mu) = \frac{M-i\mu(1-\lambda)d^*}{T}$. Any interest rate above the cutoff ensures that the short-run price level is the same as it would be if there were no run.*

2. *If $i < i^\dagger(\mu)$, then under a bailout $P_1(\mu) = P_2(\mu) = \frac{M-i\mu(1-\lambda)d^*}{T} > \frac{M}{T}$. The price level is higher in both periods than it would be if there were no run.*

If the price $P_2(\mu)$ exceeds P_1^*, then $P_1(\mu)$ must also exceed P_1^*. Otherwise, all patient households would purchase goods in period 1, more money would be spent on the same amount of goods (since banks do not liquidate investments), and we would counterfactually get $P_1(\mu) > P_1^*$. Setting i above $i^\dagger(\mu)$ pushes $P_2(\mu)$ below P_1^*. At this point, patient households store money and wait to buy goods in period 2. But then the price level in period 1 must be the same as it would be with no run, i.e., $P_1(\mu) = P_1^*$.

Since $i^\dagger(\mu) \leq 0$, with strict inequality if $T < 1-\lambda$, the central bank can maintain short-run price stability even without fully removing all of the liquidity it injects. Intuitively, this is because when there is no run, $P_1^* = \frac{M}{1-\lambda}$ strictly exceeds $\frac{M}{T} = P_2^*$. Even if we raise $P_2(\mu)$ a little above P_2^*, patient households will strictly prefer to store their cash. Stabilizing the price level in period 1 is thus consistent with allowing the long-run price level $P_2(\mu)$ to exceed, equal, or fall below its level when there is no run.

4.2 Welfare and Redistribution

So far, we have shown that a central bank that wants to ensure that its liquidity injection does not increase the short-run price level can do so by not setting the interest rate too low. The fact that short-run price stability is achievable under a wide range of interest rates suggests that the central bank may have some freedom in setting the interest rate on loans to achieve other objectives. A natural objective beyond preventing inefficient liquidation and maintaining price stability is to ensure that the welfare of households is equal to the ex-ante utility they are due under the efficient allocation.

When banks are able to avoid liquidation, changing the interest rate i that banks pay the central bank has no effect on the quantity of goods available for aggregate consumption in each period. Instead, changes in i only lead to a zero-sum reallocation of goods between three types of households: impatient households, patient households that withdraw early, and patient households that wait. This reallocation is effected through changes to market prices.

When $i \geq i^{\dagger}(\mu)$, patient households that withdraw early store money and wait to buy goods in period 2. Given that the price level in period 2 is decreasing in i, patient households that withdraw early can consume more when i rises. Since total consumption in period 2 is constant and patient households that withdrew early consume more when i is higher, patient households that wait must be worse off when i is higher. Finally, since the price $P_1(\mu)$ does not change with i when $i \geq i^{\dagger}(\mu)$, impatient households are unaffected by a higher i. Increasing i when $i \geq i^{\dagger}(\mu)$ redistributes resources from patient households that wait to patient households that withdraw early.

When $i < i^{\dagger}(\mu)$, Proposition 4 establishes that both $P_1(\mu)$ and $P_2(\mu)$ are decreasing in i. In this case, both impatient households and patient households that withdraw early benefit from a higher i. Patient households that wait must therefore be worse off.

Since $P_1(\mu) = P_1^*$ when $i \geq i^{\dagger}(\mu)$, impatient households consume c_1^* if the central bank sets $i \geq i^{\dagger}(\mu)$: the bailout ensures that impatient households are paid the full promised amount d^* even when there is a run, and since the price coincides with the price under the efficient equilibrium, the amount of goods impatient households consume, $s_0^* + \frac{m_0^*+d^*}{P_1^*}$, is the same as in that equilibrium. Setting $i \geq i^{\dagger}(\mu)$ thus ensures that impatient households receive the utility they have under the socially optimal allocation. Is there also a way to set interest rates so that even with a run, all patient households consume c_2^* whether they withdraw or not? The next proposition summarizes the welfare effects of changing i for the different types of households. It shows that there indeed exists an interest rate $i^{\dagger\dagger}(\mu) \geq 0$ that ensures that patient households consume the same amount of goods whether they withdraw early or wait.

Proposition 5. *Suppose that $\mu > 0$ and banks borrow $\mu(1-\lambda)d^*$ at rate i.*

1. *When $i < i^{\dagger}(\mu)$, raising i increases the utility of impatient households. When $i \geq i^{\dagger}(\mu)$, raising i does not change the utility of impatient households.*
2. *If $0 < \mu < 1$ so that some but not all households run, raising i decreases the utility of patient households that do not run and increases the utility of those that do run.*
3. *There exists an interest rate $i^{\dagger\dagger}(\mu) \geq 0$ for every $\mu > 0$ at which the utility for patient households is the same whether they withdraw early or wait. If $T < T^{**}$, then $i^{\dagger\dagger}(\mu) > 0$.*

The last part of Proposition 5 states that the interest rate that equates the consumption of all patient households is nonnegative. Since $i^{\dagger\dagger}(\mu) \geq 0 \geq i^{\dagger}(\mu)$, setting $i = i^{\dagger\dagger}(\mu)$ induces impatient households to consume c_1^*. Since all patient households consume the same amount, the amount they all consume must be c_2^*.

To see why the interest rate $i^{\dagger\dagger}(\mu)$ that equates the consumption of all patient households is nonnegative, observe that the wealth of patient households that run is equal to $P_1(\mu)c_1$, where c_1 is the amount of goods impatient households consume (which can in principle differ from c_1^*). The only way for the central bank to ensure that patient households with this wealth consume c_2^* is if they store the money they withdraw in period 1 and if the price $P_2(\mu)$ is such that $P_1(\mu)c_1 = P_2(\mu)c_2^*$. If patient households that run store cash, then $P_1(\mu) = P_1^*$ and $c_1 = c_1^*$. In this case, we have

$$P_1^* c_1^* = P_2(\mu) c_2^*. \tag{19}$$

Recall that a necessary condition for the efficient equilibrium to feature no runs is Equation (12), which stipulates that

$$P_1^* c_1^* \leq P_2^* c_2^*.$$

Ensuring that patient households that run consume c_2^* thus requires $P_2(\mu) \leq P_2^*$. The amount of money in circulation in period 2 must either remain unchanged or decrease.[14]

The observation that ensuring all patient households consume c_2^* requires reducing the money stock in period 2 has several implications. First, while ensuring ex-ante efficiency implies that $P_1(\mu) = P_1^*$ and is thus compatible with short-run price stability, it does not ensure long-run price stability and generally requires $P_2(\mu) < P_2^*$.

