AN ALMOST (BUT NOT QUITE) NAÏVE SEMANTICS FOR COMPARATIVES

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1. THE ALMOST (BUT NOT QUITE) NAÏVE SEMANTICS FOR DP COMPARATIVES

1.1. NAIVE THEORY OF MEASURES:
Base semantics on a 'naïve' ontology of degrees and measures: the one used in the sciences.
- measure scales based on the set of real numbers, equipped with order, supremums and infimums, and arithmetic.
- measure functions, like height in inches which assign to one individuals (in a world at a time at a place,….) one height in inches.

- No measure relations in which I am many heights simultaneously (Heim and others)
- No extents of Tallness and anti-extents of Shortness (von Stechow, Kennedy)
- No conceptual reconstructions of scales and measures (Kamp, McConnel-Ginet,Klein,…)

1.2. NAÏVE SEMANTICS OF MEASURES:
- Fred is taller than Susan if there is a difference in height between them, in Fred's favor.
- Fred is tall if there is a difference in height between Fred's height and a contextual standard, in Fred's favor.
Following von Stechow: adjective tall-Ø and comparative tall-er are not defined in terms of each other, both are defined in terms of dimension tall. (Against Bartsch and Vennemann, Kamp, Klein,…)

1.3. NOT QUITE MEANS: SEMANTICS NEED NOT BE NAÏVE.

The Principle BPR: (Bach, Partee, Rooth):
Interpret everything as low as you can, but not so low that you will regret it later.

Ideal semantics for degree phrases:
- three denotes 3.
- Keep the denotations of degree expressions at the level of degrees, predicates of degrees, etc. for as long as you can.
Example:
We want to define the meaning of *very* in *very tall* as an operation on the degree meaning of *tall*:

**Degree meaning:** The set of degrees higher than the standard for minimal Tallness.

**Adjectival meaning:** The set of individuals who are Tall in the context.

This requires type shifting principles, most importantly:

Let \( d \) be the type of individuals, \( t \) of truth values, \( \delta \) of degrees.

Let \( \alpha \) be a predicate of degrees of type \( <\delta,t> \).

Let \( M \) be a measure function (with world parameter specified) of type \( <d,\delta> \).

**TYPE SHIFT:** Compose with measure function

\[
\alpha \rightarrow \alpha \circ M \quad \text{(where } \alpha \circ M = \lambda x.\alpha(M(x)))
\]

\[
<\delta,t> \quad <d,t>
\]

\[
\lambda \delta. \delta > 3 \quad \rightarrow \quad \lambda x. M(x) > 3
\]

The set of degrees bigger than three  
The set of individuals with measure bigger than three

1.4. MEASURE ONTOLOGY

\( r \) is the type of (real) **numbers.** \( R \)

\( m \) is the type of **measures.** **Primitives:** \( H(eight) \), Age, …

\( u \) is the type of **measure units.** **Primitives:** \( m(eters) \), "(inches), …

\( \delta \) is the type of **degrees.**

A degree is a triple \( <r,u,m> \) where \( r \) is a real number, \( u \) a unit, \( m \) a measure, and \( u \) is an appropriate unit for \( m \).

**General convention:**

**mnemonic superscripts** denote the relevant element of a tuple:

example: \( <29,\ " \ , H>^r = 29; \ (^r \text{ for real value}) \)

**Measure functions:**

-H- *height in inches* is a (partial) function which assigns to an index \( w \) (world, time) and an individual \( x \) a triple \( H_w(x) = <r,\ " \ , H> \), where \( r \) is a real number.

-This function is convertible into the function \( H_m \), height in meters.

-Up to convertability of units, there is, **per measure**, only one measure function.
SCALES
s is the type of scales.

Given measure M, unit u, context k.

A basic scale $S_{M,u,k}$ is a tuple:
$S_{M,u,k} = <M, D, >_M, \sqcup_M, -_M, M_u, \text{LOW}_{M,u,k}, \text{HIGH}_{M,u,k}>$ where:

1. $M$ is the measure that $S$ is based on.
2. $D = \{<r,u,M>: r \in \mathbb{R}\}$ the domain of relevant degrees.
3. $>_M$ (bigger than), $\sqcup_M$ (supremum), $-_M$ (subtraction) specify the direction of the scale. These notions are lifted from $\mathbb{R}$:
   - e.g. $<p,u,M> >_M <q,u,M>$ iff $n >_R m$
   [Note: $>_M$ is lifted from $>_R$ and not $<_R$. This means that $\sqcup_M$ corresponds to $\sqcap_R$]
4. $M_u$ is the measure function.
5. $\text{LOW}_{M,u,k}, \text{HIGH}_{M,u,k} \in D$ and $\text{HIGH}_{M,u,k} >_M \text{LOW}_{M,u,k}$

Let $S_{M,u,k}$ be a basic scale.
The converse scale for $S_{M,u,k}$, $S_{M,u,k}^c$ is given by:

$S_{M,u,k}^c = <M, D, >_M^c, \sqcup_M^c, -_M^c, M_u, \text{LOW}_{M,u,k}^c, \text{HIGH}_{M,u,k}^c>$, where

1. Measure, domain, and measure function of the converse scale are the same as those of the basic scale.
2. The directional notions are the converse notions:
   - $<n,u,M> >_M^c <m,u,M>$ iff $<m,u,M> >_M <n,u,M>
   - $\sqcup_M^c = \sqcap_M$
   - $<n,u,M> -_M^c <m,u,M> = <m,u,M> -_M <n,u,M>
3. Contextual low and high are converted:
   $\text{LOW}_{M,u,k}^c = \text{HIGH}_{M,u,k}$; 5. $\text{HIGH}_{M,u,k}^c = \text{LOW}_{M,u,k}$

Also here I will use mnemonic superscripts.
The context $k$ may provide for a measure like H(eight) a default unit, which I will call $\Delta_{M,k}$, or $\Delta$ for short.
1.5. THE ALMOST (BUT NOT QUITE) SEMANTICS

1. *tall* has an interpretation as a *measure*, *short* does not.
2. *tall* and *short* have interpretations as *dimensions* (in essence, *scales*):
   
   *tall* denotes the **basic scale** of Height.
   *short* denotes the **converse scale** of Height.

3. The meaning of adjectives *tall/short* and comparatives *tall/short* are derived from their **dimensional meanings**.

4. *more* denotes the **difference function** (subtraction)
   *less* denotes the **converse of the difference function**

5. Function composition.

**Generalized function composition**

\[ f \circ g = \lambda x_n \ldots \lambda x_1. f(g(x_1, \ldots, x_n)) \]

(Bring function \(g\) down to the input type for \(f\), by applying it to variables, apply \(f\) to the result, and abstract over all the variables used.)

The heart of the semantics for comparatives is the following principle:

A one place number/degree predicate combines with a difference function to form a two place number/degree relation.

**SAMPLE DERIVATION**

**STEP 1:**

- **PRED**\(_{num}\): null-predicate of numbers \(\emptyset_{a \ bit} \), with semantic meaning *a bit* (or *some*)
  \[ \emptyset_{a \ bit} \rightarrow \lambda r. r > 0 \quad \text{(the set of positive real numbers)} \]

- **DIF**\(_{num}\): *more* and *less* denote the difference function and its converse (resp):
  *more* \[ \rightarrow \lambda m \lambda n. (n - R m) \]
  *less* \[ \rightarrow \lambda m \lambda n. (m - R n) \]

**STEP 2: COMPOSITION:** \(\text{REL}_{num}\)

\[ \emptyset_{a \ bit \ more} \rightarrow \lambda r. r > 0 \circ \lambda m \lambda n. (n - m) \]
\[ = \lambda m \lambda n. (n - m) > 0 \]
\[ = >_R \]

\[ \emptyset_{a \ bit \ less} \rightarrow \lambda r. r > 0 \circ \lambda m \lambda n. (m - n) \]
\[ = \lambda m \lambda n. (m - n) > 0 \]
\[ = <_R \]
I will assume a structure that follows the following semantic composition:

- **PRED** _dim_
- **REL** _dim_
- **MP**
- **DIM**
- **DP**

**STEP 3:** **PRED** _num_ : APPLY the meaning of the **REL** _num_ derived to 3:

- more than three → λm. m >ₕ 3
- less than three → λm. m <ₕ 3

**STEP 4:** **PRED** _unit_: LIFT the numerical predicate and " (inch) to predicates of degrees, semantically adjoin the latter to the first. (Formulation of the obvious operations omitted.):

- more than three inches → λδ.δ >ₕ 3 ∧ δₜ = "
- less than three inches → λδ.δ <ₕ 3 ∧ δₜ = "

Also: null degree predicate Ø _a bit_:

- Ø _a bit_ → λδ.δ > 0 ∧ δₜ = Δ

(A wrinkle: context or grammar must here pick the correct measure for the derivation)

**STEP 5:** **DIF** _unit_: more and less denote scale dependent functions of subtraction and its converse: (⁻ is a mnemonic superscript)

- more → λs.s⁻
- less → λs.(s⁻)⁻

The function that maps every scale onto its subtraction operation.

The function that maps every scale onto the subtraction operation of its converse scale.
**STEP 6:** DIM: *tall* and *short* are **dimensions**: in essence they denote **scales**, in practice functions from units to scales:

\[
tall \rightarrow \lambda u. S_{H,u,k}
\]
The function that maps unit \(u\) onto the **basic** scale \(S_{H,u,k}\)

\[
short \rightarrow \lambda u. S_{H,u,k}^c
\]
The function that maps unit \(u\) onto the **converse** scale \(S_{H,u,k}^c\)

**DERIVATION SKETCH:**

**STEP 7:** \(\text{REL}_\text{unit}: \text{compose} \) PRED\(_\text{unit}\) and DI\(_\text{unit}\) \((\text{more}/\text{less})\):

\(\text{REL}_\text{unit}: \text{function from scales into two-place relations between degrees}\)

**STEP 8:** **APPLY** DIM *tall*/*short* to the unit derivable from \(\text{REL}_\text{unit}\)

(form meanings \(S_{H,\cdot,k}\) or \(S_{H,\Delta,k}\) for *tall* and the converse scales for *short*)

**STEP 9:** \(\text{REL}_\text{dim}: \text{APPLY STEP 7 TO STEP 8.}\)

Result: relations between degrees of type \(<\delta,<\delta,t,>\>

After reduction:

\[
\begin{align*}
[\text{REL}_\text{dim} \text{more than three inches more tall than}] & \rightarrow \lambda \delta_2 \lambda \delta_1 \in D_{H,\cdot,\cdot}: \delta_1^r > \delta_2^r + 3 \\
[\text{REL}_\text{dim} \text{less than three inches more tall than}] & \rightarrow \lambda \delta_2 \lambda \delta_1 \in D_{H,\cdot,\cdot}: \delta_1^r < \delta_2^r + 3 \\
[\text{REL}_\text{dim} \text{more than three inches less tall than}] & \rightarrow \lambda \delta_2 \lambda \delta_1 \in D_{H,\cdot,\cdot}: \delta_1^r < \delta_2^r - 3 \\
[\text{REL}_\text{dim} \text{less than three inches less tall than}] & \rightarrow \lambda \delta_2 \lambda \delta_1 \in D_{H,\cdot,\cdot}: \delta_1^r > \delta_2^r - 3 \\
[\text{REL}_\text{dim} \text{Ø more tall than}] & \rightarrow \lambda \delta_2 \lambda \delta_1 \in D_{H,\Delta(k,h)}: \delta_1^r > \delta_2^r \\
[\text{REL}_\text{dim} \text{Ø less tall than}] & \rightarrow \lambda \delta_2 \lambda \delta_1 \in D_{H,\Delta(k,h)}: \delta_1^r < \delta_2^r
\end{align*}
\]

**FACT:** *β more short* is equivalent to *β less tall*

E.g.: at least three inches more short than = at least three inches less tall than

Relations between individuals: composition with the measure function (twice in the derivation):

(1) Fred is taller than Susan \(H-(\text{Fred})^r > H-(\text{Susan})^r\)

(2) Fred is more than three inches taller than Susan \(H-(\text{Fred})^r > H-(\text{Susan})^r + 3\)

(3) Fred is less than three inches taller than Susan \(H-(\text{Fred})^r < H-(\text{Susan})^r + 3\)

(4) Fred is shorter than Susan \(H-(\text{Fred})^r < H-(\text{Susan})^r\)

(5) Fred is more than three inches shorter than Susan \(H-(\text{Fred})^r < H-(\text{Susan})^r - 3\)

(6) Fred is less than three inches shorter than Susan \(H-(\text{Fred})^r > H-(\text{Susan})^r - 3\)

Case (3) (and 6): compare (7):

(7) A. Is John taller than Mary?  
B. I don’t know. But I do know that he is less than two centimeters taller than Mary. You see, Mary is 1.63. And I happen to know that John was rejected by the police because of his height, and they only accept people 1.65 and up.