Second, it may not always be possible to remove the necessary amount of money in period 2 by charging banks $i = i^{\dagger\dagger}(\mu)$. Equation (19) implies that to ensure all patient households consume c_2^*, we would need to set $P_2(\mu) = P_1^* \frac{c_1^*}{c_2^*}$. The expression on the right-hand side is independent of μ. To attain the ex-ante efficient allocation, the central bank must remove a fixed amount of cash that does not depend on μ. The net amount of money removed from circulation due to loan repayments is $i\mu(1-\lambda)d^*$, which tends to 0 as $\mu \to 0$ unless $i \to \infty$. Draining the required amount of money when few households run thus necessitates setting a large interest rate that would inevitably exceed $\bar{\imath}(\mu)$. Policymakers could still ensure the ex-ante efficient outcome in this case by charging a lump-sum tax $\tau(\mu)$ on banks that must be paid in cash in period 2 rather than imposing interest charges that are proportional to the amount borrowed. For example, banks might be asked to contribute to a deposit insurance fund following a run.

4.3 Implications for the Lender of Last Resort Policy

We now collect our results and summarize our key policy implications. A bailout rule specifies an interest rate $i(\mu)$ on loans to banks as a function of the fraction μ of patient households that withdraw early. Recall that $\bar{\imath}(\mu)$ is the maximal interest rate that banks are willing to pay, $i^{\dagger}(\mu)$ is the minimal interest rate associated with short-run price stability, and $i^{\dagger\dagger}(\mu)$ is the maximal interest rate that guarantees that patient households prefer not to withdraw in period 1. A bailout rule exhibits the following features:

Theorem 1. *Suppose that the central bank lends the amount $\mu(1-\lambda)d^*$ to banks at rate $i(\mu) \le \bar{\imath}(\mu)$ whenever $\mu > 0$.*

1. ***Financial Stability**: Since $i(\mu) \le \bar{\imath}(\mu)$, there is no inefficient liquidation, no bank failure, and no fire sales.*

[14] For $T \in [1-\lambda, T^{**})$, ensuring that patient households consume c_2^* requires setting P_2 strictly below P_2^*. That is, the central bank has to reduce the amount of money in circulation.

2. ***Price-Level Stability***: *If $i(\mu) \geq i^\dagger(\mu)$ for all μ, then $P_1(\mu) = P_1^*$ for all μ and prices are stable in the short run regardless of how many households run. If $i(\mu) = 0$ for all μ, then $P_2(\mu) = P_2^*$ for all μ and prices are also stable in the long run.*

3. ***Efficiency***: *Whether consumption is ex-ante efficient depends on the interest rate $i(\mu)$:*
   - *If $i(\mu) \geq i^\dagger(\mu)$ for all μ, then impatient households consume c_1^*.*
   - *If a fraction μ of patient households run and $i(\mu) = i^{\dagger\dagger}(\mu) \leq \bar{\imath}(\mu)$, then patient households consume c_2^* regardless of whether they withdraw early or wait.*

4. ***Equilibrium Runs***: *Whether $\mu > 0$ is a continuation equilibrium depends on $i(\mu)$:*
   - *If $i(\mu) < i^{\dagger\dagger}(\mu)$ for all μ, the only continuation equilibrium is $\mu = 0$ (no run).*
   - *If $i(\mu) = i^{\dagger\dagger}(\mu) \leq \bar{\imath}(\mu)$ for some μ, then there exists a continuation equilibrium in which a fraction μ of patient households run.*

Figure 2 illustrates the key insights from Theorem 1 graphically. It shows the range of interest rates that are consistent with price-level stability in the short and long run, as well as how interest rates affect consumption.

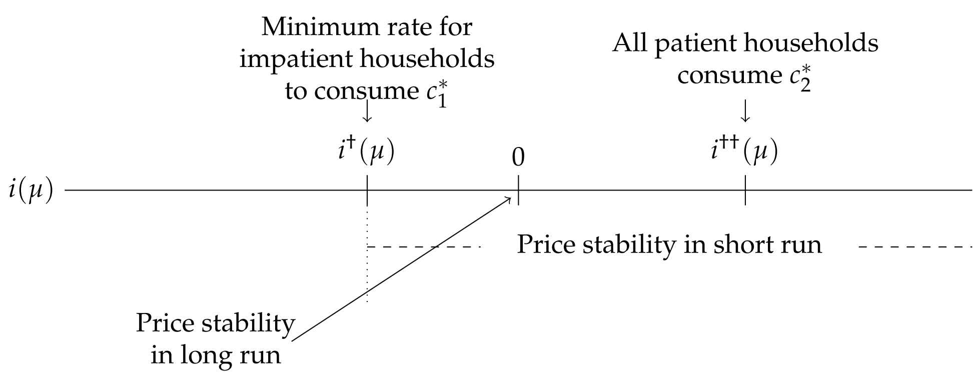

Figure 2: Interest Rates, Price Stability, and Consumption

Within the range of interest rates that avoid liquidation and ensure short-run price stability ($P_1(\mu) = P_1^*$), there is an interest rate that ensures the ex-ante efficient allocation even in the event of a run. This interest rate is given by $i(\mu) = i^{\dagger\dagger}(\mu)$ and is positive. It is therefore possible to avoid inefficient liquidation, maintain short-run price stability, and ensure that all patient households consume the ex-ante efficient amount c_2^* even if there is a run, but only by making loans sufficiently costly for banks.