This discourse is felicitous and compatible with John being **smaller** than Mary, supporting the interpretation given in (3).
2. PREDICTIONS FOR DP COMPARATIVES

2.1. QUANTIFICATIONAL DP COMPLEMENTS

John is taller than every girl. \( \forall y [\text{GIRL}(y) \rightarrow H_{\Lambda}(\text{John})^f > H_{\Lambda}(y)^f] \)

\[ H_{\Lambda}(g_1)^f \ldots \ldots \ldots \ldots \ldots \ldots \ldots \llap{\llap{\ldots H_{\Lambda}(g_n)^f}} \]

John is taller than the tallest girl.

John is taller than some girl. \( \exists y [\text{GIRL}(y) \land H_{\Lambda}(\text{John})^f > H_{\Lambda}(y)^f] \)

\[ H_{\Lambda}(g_1)^f \ldots \ldots \ldots \ldots \ldots \llap{\llap{\ldots H_{\Lambda}(g_n)^f}} \]

John is taller than the shortest girl.

John is taller than exactly three girls. \( [\text{GIRL} \cap \lambda y. H_{\Lambda}(\text{John})^f > H_{\Lambda}(y)^f] = 3 \)

\[ H_{\Lambda}(g_1)^f \ H_{\Lambda}(g_2)^f \ H_{\Lambda}(g_3)^f \ldots \ldots \ldots \ldots \ldots \ldots \llap{\llap{\ldots H_{\Lambda}(g_n)^f}} \]

John is taller than the shortest three girls, but not taller than any other girls.

(Many theories have problems getting this)

John is taller than no girl. \( \neg \exists y [\text{GIRL}(y) \land H_{\Lambda}(\text{John})^f > H_{\Lambda}(y)^f] \)

\[ H_{\Lambda}(g_1)^f \ldots \ldots \ldots \llap{\llap{\ldots H_{\Lambda}(g_n)^f}} \]

John's height is at most that of the shortest girl.

(Stilted, because English prefers auxiliary negation, but felicitous.)

John is at least two inches taller than every girl. \( \forall y [\text{GIRL}(y) \rightarrow H-(\text{John})^f \geq H-(y)^f + 2] \)

\[ H-(g_1)^f \ H-(g_2)^f \ H-(g_3)^f \ H-(g_4)^f \ldots \ldots \llap{\llap{\ldots H-(g_n)^f}} \]

John's height is at least the height of the tallest girl plus two inches.

John is at most two inches taller than every girl. \( \forall y [\text{GIRL}(y) \rightarrow H-(\text{John})^f \leq H-(y)^f + 2] \)

\[ H-(g_1)^f \ H-(g_2)^f \ H-(g_3)^f \ H-(g_4)^f \ldots \ldots \llap{\llap{\ldots H-(g_n)^f}} \]

John is at most the height of the shortest girl plus two inches.

Cf. the following valid inference:

a. John is at most two inches taller than every girl.
b. Mary is the shortest girl.
c. Hence, John is at most two inches taller than Mary.
2.2. DP-COMPARATIVES AND POLARITY

DP-comparatives seem to allow polarity sensitive (PS) items and seem to be downward entailing (DE):

(1) a. Mary is more famous than anyone.
   b. (1) Mary is more famous than John or Bill.
   (2) Hence, Mary is more famous than John.

Hoeksema 1982:
1. anyone in (1a) and or in (1b) allow free choice interpretations (FC).
   Hence: the facts in (1) are consequences of FC interpretations, not PS interpretations.
   (Certainly DP-comparatives allow FC-any: only FC-any can be modified by almost)
   (2) Mary is more famous than almost anyone.)

2. DP comparatives are not downward entailing: (3) is invalid:
   (3) (1) John is more famous than Mary.
   (2) Mary is a girl.
   (3) Hence, John is more famous than every girl.

3. Dutch has PS items that are not FC items, and these are not felicitous in DP comparatives.
Hoeksema: ook maar iemand is PS but not FC (cf. FC item wie ook maar):
   (4) a. Ik leen geen boeken uit aan ook maar iemand. DE context:
      I lend no books out to ook-maar-someone PS felicitous
      I don't lend books to anyone
   b. #Dat kan je ook maar iemand vragen. Modal context:
      That can you ook-maar-someone ask PS infelicitous
      That, you can ask anyone
   c. Dat kan je wie dan ook vragen. Modal context:
      That can you who-dan ook ask FC felicitous
      That, you can ask anyone

PS items are felicitous in CP comparatives but not in DP comparatives:
   (5) a. Marie is beroemder dan ook maar iemand ooit geweest is. CP comparative
      Marie is more famous than ook-maar-someone ever been is PS felicitous
      Marie is more famous than anyone has ever been.
   b. #Marie is beroemder dan ook maar iemand. DP comparative
      Marie is more famous than ook-maar-someone PS infelicitous
      Marie is more famous than anyone
   c. Marie is beroemder dan wie dan ook. DP comparative
      Marie is more famous than who-dan ook FC felicitous
      Marie is more famous than anyone

Hoeksema's claim can be strengthened by looking at stressed éénige. As a plural, not-necessarily stressed item éénige means a few, and is not at all a polarity item:
(6) Ik heb hem enige boeken uitgeleend.  
I lent him a few books.

But as a singular, stressed element, énige is a PS item, and it means any, PS any, and nor FC any:

(7) a. Ik leen geen boeken uit aan énige filosoof.  
I lend no books to any philosopher  
DE context  
PS felicitous

b. #Dat kan je énige filosoof vragen.  
That, you can ask any philosopher.  
UE modal context:  
PS infelicitous

And we find that énige is infelicitous in DP comparatives:

(8) a. Marie is beroemder dan énige filosoof ooit geweest is.  
Marie is more famous than any philosopher has ever been.  
CP comparative  
PS felicitous

b. #Marie is beroemder dan énige filosoof  
Marie is more famous than any philosopher  
DP comparative  
PS infelicitous

Comment: (5b) and (8b) improve in felicity if we tag on them a FC appositive phrase:

(8) a. Marie is beroemder dan ook maar iemand, wie dan ook.  
Marie is more famous than whichever one you choose.  

This supports: FC is licensed in DP-comparatives, PS is not.

**Prediction of the almost (but not quite) naïve semantics for DP comparatives** (following Hoeksema 1982):

**Montague's generalization applies to DP comparatives:**
The DP complement of an extensional transitive verb/DP-comparative relation takes semantic scope over the meaning of the transitive verb/comparative relation.