Part (4) of Theorem 1 establishes that if $i(\mu) < i^{\dagger\dagger}(\mu)$ for all μ, there is no continuation equilibrium in which patient households run: given there will be a bailout, patient households are better off not running. If the central bank only wants to maintain price stability and ensure the ex-ante efficient outcome for all continuation equilibria in period 1, it can set $i < i^{\dagger\dagger}(\mu)$ so that $\mu = 0$ is the unique continuation equilibrium. But if the central bank wants to guard against the off-equilibrium scenario that a fraction μ of patient agents run, it must set $i(\mu) = i^{\dagger\dagger}(\mu)$ to ensure that all patient households consume c_2^*. For example, the central bank might worry that not all households believe it will bail out banks that those households might still run. Setting $i(\mu) = i^{\dagger\dagger}(\mu)$ ensures that all patient households consume c_2^* even if some of them run. However, there are two important caveats. First, this intervention is effective only if $i^{\dagger\dagger}(\mu) \leq \bar{\imath}(\mu)$, which is not true for small values of μ. Central banks cannot ensure ex-ante efficiency when a small fraction of patient households run, at least not by using bank loans. Second, part (4) of Theorem 1 shows that by guarding against the scenario where a fraction μ of households run, the central bank allows that run to become a continuation equilibrium. Intuitively, if patient households receive c_2^* regardless of what they or other patient households do, nothing discourages them from running. Safeguarding against off-equilibrium scenarios can turn them into equilibrium scenarios.[15]

5 Discussion of Results

In this section, we discuss (1) how our results on central bank bailouts relate to other types of interventions, (2) how our results relate to the Bagehot rule, and (3) the importance of several of our assumptions for our results.

[15]For more on how the desire by policymakers to mitigate the effects of a run can lead to dynamic inconsistency and reinforce runs as equilibrium phenomena, see Ennis and Keister (2009).

5.1 Alternative Interventions

While our discussion so far has focused on liquidity injections by the central bank, the same logic applies if the bailout is done by a fiscal authority. Intuitively, when government finances decline and the government collects less money from the private sector, it has to collect more money when it sells its T goods in order to balance its budget. More generous loans to banks thus require a higher long-run price level in period 2, which can in turn lead to a higher price level in period 1.

Under a fiscal bailout, the government provides resources to banks in exchange for some future promise of payment. This was the logic behind the Troubled Asset Relief Program (TARP) during the financial crisis that started in 2007, which empowered the U.S. Treasury to purchase preferred shares from banks. These shares came with a promised rate of return and were junior to demand deposits but senior to equity, meaning that the government was repaid before equity holders.

Assuming that only the central bank can issue cash, a fiscal bailout requires the government to raise funds by selling government bonds. Specifically, the government sells bonds to patient households in exchange for the cash that households withdraw from banks. The money that patient households withdraw is then absorbed by the government in period 1 rather than held by the private sector. Patient households can save outside of the banking system by buying bonds rather than by storing cash. Beyond this difference, a fiscal intervention is equivalent to a central bank lending money to banks. As long as the promised payment on preferred shares is high enough, the bailout does not increase P_1, similarly to how a sufficiently high interest rate on loans to banks ensures price stability in our framework.

We can interpret a fiscal bailout as giving cash to banks to pay off depositors while simultaneously removing the newly generated cash from the economy by issuing debt. If the central bank owns government debt, it can similarly lend to banks and then sell its debt holdings to soak up the newly generated cash. This is known as a sterilized intervention. By contrast, creating money to lend to banks that is then held by the public as in our model corresponds to an unsterilized intervention. A fiscal bailout and a sterilized intervention are therefore equivalent in our model. Since patient households in our model are happy to store money under the mattress, they effectively sterilize the monetary intervention themselves by removing it from active circulation.[16]

An alternative to sterilization is for the fiscal authority to offset any profits the central bank earns on its loans by cutting taxes or rebating any profits it earns to households. Such policies are considered by Antinolfi, Huybens and Keister (2001) and Robatto (2019). In our framework there are no taxes. However, we can allow the government to sell a smaller amount of goods and give some of its endowment to households as a form of negative taxes. Suppose that if a bailout is profitable, the government commits to give households the real value of the the central bank's profits, i.e., $\frac{i(1-\lambda)\mu d^*}{P_2}$. This leaves the government with $T - \frac{i(1-\lambda)\mu d^*}{P_2}$ to sell in the regular market. Patient households would buy these goods with the $M - i(1-\lambda)\mu d^*$ cash they hold. It follows that

$$P_2 = \frac{M - i(1-\lambda)\mu d^*}{T - i(1-\lambda)\mu d^* / P_2}. \tag{20}$$

Solving for P_2 reveals that $P_2 = \frac{M}{T}$. A commitment to remit central bank profits to households thus ensures long-run price stability. It is the lack of commitment to offset central bank profits that allows a bailout to change the long-run price level. In that case, a sufficiently high interest rate that limits central bank losses is necessary to prevent both short-run and long-run price levels from rising.

5.2 Relationship to the Bagehot Rule

Our results on the consequences of different interest rates on loans are reminiscent of the Bagehot (1873) rule, which states that the central bank should lend freely to banks during a crisis at a high rate of interest. Bagehot argues that central banks should charge high rates during a crisis to discourage unprofitable borrowing and screen risky borrowers to reduce losses for the central bank. An alternative justification is provided by Goodhart (1999), who argues that the purpose of discouraging borrowing is to avoid an excessive expansion of the money stock. Bignon, Flandreau and Ugolini (2012) provide a related explanation that focuses on stabilizing the exchange rate.

Our analysis offers two reasons to charge banks a high interest rate. First, not

[16] Humphrey (1989) discusses the importance of sterilizing bank bailouts, and attributes the idea to Thorton (1802) and Bagehot (1873).

charging too low an interest rate helps to avoid an expansion of the money stock that would lead to higher prices. This works by draining enough of the initial liquidity injection rather than by discouraging banks from borrowing. Second, charging a high interest rate robustly ensures ex-ante efficiency even if there is a run. However, our model also suggests that setting a high interest rate can encourage runs even as it mitigates the damage a run inflicts.