Consequently:

**DP-comparatives are not downward entailing on their DP complement argument, and polarity items are not licensed.**

**CONCLUSIONS FOR THE SEMANTICS OF DP COMPARATIVES:**
-Hoeksema 1982 was on the right track for DP comparatives (not to reduce them to CP comparatives, but treat them semantically on a par with extensional transitive verbs).
-His theory can be 'modernized' in a type shifting semantics of degrees with composition.
-The almost (but not quite) naïve theory adds to this an analysis of **converse orders** and **converse operations**.
-The resulting theory smoothly predicts the right interpretations for quantificational complements and makes the right predictions about polarity items in the complement.
3. CP COMPARATIVES

3.1. GENERAL SEMANTICS FOR CP COMPARATIVES

Terminology: DP-comparative and CP-comparative in (1): **comparative correlates.**

(1) 
- a. John is taller than DP 
- b. John is taller \([\text{CP than DP is } - ]\)

\[
[\alpha \text{ [MP than DP is } - ] ]
\]

**PRED_{dim}**

**REl_{dim}**

\(\alpha\)

**MP**

\(\text{Than}\)

\(\lambda \delta_n\)

\(\text{DP}\)

\(\text{I'}\)

\(\text{PRED}\)

\(\lambda \delta \cdot R(\delta_n, \delta)^{\circ} H_\alpha\)

-CP: Operator gap construction: semantically interpreted (variable binding)
-CP level: Abstraction over a degree variable \((\delta_n)\) introduced in the gap.
-Gap: predicate gap. With BPR: degree predicate \((\lambda \delta \cdot R(\delta_n, \delta))\) for some relation \(R\), shifted to a predicate of individuals with composition with the measure function.

**GENERAL SEMANTICS FOR COMPARATIVE COMPLEMENTS:**

\[
[\alpha \text{ [MP than DP is } - ] ]
\]

\(\alpha + \text{ M} (\lambda \delta \cdot \text{DP} (\lambda x. \delta \cdot R H_\alpha(x) )\)

1. What is relation \(R\)?
2. What is operation \(M\)?

**Von Stechow:**

\(R = (\text{identity})\)

\(M \sqcup \varsigma \quad (\text{supremum, maximalization operation})\)

**Heim**

\(R = \lambda \delta \gamma \lambda \delta_1. 0 < \delta_1 \gamma \leq \delta_2 \gamma \quad (\text{monotonic closure down})\)

\(M \sqcup \varsigma \quad (\text{supremum, maximalization operation})\)

**The Naïve (but clever) Theory (→Schwarzschild and Wilkinson)**

\(R \quad \alpha \quad (\text{the external comparative relation})\)

\(M \quad \lambda P. P \quad (\text{identity})\)
3.2. VON STECHOW’S SUPREMUM THEORY.

PROBLEM: (Schwarzschild and Wilkinson):
Maximalization theories predict unnatural readings and fail to predict natural readings.

(1) John is taller than some girl is –

von Stechow: $H_A(\text{John}) >_H \cup_\nu (\exists \delta. \exists x \{ \text{GIRL}(x) \land \delta = H_A(x) \} )$

Wrong meaning: John is taller than the tallest girl

(2) John is taller than every girl is –

Heim: $H_A(\text{John}) >_H \cup_\nu (\lambda \delta. \forall x \{ \text{GIRL}(x) \rightarrow 0 < \delta^f \leq H_A(x)^r \} )$

Wrong meaning: John is taller than the shortest girl

von Stechow: $H_A(\text{John}) >_H \cup_\nu (\lambda \delta. \forall x \{ \text{GIRL}(x) \rightarrow \delta = H_A(x) \} )$

John is taller than the degree to which every girl is tall.

Unwarranted presupposition: all girls have the same degree of height

PROPOSED SOLUTION (Larsons, Von Stechow):

Give the CP internal DP scope over the comparative.

Larsons: standard scope mechanism; von Stechow: non-standard scope mechanism

PROBLEM 1:

This requires systematic scoping of all kinds of DPs out of the CP, which is problematic. Cf. scoping out of relative clauses, wh-clauses, or even propositional attitude complements: in all other CPs scoping out is severely restricted.

PROBLEM 2: Intensional contexts (Schwarzschild and Wilkinson)

(3) John is taller than [CP Bill believes that every girl in Dafna's class is –]

$\lambda \delta. \text{BELIEVE}(\text{BILL, } \land \forall x \{ \text{GIRL-IN-DC}(x) \rightarrow \delta^f > H_A(x)^f \} )$ (H_A(\text{John}))

John's actual height has the property that Bill believes it to be bigger than what he thinks is the height of what he thinks is the tallest girl in Dafna's class.

-No presupposition that Bill believes that the girls have the same height.
- von Stechow: scope every girl over the comparative.
- But every girl takes narrow scope under believe, which is inside the comparative.

Problem 3: Polarity items:

Unlike DP-comparatives, CP-comparatives allow polarity items inside the CP-complement:

(4) Marie is beroemder dan enige filosoof en enige psycholoog ooit geweest zijn.

Marie is more famous than any philosopher and any psychologist have ever been.

-No presupposition that any philosopher has the same degree of fame as any psychologist.
- von Stechow: scope enige filosoof en enige psycholoog over the comparative.
- But conjunction of PS items: not licensed if scoped over the comparative.

CONCLUSION (Schwarzschild and Wilkinson):

Supremum theories like von Stechow's (and Heim's) are untenable.
3.3. THE NAÏVE (BUT CLEVER) THEORY:

\[ \alpha \ [\text{MP than DP is } \lambda \delta. \text{DP( } \lambda x. \alpha(\delta,H_a(x)) \text{)}] \]

\[ \text{PRE}_{\text{dim}} \]

\[ \text{RE}_{\text{dim}} \]

\[ \alpha \]

\[ \text{MP} \]

\[ \text{CP} \]

\[ \emptyset \]

\[ \text{IP} \]

\[ \text{I'} \]

\[ \text{DP} \]

\[ \text{PRED} \]

\[ \varnothing \]

\[ M \text{ than} \]

\[ \lambda \delta. \text{DP( } \lambda x. \alpha(\delta,H_a(x)) \text{)} \]

\( \alpha \) is interpreted inside the comparative CP as part of the interpretation of the gap.

The clever intuition:

(5) John is taller than Mary/ every girl is —

Everybody else: The CP denotes a set of degrees to which Mary, every girl is tall.

Schwarzschild and Wilkinson: The CP denotes the set of degrees bigger than Mary’s height/every girl’s height.

This is the clever bit.

PREDICTIONS:

1. Correct readings:

CP-comparatives that have correlates have the same interpretation as their correlates: (1) – (2) get the correct readings.

2. No unwanted presuppositions: in situ readings for (2), (3) and (4) do not presuppose the same height, belief of same height, or the same degree of fame. Consequently, no scoping out of the CP is necessary to get the correct readings.
3.4. TWO PROBLEMS FOR THE NAÏVE (BUT CLEVER) THEORY

PROBLEM ONE: downward entailing DPs in CP-comparatives.
Hoeksema, Rullmann: Downward entailing DPs in CP-complements are infelicitous.

(1)  a. Mary is taller than nobody.
    b. #Mary is taller than nobody is.
    c. #Mary is taller than nobody ever was.

Cf, also

(2)  a. #Mary is more famous than John isn’t.
    b. #Mary is more famous than John will never be.
    c. #Bill is taller than at most three girls ever were.
    d. #Bill is at least two inches taller than nobody ever was.
    e. #Bill is at most two inches taller than nobody ever was.
    f. #Bill is exactly two inches taller than nobody every was.

-(1a) is felicitous, if stilted, and nobody has a wide scope reading:
Nobody is such that Mary is taller than them., i.e. Mary is the shortest.
-(1b) and (1c) are baffling.

What does (1c) mean? Are you trying to say that nobody ever was as tall as Mary is?
Mary's height has boldly gone where nobody's height has ever gone before?
It is not clear how (1c) can mean that compositionally, if it should mean that at all!

My own judgement: My brain is trying several interpretation strategies simultaneously and gets hopelessly muddled.

The naïve (but clever) theory makes a wrong prediction:
(1b) means the same as (1a).

PROBLEM TWO: Polarity items in CP-comparatives.
PS items are felicitous in CP-comparatives, but CP comparatives are not downward entailing (Schwarzschild and Wilkinson):

(3)  a. John is more famous than Mary is.
    b. Mary is a girl.
    c. Hence, John is more famous than every girl is.

The naïve (but clever) theory has no stage in the semantic derivation on which to hang the difference in polarity licensing between CP comparatives and DP comparatives.
4. THE DIMENSIONAL SUPREMUM THEORY OF CP COMPARATIVES

4.1. DIMENSIONAL SUPREMUMS
Back to the almost (but not quite) naïve theory of DP comparatives.
Let us use $+$ for elements that derive from the basic scale $S_H$,
$-$ for elements that derive from the converse scale $S_H^c$:
We stipulate: $>_R$ is $+$, $<_R$ is $-$.
For simplicity and clarity I will in the following discussion ignore cases involving $=_{R^*}$
(The discussion is easily amended to include these cases.)

\[
\begin{align*}
\text{more than three inches more tall than} & \quad \delta_1^r >_R \delta_2^r + 3 \\
\text{less than three inches more tall than} & \quad \delta_1^r <_R \delta_2^r + 3 \\
\text{less than three inches more short than} & \quad \delta_1^r >_R \delta_2^r - 3
\end{align*}
\]

The relation $\alpha$ derived by the semantics consist of a comparison relation $>_R$, $<_R$,
and a differential $+3$, $-3$. Let us call this relation the $R$-relation of $\alpha$.

**FACT:** For $\alpha$ derived above, the R-relation of $\alpha$ is $+(-)$ iff the
number of $-$ elements used in the semantic composition is
even (odd).
(This means that the notion of $R$-relation of $\alpha$ can be defined derivationally.)

**DIMENSIONAL ORDER AND DIMENSIONAL SUPREMUM OF $\alpha$**

Let $\alpha$ be derived as above

**The dimensional order for $\alpha$, $>_\alpha$, and the dimensional supremum for $\alpha$, $\sqcup_\alpha$ are given by:**

\[
\begin{align*}
>_\alpha &= \begin{cases} 
>_H & \text{if the R-relation of $\alpha$ is $+$} \\
<_H & \text{if the R-relation of $\alpha$ is $-$}
\end{cases} \\
\sqcup_\alpha &= \sqcup_{>_\alpha}
\end{align*}
\]

**Examples:**

\[
\begin{align*}
\text{more than three inches more tall than} & \quad \alpha = \delta_1^r >_R \delta_2^r + 3 & \sqcup_\alpha = \sqcup_H \quad \text{(which corresponds to $\sqcup_{R^*}$)} \\
\text{less than three inches more tall than} & \quad \alpha = \delta_1^r <_R \delta_2^r + 3 & \sqcup_\alpha = \sqcup_H \quad \text{(which corresponds to $\sqcup_R$)}
\end{align*}
\]
4.2 THE DIMENSIONAL SUPRENUM THEORY

Naïve (but clever) theory:
- Comparison relation $\alpha$ is interpreted inside the comparative CP.

Dimensional supremum theory: comparison relation $\alpha$ has a double effect:
- Comparison relation $\alpha$ is interpreted inside the comparative CP.
- Comparison relation $\alpha$ determines the interpretation of the head M as operation $M_\alpha$.

DIMENSIONAL SUPRENUM AS A PRESUPPOSITIONAL CHECK OPERATION $M_\alpha$

\[
M_\alpha(\beta) = \begin{cases} 
\beta & \text{if } \sqcup_\alpha(\beta) \text{ is defined} \\
\text{undefined} & \text{otherwise}
\end{cases}
\]

$M_\alpha$ is a presuppositional version of the identity function:
- $M_\alpha$ has no semantic effect on input set of degrees $\beta$, $M_\alpha(\beta)$ has the same meaning as $\beta$,
- $M_\alpha$ presupposes that the dimensional supremum of $\alpha$ of set of degrees $\beta$ exists.

I assume here that $\sqcup_\mathbb{R}$ and $\cap_\mathbb{R}$ are operations on $\mathbb{R}$, and since $\infty$ and $-\infty$ are not in $\mathbb{R}$, for our purposes here, $\sqcup_\mathbb{R}(X)$ and $\cap_\mathbb{R}(X)$ are undefined when infinite.

$M_\alpha$: like a definiteness operation, but without the type change from $<d,t>$ to $d$:

$M_\alpha(\beta)$ presupposes that the unique object $\sqcup_\alpha(\beta)$ exists, even if it doesn't denote it.
CONSEQUENCE:

If $\beta_{CP}$ and $\beta_{DP}$ are comparative correlates and the meaning of $\beta_{CP}$ is defined, then $\beta_{CP}$ and $\beta_{DP}$ have the same meaning.