5.3 Discussion of Selected Assumptions

Storage of goods by banks. We assume banks cannot store goods, effectively restricting the set of investments they can undertake. However, since $P_0 = P_1$ in the efficient equilibrium, the return on buying goods at the end of period 0 and storing them is the same as the return on holding money. Thus, our restriction does not prevent banks from undertaking investments that would net them higher returns than they can already earn. Rather, our assumption ensures that any money that households withdraw in period 1 remains in the private sector and does not flow back to banks. Since banks hold just enough money to meet their obligations, this uniquely pins down the amount of money households have at the end of period 0 and thus the amount of money that impatient households trade for goods in period 1. If we let banks buy goods after making their investments, there could be additional equilibria in which banks hold some money in period 1 and the amount traded for goods in period 1 is different. Preventing banks from storing goods essentially serves as an equilibrium selection to avoid such indeterminacy. This indeterminacy is orthogonal to whether bailouts must be inflationary: regardless of the price level in periods 0 and 1, the central bank can keep the price level in period 1 unchanged even if there is a run by setting the interest rate on loans appropriately.

While the main reason for restricting banks to holding money rather than goods is to ensure a unique price level, we also view this assumption as plausible. In practice, banks engage in financial transactions rather than in the production or distribution of goods. Non-financial firms rather than banks would in reality store the goods between periods 0 and 1 needed to meet the demand for goods in period 1 by impatient households or operate any real long-run investments. Banks only serve as intermediaries. Allowing banks to either hold money or undertake long-run investments is a shortcut to avoid modeling how all real activity is undertaken by the non-financial sector and banks only shuttle cash payments between agents (and possibly monitor borrowers).

Private solutions to bank runs. Given our focus on the potential tradeoff between price stability and lending of last resort, we have abstracted from alternative policies that can be used to prevent inefficient liquidation during a run, e.g., suspension of convertibility or deposit insurance. However, in principle, there can be private sector solutions as well. Here, we briefly comment on these possibilities and their connection to our model.

First, households who want cash could potentially sell their bank claims to other households, potentially eliminating the need for banks to liquidate projects to raise cash. We rule out this possibility by assuming that bank contracts are exclusive and cannot be traded. Allowing for trade may mitigate costly liquidation, but Jacklin (1987) shows that allowing households to trade bank contracts limits the ability of banks to share risks. Maintaining ex-ante efficiency requires that contracts be exclusive.

Second, patient households could potentially lend their cash to banks directly instead of the central bank. Bailouts do not necessarily require government intervention and can in principle be done privately, as was in fact the practice before the rise of central banks. Such bailouts do not involve the creation of new money, in contrast to the case of loans made by the central bank. However, such bailouts may require potential lenders to coordinate, which may be difficult to implement. Nevertheless, our focus on central bank loans should not be interpreted to mean that addressing bank runs requires public rather than private intervention.

6 Conclusion

This paper investigated the view advanced by some policymakers and researchers that there is an inherent conflict between the central bank's role as a guarantor of price stability and its role as lender of last resort during a credit crunch. Our setup allows these goals to conflict, but suggests that the conflict is not inevitable as some of the discussion suggests. The terms at which the central bank lends during a bank run affect what households that run on a bank do with the cash they withdraw, which determines what happens to the price level following the bailout. If the central bank avoids setting a rate that is too low, lending to banks should not lead to higher prices. Our analysis further suggests that by setting a high rate, the central bank can ensure both short-run price stability and ex-ante efficiency even in the case of a run.

We conclude with two observations. First, in our setup, the various ways we discussed to prevent inefficient liquidation during a run such as fiscal bailouts, sterilized interventions, and bailouts offset by fiscal interventions are all equivalent. However, there may be reasons outside of our model to prefer one approach to another. These include the ease of communicating policy to the public or features that our model ignores that can break the equivalence between these different policies. These issues are something for future work to explore.

Second, while our results suggest that lender-of-last-resort loans do not necessarily conflict with price stability, they do not speak to broader potential conflicts between price stability and financial stability. For example, our model only considers runs due to coordination failures rather than those driven by fundamentals, e.g., a shock to the fraction of impatient households. As another example, while we show that bailouts need not threaten price stability, policies intended to ensure price stability may lead to runs. A move by the central bank to raise the short-term interest rate in order to dampen economic activity and lower inflation may encourage depositors to run by raising concerns about a bank's net worth, as appears to have occurred with the run on Silicon Valley Bank. Exploring the connection between policies that promote price stability and the possibility of runs is left for future work.

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A Appendix

Proof of Lemma 1. In optimum the budget constraint (2) is clearly binding. If the feasibility constraint (3) is binding, then $c_1 = 1/\lambda$, and by (2), $c_2 = T/(1-\lambda)$. If it were the case that $T \leq (1-\lambda)/\lambda$, then $c_1 \geq c_2$, which, by the concavity of $u(\cdot)$, would violate the first-order condition of Equation (1). Furthermore, there exists a unique $T^* > (1-\lambda)/\lambda$ for which the first-order conditions hold when the feasibility constraint is binding. It follows that for any $T < T^*$, the solution to the social planner's problem is interior. As the objective is concave, the solution is unique and given by the solution of Condition (4).

Next, we show that $1 < \frac{c_2^*}{c_1^*} < 1 + R$ and $c_1^* > 1$. The inequality $1 < \frac{c_2^*}{c_1^*}$ follows immediately from the concavity of $u(\cdot)$ and Equation (4). Assume by way of contradiction that $\frac{c_2}{c_1} \geq 1 + R$. Then,

$$\begin{aligned}
(1+R)u'(c_2) &\leq (1+R)u'((1+R)c_1) \\
&= u'(c_1) + \frac{1}{c_1}\int_{c_1}^{(1+R)c_1} \frac{\partial}{\partial c}\left(cu'(c)\right) dc \\
&= u'(c_1) + \frac{1}{c_1}\int_{c_1}^{(1+R)c_1} \left(u'(c) + cu''(c)\right) dc \\
&< u'(c_1),
\end{aligned}$$

where the first inequality follows from the concavity of u and the second inequality follows from the assumption that the relative risk aversion coefficient is strictly greater than unity. Thus, the FOC does not hold, a contradiction. Therefore, $\frac{c_2^*}{c_1^*} < 1 + R$.

Finally, if it were the case that $c_1 \leq 1$, then by (2), $c_2 \geq 1 + R + T/(1-\lambda)$. This, in turn, would imply that $\frac{c_2}{c_1} > 1 + R$, a contradiction. Thus, $c_1^* > 1$. ■

Proof of Proposition 1. We begin by fully characterizing the equilibrium candidate derived in the main text. We then verify that this is indeed an equilibrium. Finally, we show that there is no equilibrium that implements the efficient allocation with $\mu > 0$.