1a) and (1b) are predicted to be equivalent, whenever (1b) is defined:

(1)  
a. John is $\alpha$ than DP  
b. John is $\alpha$ than $[CP\text{DP is } \neg]$  

Thus: the dimensional supremum theory of CP comparatives is a presuppositional variant of the naïve (but clever) theory.
4.3. DOWNWARD ENTAILING EXPRESSIONS INSIDE CP COMPARATIVES

(1) John is taller than every girl is —
taller: \( \alpha = \exists H \setminus \alpha = \exists H \) derived from \( \cap R \)
CP: than every girl is — \( \beta = \lambda \delta. \forall y[\text{GIRL}(y) \rightarrow \delta > H_{\alpha(y)}] \)
\( \cap \beta^r \) defined \( \text{FELICITOUS} \)

\[ H_{\alpha(g_1)^r} \ldots H_{\alpha(g_n)^r} \]

(2) #John is taller than no girl is —
taller: \( \alpha = \exists H \setminus \alpha = \exists H \) derived from \( \cap R \)
CP: than no girl is — \( \beta = \lambda \delta. \exists y[\text{GIRL}(y) \wedge \delta > H_{\alpha(y)}] \)
\( \cap \beta^r \) undefined \( \text{INFELICITOUS} \)

\[ H_{\alpha(g_1)^r} \ldots H_{\alpha(g_n)^r} \]

(3) John is at most two inches taller than every girl is —.
\( \cup \beta^r \) defined \( \text{FELICITOUS} \)

\[ H_{\alpha(g_1)^r} + 2 \ldots H_{\alpha(g_n)^r} + 2 \]

(4) #John is at most two inches taller than at most three girls are —.
\( \text{INFELICITOUS} \)

\[ H_{\alpha(g_1)^r} + 2 \ldots H_{\alpha(g_{n-2})} + 2 \ldots H_{\alpha(g_n)^r} + 2 \]

(5) #John is less tall than no girl is —
\( \beta^r \) \( \text{INFELICITOUS} \)

\[ H_{\alpha(g_1)^r} \ldots H_{\alpha(g_n)^r} \]

CONSEQUENCE: Downward entailing noun phrases in the CP complements are predicted to be infelicitous.
4.4. POLARITY ITEMS IN THE DIMENSIONAL SUPREMIUM THEORY

(1) Mary is more famous than John ever was.

FAME: $F_\Delta : \text{individuals} \times \text{moments of time} \rightarrow \text{degrees of fame (measured in units of } \Delta) \text{.}$

$$[\text{CP } John \text{ ever was } \neg \text{famous }]$$
$$\lambda \delta. \exists t[\delta \succ_\text{F} F_\Delta(John,t)]$$

The set of degrees $\delta$ for which there is a time $t$ such that $\delta$ is bigger on the scale of fame than John's degree of fame at $t$.

We assume, with Kadmon and Landman 1993, that the semantic effect of ever is widening. Thus, you may have said (3A), and I reply (3B):

(2) A. Mary is more famous than John is now.
   B. Mary is more famous than John ever was.

WIDENING IN CP COMPARATIVES

The widening is temporal, and involves a wide and a narrow interpretation:

(3) a. $\lambda \delta. \exists t \in \text{WIDE SET} \land \delta \succ_\text{F} F_\Delta(John,t)$
   b. $\lambda \delta. \exists t \in \text{NARROW SET} \land \delta \succ_\text{F} F_\Delta(John,t)$

Scalar comparison construction:
Standard assumption: widening is not unconstrained, but is along the scale.

$max_{JOHN,F}$ is the moment of time where John's fame is maximal (for simplicity assume that there is one such time).

Pragmatic assumption: there must be a point to using ever, and hence to widening. So:

Implicature: $max_{JOHN,F} \notin \text{NARROW SET}$

Simplest assumption:
Widening done by ever just adds $max_{JOHN,F}$ to the narrow set:

Widening: $\text{WIDE SET} = \text{NARROW SET} \cup \{ max_{JOHN,F}\}$

Interpretation of the CP:

$$[\text{CP } John \text{ ever was } \neg \text{famous }]$$
$$\lambda \delta. \exists t \in \text{NARROW SET} \cup \{ max_{JOHN,F}\} \land \delta \succ_\text{F} F_\Delta(John,t)]$$

By the assumptions made, John's degree of fame at $max_{JOHN,F}$ is higher than John's degree of fame at any of the points of time in the narrow set: so:
Given these assumptions, in a natural context (1) is interpreted as:

(1) Mary is more famous than John ever was.

\[ F_\delta(Mary)^f >_F F_\delta(John, \max_{JOHN,F})^f \]

Mary's degree of fame (now) is higher than John's degree of fame when it was maximal.

This is an adequate account of the meaning of (1).

**THE LICENSING OF THE PS ITEM.**

Kadmon and Landman: a PS item is licensed if **widening leads to strengthening** at the level of the closest relevant operator the PS item is in the scope of.

**STRENGTHENING IN CP COMPARATIVES**

Main assumption: **strengthening is defined at the level of the comparative scale** (relation between elements of type \( \delta \))

**SCALAR STRENGTHENING:**

On scale \( S_M \): \( \delta_1 \) **strengthens** \( \delta_2 \) iff \( \delta_1 \geq_M \delta_2 \)

On scale \( S_M^c \): \( \delta_1 \) **strengthens** \( \delta_2 \) iff \( \delta_1 \leq_M \delta_2 \)

**DP comparatives:**

-No grammatical level where the interpretation of the DP is of type \( \delta \). (type d or \(<d,t>,t>\))

(Composition with the measure function takes place in the comparison relation, not in its object.)

**Hence, scalar strengthening is irrelevant and PS items are not licensed in DP comparatives.**
**CP comparatives:**
-No grammatical level where the interpretation of the CP is of type $\delta$.
-But the presuppositional check operation brings in an interpretation of type $\delta$ as a presupposition: $\cup_a(\beta)$ of type $\delta$.

**Assumption:** The polarity item is licensed if widening leads to strengthening at the presuppositional level $\cup_a(\beta)$

This means that we check whether (2a) strengthens (2b):

(2) a. $\cup_\delta(\lambda \delta. \delta > F \alpha(j, \max_{\text{JOHN,F}}))$
   b. $\cup_\delta(\lambda \delta. \exists t [t \in \text{NARROW SET} \land \delta > F \alpha(\text{John},t)])$

$\cup_\delta$ corresponds to $\cap_R$, hence:

$$\cup_\delta(\lambda \delta. \delta > F \alpha(j, \max_{\text{JOHN,F}})) = F\alpha(j, \max_{\text{JOHN,F}})$$

Let $\min_{\text{narrow,JOHN,F}}$ be the time in NARROW where John's fame is minimal in comparison to the other times in the narrow set.