The market-clearing condition in the goods market in period 1 implies that banks do not liquidate, i.e., $\ell = 0$. Moreover, market clearing in the money market implies that the cash holding of banks at the end of periods 0 and 1 are given, respectively, by $m_0^B = M - m_0^*$ and $m_1^b = 0$.

Satisfying Condition (10) requires that $\frac{M}{T} \leq \frac{M}{1-\lambda}$. This implies that for an equilibrium that implements the ex-ante efficient allocation to exist, we need

$$T \geq 1 - \lambda.$$

Next, we show that Condition (12) holds if T is not too large. From the solution of the social planner's problem, we have that $c_1^*(T) < c_2^*(T)$ for any T, where $c_i^*(T)$

denotes the solution of the social planner's problem as a function of T. Suppose that $T = 1 - \lambda$. Then $T = 1 - \lambda < (1-\lambda)\frac{c_2^*(T)}{c_1^*(T)}$. The social planner's problem is continuous in T, implying that $c_1^*(T), c_2^*(T)$ are also continuous in T. By continuity, there exists a maximal $T^{**} \in (1-\lambda, T^*]$ such that $T \leq (1-\lambda)\frac{c_2^*(T)}{c_1^*(T)}$ for all $T \in [1-\lambda, T^{**}]$. Thus, Condition (12) is satisfied if $T \leq T^{**}$.

Finally, we need to verify that the equilibrium is consistent with households and banks acting optimally. Since $P_0^* = P_1^*$, households are indifferent between storing cash and goods between periods 0 and 1. Since $P_2^* \leq P_1^*$, patient households would rather store cash in period 1. Finally, since Condition (12) holds, patient households do not want to withdraw early.

Next, consider the banks. Since there is no need to trade in the liquidation market, we have that $P_1^L = P_1^* = P_0$. Since $\kappa > 0$ this implies that liquidating investments is dominated by holding cash. Banks are already providing households with the maximal expected utility, and so there is no better contract a bank could offer to steal depositors away. Moreover, Condition (12) implies that $\frac{P_0^*}{P_2^*} < 1 + R$. Thus, a bank maximizes the value of its equity by investing all the money it does not need to pay out deposits.

We now show that for $T < T^{**}$ the efficient allocation cannot be implemented by some other equilibrium in which $\mu > 0$. Suppose by way of contradiction that there were an equilibrium that implemented the efficient allocation in which $\mu > 0$. For impatient households to obtain the efficient allocation, their budget in period 1 must equal $P_1 c_1^*$. If it is optimal for patient households to withdraw early, then it must be the case that $\frac{P_1 c_1^*}{P_2} \geq c_2^*$. Since the equilibrium implements the efficient allocation, patient households must consume c_2^*, and so

$$\frac{P_1 c_1^*}{P_2} = c_2^*. \tag{21}$$

As households hold both cash and goods between periods 0 and 1, we must have $P_0 = P_1$. In combination with the fact that there is no liquidation and banks cannot store goods, under such an equilibrium we must have that

$$\lambda d + \mu(1-\lambda)d = M - m_0. \tag{6a}$$

By an analogous argument to the one used in the main text, such an equilibrium would be given by the solution of (6a), (7), (9), and (13) and the added constraint (21). Solving this system of equations yields

$$\mu = \frac{c_2^*(1-\lambda) - c_1^* T}{(c_1^*-1)c_2^*(\lambda-1)}.$$

Since $c_2^* > c_1^* > 1$ and $\lambda \in (0,1)$, we can get $\mu > 0$ only if $c_2^*(1-\lambda) < c_1^* T$ or

$$\frac{c_2^*}{c_1^*}(1-\lambda) < T.$$

However, by the definition of T^{**} this inequality does not hold for any $T < T^{**}$. ■

Proof of Proposition 2. If μ patient households demand their deposit, then banks must raise $\mu(1-\lambda)d^*$ units of cash by liquidating assets. Proposition 3 below shows that the price in the liquidation market is at most P_1^*. Therefore, an upper bound on what banks can raise by liquidation is $(1-s_0^*)(1-\kappa)P_1^*$. A bank fails if

$$(1-s_0^*)(1-\kappa)P_1^* < \mu(1-\lambda)d^*.$$

Define $\overline{\kappa}(\mu)$ as the solution to

$$(1-s_0^*)(1-\kappa(\mu))P_1^* = \mu(1-\lambda)d^*.$$

Clearly, $\overline{\kappa}(\mu) < 1$. ■

Proof of Lemma 2. Suppose by way of contradiction that $P_1^L(\mu) > P_1(\mu)$. In this case, no household are willing to buy goods in the liquidation market. Since banks must sell goods in the liquidation market in order to raise cash, the liquidation market would not clear. ■

Proof of Proposition 3. In period 2, households spend all available money to buy goods from the government, implying that $P_2(\mu) = \frac{M}{T}$.

Suppose that $P_1(\mu) \geq P_1^*$. By Proposition 1, we know that $P_1^* = \frac{M}{1-\lambda} \geq \frac{M}{T} = P_2^*$.

If $T > 1 - \lambda$, then $P_1(\mu) \geq P_1^* > P_2(\mu)$ and only impatient households buy goods in period 1. That is, patient households strictly prefer to store money in period 1.

We now consider two different cases, depending on whether $P_1^L(\mu) = P_1(\mu)$ or $P_1^L(\mu) < P_1(\mu)$ (by Lemma 2, these are the only two possible cases). If $P_1^L(\mu) = P_1(\mu)$, the price level that clears both markets (the liquidation market and the regular market) in period 1 is given by

$$P_1(\mu) = \frac{\lambda(\tilde{d} + m_0)}{(1-\lambda)s_0^* + (1-\kappa)\mu(1-\lambda)\ell}$$

where $\tilde{d}$ is the deposit paid out to households, and $\ell > 0$ is the amount of assets liquidated by banks. Note that $\ell > 0$ as banks do not hold excess cash reserves and therefore must liquidate some assets whenever $\mu > 0$. Recall that $P_1^* = \frac{\lambda(d^* + m_0^*)}{(1-\lambda)s_0^*}$. Since $\tilde{d} \leq d^*$ and $\ell > 0$, it follows that $P_1(\mu) < P_1^*$, in contradiction to the assumption that $P_1(\mu) \geq P_1^*$.