Obviously, the infimum of the degrees of John's fame corresponding to the times in the narrow set is $F\alpha(j, \min_{\text{narrow,JOHN,F}})$.

$$\cup_\delta(\lambda \delta. \exists t [t \in \text{NARROW SET} \land \delta > F \alpha(\text{John},t)]) = F\alpha(j, \min_{\text{narrow,JOHN,F}})$$

Thus, we are checking whether (3a) strengthens (3b):

(3) a. wide degree $F\alpha(j, \max_{\text{JOHN,F}})$
   b. narrow degree $F\alpha(j, \min_{\text{narrow,JOHN,F}})$

Clearly, $F\alpha(j, \max_{\text{JOHN,F}}) \geq_F F\alpha(j, \min_{\text{narrow,JOHN,F}})$, hence (3a) strengthens (3b):

**CONSEQUENCE:**
The polarity item *ever* is licensed in the CP comparative (1):

(1) Mary is more famous than John *ever* was.
We look at (4):

(4) Mary is less famous than John ever was.

-Widening involves $\text{min}_{\text{JOHN,F}}$, the time where John's fame was minimal.
-The pragmatic assumption is that John's fame at $\text{min}_{\text{JOHN,F}}$ is smaller than all the times in the narrow set.

The supremums we get here are as in (5):

(5) a. $\sqcup_{\text{f}}(\lambda \delta. \delta <_{\text{f}} F_{\Delta}(\text{John}, \text{min}_{\text{JOHN,F}})) = \text{min}_{\text{JOHN,F}}$
   b. $\sqcup_{\text{f}}(\lambda \delta. \exists t [t \in \text{NARROW SET} \land \delta <_{\text{f}} F_{\Delta}(\text{John}, t)]) = \text{max}_{\text{narrow,JOHN,F}}$

The relevant dimension is the converse dimension, i.e., we are in $S_{\text{F}}^c$, so we use $S_{\text{F}}^c$-\textit{strengthening} as defined above, and we see that indeed:

$\text{min}_{\text{JOHN,F}} \leq_{\text{F}} \text{max}_{\text{narrow,JOHN,F}}$ (5a) strengthens (5b)

\textbf{CONSEQUENCE:}
\textbf{The polarity item ever is licensed in the CP comparative (4):}

(4) Mary is more famous than John ever was.

\textbf{CONCLUSIONS FOR THE SEMANTICS OF CP COMPARATIVES:}
The dimensional supremum theory is a presuppositional variant of the naive (but clever) there.
-For CP comparatives that have a DP correlate, when defined, the CP comparative has the same meaning as its DP correlate.
-But negation and downward entailing noun phrases inside the CP complement make the CP comparative undefined (hence infelicitous), unlike their DP correlate (if they have one).
-Polarity licensing is defined directly on the level of the comparative scale and is sensitive to the direction (+/−) of the dimensional order involved.
-On the dimensional supremum theory, the type of the scale is invoked directly at the presuppositional level invoked by the operation $M_{\alpha}$, and polarity items inside the CP complement are licensed at that level.
APPENDIX 1: INTRANSITIVE MEASURE PHRASES

\[ \text{measure } tall \rightarrow H \]

The measure meaning of tall combines with more than three inches to give a degree predicate (after composition with the measure function):

\[ [\text{pred } \text{more than three inches tall}] \rightarrow \lambda x. H_{\geq w}(x)^r > 3 \]

(1) a. Wiplala is more than three inches tall.
   b. \( H_{\geq w}(\text{Wiplala})^r > 3 \)
   c. \#Wiplala is more than three inches short.

No interpretation for (1c), because there is no lexical item \[ \text{measure } short \] with meaning H.

Trivial account. But better than von Stechow/Kennedy.

von Stechow/Kennedy: (1c) is infelicitous because the human brain cannot convert supremums correctly, hence the human brain uses faulty mathematics.

APPENDIX 2: ADJECTIVES

Dimensions with default unit:

\[ [\text{dim } tall] \rightarrow S_{H,\Delta k} \]
\[ [\text{dim } short] \rightarrow S_{H,\Delta k}^c \]

**Adjective formation:**

\[ \text{dim} \rightarrow \lambda \delta. \; \delta \text{dim}^r \text{dim}^{\text{HIGH}} \]

\[ [\text{adj } tall] \rightarrow \lambda \delta. \; \delta^r > (\text{HIGH}_{H,\Delta k})^r \quad (\text{tall}^r = >_H \quad \text{tall}^{\text{HIGH}} = \text{HIGH}_{H,\Delta k}) \]
\[ [\text{adj } short] \rightarrow \lambda \delta. \; \delta^r < (\text{LOW}_{H,\Delta k})^r \quad (\text{short}^r = <_H \quad \text{short}^{\text{HIGH}} = \text{LOW}_{H,\Delta k}) \]

(1) Borremans is tall iff \( H_{\Delta}(\text{Borremans})^r > (\text{HIGH}_{H,\Delta k})^r \)

Borremans is tall iff his height in \( \Delta \) exceeds the contextual height value, which is likely (dependent on \( k \), of course), since Borremans is a giant.

(2) Wiplala is short iff \( H_{\Delta}(\text{Wiplala})^r < (\text{LOW}_{H,\Delta k})^r \)

Wiplala is short iff his height in \( \Delta \) is below the contextual height value, which is likely (dependent on \( k \)), since Wiplala is a wiplala, and wiplalas fit in your coat pocket.
APPENDIX 3. MEASURES INSIDE CP COMPARATIVES

(1) The tower is taller than it is — wide.

The naïve assumption for CP comparatives with internal measures: Wide inside the CP comparative is a measure: $[\text{measure wide}] \rightarrow W_\Lambda$

Prediction: We find for measure phrases inside the complement of CP comparatives the same contrast as we found for five feet tall/#short.

(2) a. The gate is higher than it is wide
b. The gate is higher than it is narrow
b. The gate is wider than it is high.
c. The gate is lower than it is wide.
c. The gate is narrower than it is high.
d. The gate is lower than it is narrow.
d. The gate is narrower than it is low.

CP interpretation with internal measures:
External comparative $\alpha$ makes measure function $M_\alpha$ available.

be $\alpha$ than DP is $- [\text{measure wide}]$
\[ \lambda \delta. \ DP(\lambda x. \ alpha(\delta, [W_\Lambda \rightarrow M_\alpha](x))) \] (In our examples: $[W_\Lambda \rightarrow H_\Lambda])$

$[W_\Lambda \rightarrow H_\Lambda]$ is the result of converting $W_\Lambda$ to a measure function that maps individuals onto heights degrees.

In the case of width and height, the conversion is trivial:
$[W_\Lambda \rightarrow H_\Lambda] = \lambda x. \ W_\Lambda(x) [H/W]$
The function that differs only from $W_\Lambda$ in that at each third element in the triple it has $H$ instead of $W$.

(3) a. The tower is taller than it is — wide
b. $H_\Lambda$(Tower) $>_H [W_\Lambda \rightarrow H_\Lambda]$(Tower)

If the tower is 20 meters tall and 4 meters wide, then
$H_m$(Tower) = $<20,m,H>$ $W_m$(Tower) = $<4,m,W>$
$[W_m \rightarrow H_m]$(Tower) = $<4,m,H>$
$<20,m,H>$ $>_H <4,m,H>$. 
APPENDIX 4: INVERSE MEASURES

Converse antonym pairs:
tall: basic scale of measure H short: converse scale of measure H.

Inverse antonym pairs: fat-thin, concave-convex, flat-sharp (for musical keys).

Inverse antonyms: both elements of the pair have an interpretation as a measure:

\[ \text{[measure flat]} \rightarrow F \quad \text{[measure sharp]} \rightarrow S \]

Both (6a) and (6b) are felicitous:
(6)  a. A major is three notches sharp. \( (###) \)
    b. F minor is four notches flat. \( (bbbb) \)

Both sharp and flat can occur inside the complement of CP comparatives, as in (7):
(7)  a. F minor is flatter than A major is sharp \( bbb \) vs. \( ### \)
    b. C-sharp minor is sharper than E-flat major is flat \( ### \) vs. \( bbb \)

Hence: degrees of sharpness \( \langle r,n,S \rangle \) and degrees of flatness \( \langle r,n,F \rangle \)

INVERSE MEASURES

Measures A and B are inverse measures iff
For every unit u appropriate for A and B, for every x, w:
\[ A_u(x,w) = -B_u(x,w) \]

Flatness is negative sharpness, sharpness is negative flatness.

Example: \( F_n(F \text{ minor}) = <4,n,F> \), hence \( S_n(F \text{ minor}) = <-4,n,S> \)

(11) a. F minor is seven notches flatter than A major is
\( F_n(F \text{-minor}) = <4,n,F> \) \( (bbbb) \)
\( S_n(A \text{-major}) = <3,n,S> \) \( (###) \).
\( F_n(A \text{-major}) = <-3,n,F> \) \( (-bbb) \)

Comparison of inverse antonyms does not take place on a superscale but is a comparison between scales.
APPENDIX 5: THE INTERVAL THEORY OF SCHWARZSCHILD AND WILKINSON AND THE NAÏVE (BUT CLEVER) THEORY

DP₁ is α than DP₂ is ¬

1. We assume:
   - Charity in correcting minor mistakes
   - Charity in interpretation of undefined notions
   - Charity in obvious extensions to other cases

2. We ignore vagueness (i.e. H₃(John) is a point degree, not an interval of degrees)

3. We ignore quantificational external subjects (i.e. DP₁ is, say, a proper name)

THEOREM: UNDER THE CONDITIONS 1-3, THE INTERVAL SEMANTICS OF SCHWARZSCHILD AND WILKINSON FOR DP₁ is α than DP₂ is ¬ IS EQUIVALENT TO THE NAÏVE (BUT CLEVER) THEORY.

What follows is an outline of how the proof goes.

Schwarzschild and Wilkinson:

(1) DP₁ is β-taller than DP₂ is ¬.

where β is a numerical predicate of the form:

- at least two inches,
- at most two inches,
- exactly two inches,
- Ø…..

∃j[ DP₁ is j-tall ∧ DP₂ is max(λi. β(j-i))-tall ]

There is a degree interval j such that DP₁ is j-tall and DP₂ is k-tall, where k is the maximal interval in the set of intervals i such that j-i is β (at least two inches, at most two inches…)

With all the assumptions under (1-3), the interval meaning for (1) can be massaged into the following meaning:

(2) DP₁ is β-taller than DP₂ is ¬.

DP₁(λx. [λδ. DP₂ is max(λi. β(δ,δ)-i))-tall] (H₃(x)))

With the principle BPR, and some more charity, we can derive the comparative meaning:

(3) β-taller than DP is ¬.

λδₙ.(DP(λx. R(δₙ, H₃(x)))

where R = λδ₂λδ₁. δ₂ε max(λi. β([δ₁, δ₁]-i))
max(λi. β([δ1, δ1]−i)) is defined in Schwarzschild and Wilkinson's paper.
Relevant for the theorem is the following central lemma:

**Central Lemma:** \( \delta_2 \in \max(\lambda_i. \beta([\delta_1, \delta_1]−i)) \iff \beta([\delta_1, \delta_1]− [\delta_2, \delta_2]) \)

Proof: omitted

With the lemma, (3) is equivalent to (4):

(4) \( \text{β-taller than DP is } \neg. \lambda \delta_n. (\text{DP}(\lambda x. R(\delta_n, H_\Delta(x))) \)

where \( R = \lambda \delta_2 \lambda \delta_1. \beta([\delta_1, \delta_1]− [\delta_2, \delta_2]) \)

**Adequacy constraint:** for point intervals, Schwarzschild and Wilkinson's notion (\( \beta_{sw} \)) should be equivalent to the corresponding almost (but not quite) naïve notion (\( \beta \)) \( \beta_{sw}([\delta_1, \delta_1]− [\delta_2, \delta_2]) \iff \beta(\delta_1 \neg_H \delta_2) \)

Hence (4) is equivalent to (5):

(5) \( \text{β-taller than DP is } \neg. \lambda \delta_n. (\text{DP}(\lambda x. R(\delta_n, H_\Delta(x))) \)

where \( R = \lambda \delta_2 \lambda \delta_1. \beta(\delta_1 \neg_H \delta_2) \)

This means that \( R = \beta \circ \neg_H \), which is \( \alpha \) of the almost (but not quite) naïve theory.

Hence (5) is equivalent to (6):

(6) **The Naïve (but Clever) Theory**

\( \text{β-taller than DP is } \neg. \lambda \delta_n. (\text{DP}(\lambda x. \alpha(\delta_n, H_\Delta(x))) \)