Next, suppose that $P_1^L(\mu) < P_1(\mu)$. All households spend all of their cash in the liquidation market (since there is an "arbitrage" between the liquidation and regular markets) to buy the $(1-\kappa)\ell$ goods supplied by the banks. In the regular market, patient households sell both the goods that they stored and the goods that they purchased in the liquidation market (since $P_1(\mu) > P_2$). Furthermore, impatient households purchase goods with only $\tilde{d}$ units of cash (as they spend their stored cash in the liquidation market). It follows that the market-clearing price in the regular market is $P_1(\mu) = \frac{\tilde{d}}{(1-\lambda)(s_0^* + (1-\kappa)\ell)} < P_1^*$. This contradicts the assumption that $P_1(\mu) \geq P_1^*$.

Finally, were it the case that $P_1(\mu) < \frac{M}{T}$, no household would hold money at the end of period 1, which cannot be an equilibrium. This ensures that $P_1(\mu) \geq \frac{M}{T}$.

If $T = 1 - \lambda$, we can use the above arguments to establish that $\frac{M}{T} \leq P_1(\mu) \leq P_1^* = \frac{M}{1-\lambda} = \frac{M}{T}$. Hence, if $T = 1 - \lambda$, the equilibrium price $P_1(\mu) = \frac{M}{T}$, as claimed. ■

Proof of Lemma 3. Since impatient households spend money to buy goods in period 1, patient households must hold a positive amount of money at the end of period 1, i.e., $m_1 > 0$. Patient households can choose whether to spend their money holdings m_1 to buy goods from banks at a price $P_2^L(\mu)$ in the preliminary market or from the government at a price $P_2(\mu)$ in the regular market. If $P_2^L(\mu) \neq P_2(\mu)$, households only buy goods in the market where goods are cheaper. That is, there exists one market in which demand for goods is zero but supply is positive: the government sells T goods, while banks need to sell goods whenever their obligation to the central bank $(1+i)\mu(1-\lambda)d^*$ is strictly positive. The latter expression is strictly positive since $i > -1$.

Finally, the market-clearing price in period 2 is given by $P_2(\mu) = \frac{M - i\mu(1-\lambda)d^*}{T}$ as the government sells its endowment of T against the outstanding cash in circulation $M - i\mu(1-\lambda)d^*$.

Proof of Proposition 4. Part 1: Consider the case where $i \geq i^\dagger(\mu)$. Since banks do not liquidate investments or fail, impatient households receive their full deposit. Hence, their demand for goods in period 1 is given by $\frac{d^* + m_0}{P_1(i,\mu)}$, where $P_1(i,\mu)$ is the period-1 price when μ patient households withdraw early, and the central bank sets the interest rate at i. The amount of goods that impatient households purchase is at most $s_0^*(1-\lambda)$. It follows that the market-clearing price is at least $\frac{d^* + m_0}{s_0^*(1-\lambda)}$.

If $i > i^\dagger(\mu)$, then, by the definition of $i^\dagger$, it holds that $P_1(i,\mu) > P_2(i,\mu)$, and so patient households want to sell the goods stored in period 1 and hold cash, regardless of whether they withdrew their deposit. It follows that the market-clearing price must be

$$P_1(i,\mu) = \frac{d^* + m_0}{s_0^*(1-\lambda)} = P_1^*.$$

If, on the other hand, $i = i^\dagger(\mu)$, then patient households are indifferent between storing goods and money. However, should a strictly positive measure of them choose to store goods, the market-clearing price in period 1 exceeds P_1^*, which, by the definition of $i^\dagger(\mu)$, implies that patient households strictly prefer to store money.

Part 2: Next, consider the case where $i < i^\dagger(\mu)$. By the definition of $i^\dagger$, in this case $P_2(i,\mu) > \frac{M}{1-\lambda}$. For the cash market to clear in period 1, it must be the case that $P_1(i,\mu) \geq P_2(i,\mu)$, as otherwise no one holds cash. If $P_1(i,\mu) > P_2(i,\mu)$, only impatient households want to buy goods, setting the market-clearing price to P_1^*, which, in this case, is below $P_2(i,\mu)$. Hence, clearing the market requires that $P_1(i,\mu) = P_2(i,\mu) > \frac{M}{1-\lambda}$. ∎

Proof of Proposition 5. Consider the case where $i < i^{\dagger}(\mu)$. In this case, by Proposition 4, the price levels are the same in both periods, and, moreover, the price levels are decreasing in i. Hence, the consumption of impatient households and patient households that withdraw early is increasing in i. Consequently, the consumption of patient households that do not withdraw early is decreasing in i.

Next, consider the case where $i \geq i^{\dagger}(\mu)$. By Proposition 4, we know that $P_1(\mu) = P_1^*$ and $P_2(\mu) = \frac{M - i\mu(1-\lambda)d}{T}$. Therefore, in this case, increasing the interest rate does not change the utility of impatient households. This establishes part 1 of the proposition.

From Proposition 4 it follows that, in this case, patient households weakly prefer storing cash to storing goods in period 1. Hence, to compare the utilities of patient households that withdraw early and those that do not, it is sufficient to compare the nominal value of the assets that each type of household has in period 2. Households that withdraw early have $d^* + m_0^* + P_1^* s_0^*$, whereas those that wait have $\pi(\mu, i) + m_0^* + P_1^* s_0^*$, where $\pi(\mu, i)$ represents the nominal value of an equity share in a bank.

The value of a bank's stored goods is

$$(1 - s_0^*)(1+R)P_2(i,\mu) = (1 - s_0^*)(1+R)\frac{M - i(1-\lambda)\mu d^*}{T}.$$

As it must also repay the loan, its overall equity is worth

$$(1 - s_0^*)(1+R)\frac{M - i(1-\lambda)\mu d^*}{T} - (1+i)(1-\lambda)\mu d^*.$$

Hence, patient households that wait are better off than those that withdraw early if and only if $\pi(\mu, i) > d^*$. Formally,

$$\frac{(1 - s_0^*)(1+R)\frac{M - i(1-\lambda)\mu d^*}{T} - (1+i)(1-\lambda)\mu d^*}{(1-\mu)(1-\lambda)} > M\frac{c_1^* - 1}{1-\lambda}.$$

The left-hand side of this expression is decreasing in i, whereas the right-hand side is constant in i. Since there is no liquidation and impatient households consume c_1^* for any interest level in this case, patient households consume $(1-\lambda)c_2^*$ in aggregate. The amount consumed by patient households that withdraw is proportional to the ratio of their net value to the net value of all patient households. It follows that as i increases patient households that withdraw consume more, whereas those that do not withdraw consume less. This establishes part 2 of the proposition.

To establish part 3 of the proposition, note that if $i = 0$, then by Proposition 4 the price levels in both periods are the same as in the efficient equilibrium. Hence, patient households that withdraw early consume $c_1^* \frac{P_1^*}{P_2(\mu)}$. If $T < T^{**}$, this level of consumption is strictly less than c_2^*. That is, if $i = 0$ the consumption of patient households that withdraw early is less than their consumption in the efficient equilibrium. Since there is no liquidation and the consumption of impatient households remains the same, it follows that for $i = 0$ the nominal value of an equity share is greater than the nominal value of a deposit. Since the former is linear and decreasing in i, whereas the latter is independent of i, it follows that there exists $i^{\dagger\dagger}(\mu) > 0$ for which the nominal value of a deposit is exactly equal to the nominal value of an equity share. In such a case, patient households' consumption does not depend on their time of withdrawal. ■

Proof of Theorem 1. The only part of this result that does not follow from our previous results is part (4). By Proposition 5, the utility of households that withdraw early is lower than those that wait for $i < i^{\dagger\dagger}(\mu)$. Hence, if $i < i^{\dagger\dagger}(\mu)$, there cannot be a continuation equilibrium in which $\mu > 0$. If $i = i^{\dagger\dagger}(\mu) \leq \bar{\imath}(\mu)$, then for the equilibrium prices given μ and the intervention, patient households are indifferent between withdrawing early and waiting. This confirms that μ is a continuation equilibrium. ■

B Online Appendix

In this appendix we formally derive the optimization problems of the banks and households, as well as the equilibrium conditions.

Banks' optimization problem

A (representative) bank takes as given the market prices $\mathbb{P} = \langle P_0, P_1^L, P_1, P_2 \rangle$. Its objective is to provide the contract that maximizes the expected utility its depositors can attain (in combination with the depositors' own trades). In this section, we derive the feasible contracts $\langle d, \pi \rangle$ a bank can offer, where d is the money households receive if they withdraw early and π is the expected real value of an equity share in the bank they receive if they wait. We then analyze the households' problem, and derive their preferences over contracts of the form $\langle d, \pi \rangle$ given prices $\mathbb{P}$. Note that a bank's optimization problem depends on its beliefs about households' withdrawal strategies. Let η denote the measure of households the bank believes will withdraw in period 1.

A bank begins period 0 with M units of cash. In period 0, it must choose how many goods to purchase. We denote the amount of goods the bank purchases and invests in period 0 by z.

In period 1, a bank chooses how many goods to liquidate and sell in the liquidation market. Denote the cash holdings of the bank at the end of period 0 by m_0^B , and recall that the amount of goods the bank liquidates in period 1 is denoted by ℓ. The bank's budget constraint in period 0 is

$$m_0^B = M - P_0 z. \tag{BB}$$

Fix $\mathbb{P}$ and η. To specify what contracts are feasible, we must enable a bank to sell liquidated investments in the regular market in period 1. We denote the amount of investments a bank liquidates and sells in this market by x. A contract $\langle d, \pi \rangle$ is feasible if there exist $z \in [0, \frac{M}{P_0}]$ and $\ell, x \in [0, z]$, such that

$$x + \ell \leq z, \tag{Feasibility 1}$$

$$\eta d \leq m_0^B + P_1^L \ell (1-\kappa), \qquad \text{(Feasibility 2)}$$

$$(1-\eta)\pi = (1+R)(z-\ell) + \frac{m_0^B + P_1^L(1-\kappa)\ell + P_1(1-\kappa)x - \eta d}{P_2}. \qquad \text{(Feasibility 3)}$$

The first constraint states that the bank doesn't liquidate more goods than it invested. The second constraint states that the bank has enough cash to pay the η households that demand their deposits. The third constraint equates the aggregate value of the equity shares to the real value of the bank's equity. If the demand for deposits is $\tilde{\eta} > \eta$, we assume that the bank must either choose ℓ such that $\tilde{\eta} d \leq m_0^B + P_1^L(1-\kappa)\ell$ or set $\ell = z$. That is, the bank must continue to liquidate as long as its cash reserve falls short of the amount it is obligated to pay households that withdraw early.

Next, we show how a bank will maximize its equity value for a given set of prices. We begin with how the bank obtains the cash to pay out deposits. The nominal cost of carrying cash from period 0 to period 1 is 1. Alternatively, the bank can purchase a good, invest it in period 0, and sell it in period 1. If the bank does so, it will sell it in the market in which the price of goods is higher. Let $\overline{P}_1 = \max\{P_1^L, P_1\}$. The cost of the latter option is $\frac{P_0}{\overline{P}_1(1-\kappa)}$. Thus, if $(1-\kappa)\frac{\overline{P}_1}{P_0} < 1$, the bank will store money to pay deposits, while if $(1-\kappa)\frac{\overline{P}_1}{P_0} > 1$, the bank will not store money at the end of period 0.

Second, consider how the bank obtains the promised equity π. There are three ways in which the bank can generate real goods in period 2. (i) The bank can purchase goods in period 0, invest them, and wait for the investment to mature. The nominal time-0 cost of obtaining a (marginal) unit of goods in period 2 in this way is $\frac{P_0}{1+R}$. (ii) The bank can hold cash from period 0 to period 2. The nominal time-0 cost of obtaining a (marginal) unit of goods in period 2 in this way is P_2. (iii) The bank can purchase goods in period 0, invest them, liquidate them, and give the cash to the households. The nominal time-0 cost of obtaining a (marginal) unit of goods in period 2 in this way is $\frac{P_0}{\overline{P}_1(1-\kappa)}P_2$. The bank will implement the feasible contract by choosing the cheapest of these three options.

Households' optimization problem

The households take $\mathbb{P}$ as given as well as the contract offered by their bank: namely, a nominal deposit d, and the expected real value of their equity share (in real goods) π.

Let $\theta = \langle \mathbb{P}, d, \pi \rangle$ denote the state variables that are relevant to a household's decisions. It is convenient to describe and solve households' utility maximization problem using backward induction, starting from period 2.

In period 2, households consume all the goods that they have stored or received from banks and use all of their cash to purchase goods from the government. The household's period-2 budget constraint is

$$m_1 + P_2(\pi \cdot \mathbb{1}_{equity} + s_1) = P_2 c_2, \tag{BC_2}$$

where m_1 (s_1) is the cash (real good) holding of a patient household at the end of period 1, and $\mathbb{1}_{equity}$ is an indicator for whether the household has an equity share of a bank. Hence, a patient household's utility in period 2 is

$$V_2^p(m_1, s_1; \theta) = u(\pi \cdot \mathbb{1}_{equity} + s_1 + \frac{m_1}{P_2}).$$

Next, consider period 1 and assume that there is no need to trade in the liquidation market. A household that learns it is impatient will withdraw its deposit, use all of its cash to purchase goods, and consume all of its goods (stored or purchased). An impatient household's period-1 budget constraint is

$$m_0 + d + P_1 s_0 = P_1 c_1, \tag{$BC_1 - impatient$}$$

and so such a household has the following continuation utility:

$$V_1^I(m_0, s_0; \theta) = u\left(s_0 + \frac{d + m_0}{P_1}\right).$$

A household that learns that it is patient must decide how to allocate its resources between goods and cash. In particular, such a household's continuation utility, given its choice of whether to withdraw or not, is given by the solution to the following maximization problem:

$$W(m_0, s_0; \theta) = \max_{m_1, s_1} V_2^p(m_1, s_1; \theta)$$

s.t.

$$P_1 s_1 + m_1 = P_1 s_0 + m_0 + (1 - \mathbb{1}_{equity}) d. \qquad (BC_1 - patient)$$

Denote the values of m_1 and s_1 that solve this problem by $m^*(m_0, s_0, \mathbb{1}_{equity}; \theta)$ and $s^*(m_0, s_0, \mathbb{1}_{equity}; \theta)$, respectively.

This solution is "bang-bang:" if $P_1 < P_2$ the household stores only goods ($m^* = 0$) and if $P_1 > P_2$ the household stores only cash ($s^* = 0$). Hence, the value of a patient household in period 1, as a function of its resources at the end of period 0, is equal to

$$V_1^p(m_0, s_0; \theta) = \max\{W(m_0 + d, s_0; \theta), V_2^p\,(m^*(m_0, s_0; \theta), s^*(m_0, s_0; \theta) + \pi; \theta)\}.$$

Note that a patient household withdraws if

$$V_1^p(m_0 + d, s_0; \theta) > V_2^p(m^*(m_0, s_0; \theta), s^*(m_0, s_0; \theta) + \pi; \theta).$$

In period 0, the household's problem boils down to

$$\max_{m_0, s_0} \lambda V_1^I(m_0 + d, s_0; \theta) + (1 - \lambda) V_1^p(m_0, s_0; \theta),$$

$$\text{s.t. } P_0 s_0 + m_0 = P_0. \qquad (BC_0)$$

Note that since there is another market in period 1, the solution of this problem is again "bang-bang:"if $P_0 < P_1$ the household stores only goods ($m_0 = 0$), whereas if $P_0 > P_1$ the household stores only cash ($s_0 = 0$).

Finally, consider the household's problem should there be a need to trade in the liquidation market. The household's budget constraint in this case is given by

$$\text{s.t. } P_1^L s_1^L + m_1^L = P_1^L s_0 + m_0, \qquad (BC_1 - liq)$$

where m_1^L and s_1^L are, respectively, the household's holdings of cash and goods when the liquidation market closes. Since the liquidation market is followed by a regular market, the objective of the household in the liquidation market is to maximize its net wealth in the regular market, regardless of its type. Hence, if $P_1^L < P_1$ the household stores only goods ($m_1^L = 0$), whereas if $P_1^L > P_1$ the household stores only cash ($s_1^L = 0$).

Equilibrium

The following components must be specified by an equilibrium:

- A price vector: $\langle P_0, P_1^L(\mu), P_1(\mu), P_2 \rangle$.
- A strategy for the bank: $\langle m_B, d, z, \ell(\mu), x(\mu) \rangle$.
- A strategy for the households $\langle m_0, s_0, m_1(\mathbb{1}_{equity}, \mu), s_1(\mathbb{1}_{equity}, \mu), m_1^L(\mu), s_1^L(\mu) \rangle$, where μ represents the probability with which patient households choose to withdraw in period 1, and the proportion of households that choose to do so;

these components constitute an equilibrium if the following conditions hold:

- The induced contract $\langle d, \pi \rangle$ is feasible and maximizes the households' expected utility over all feasible contracts.
- The banks' strategy is optimal given the prices.
- The households' strategy is optimal given the prices.
- All budget constraints are satisfied.
- The goods market clears in every period, that is,

$$s_0 + z = 1, \tag{MC_0}$$

$$s_1^L(\mu) = s_0 + (1-\kappa)\ell(\mu), \tag{MC_1^L}$$

$$\lambda c_1(\mu) + (1-\lambda)\mu s_1(0,\mu) + (1-\lambda)(1-\mu)s_1(1,\mu) = s_0 + (1-\kappa)(\ell(\mu) + x(\mu)), \tag{MC_1}$$

$$\begin{aligned}(1-\lambda)\mu c_2(0,\mu) + (1-\lambda)(1-\mu)c_2(1,\mu) = \\ (z - \ell(\mu) - x(\mu))(1+R) + T + (1-\lambda)\mu s_1(0,\mu) + (1-\lambda)(1-\mu)s_1(1,\mu).\end{aligned} \tag{MC_2}$$