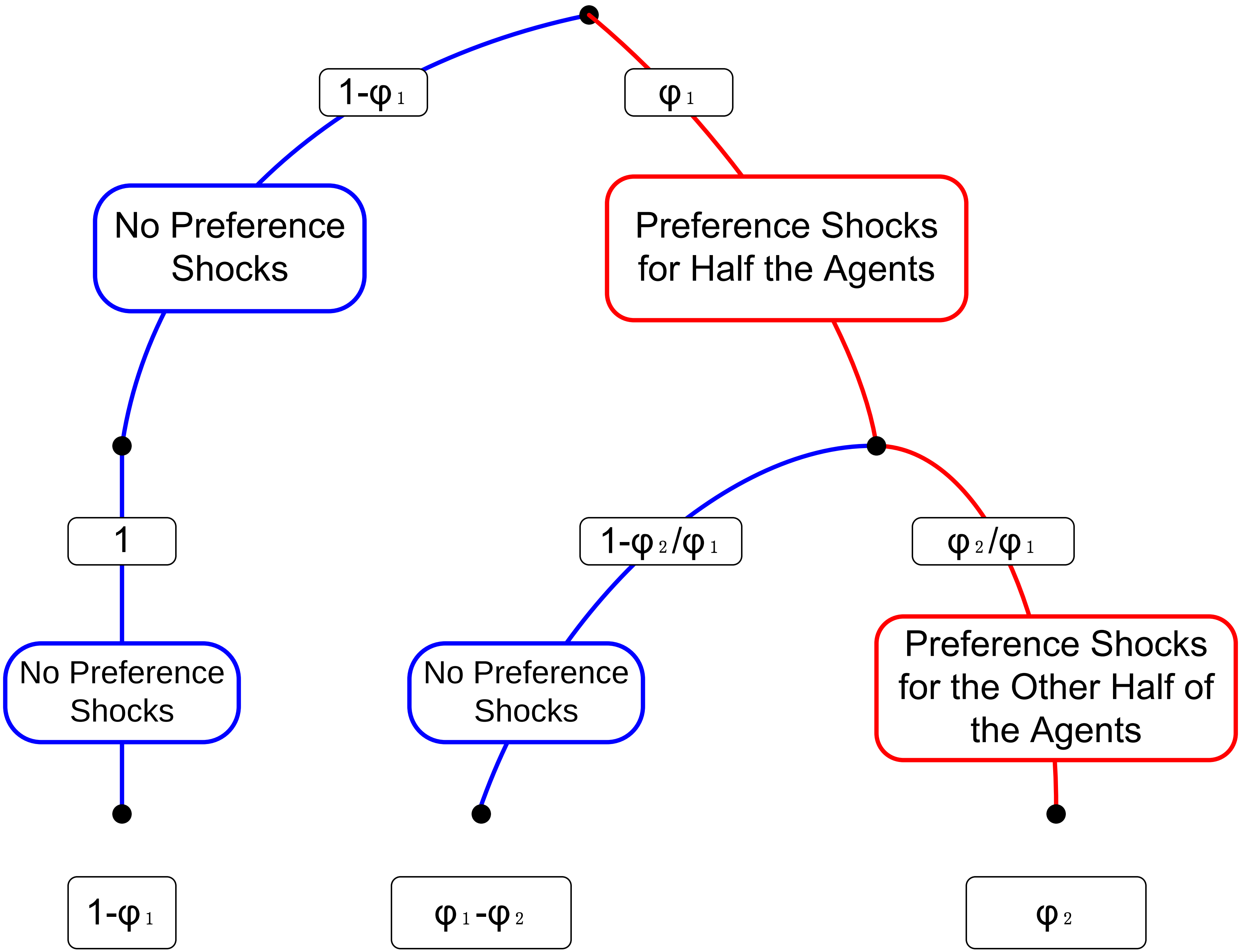
Model Structure

- Three-period model

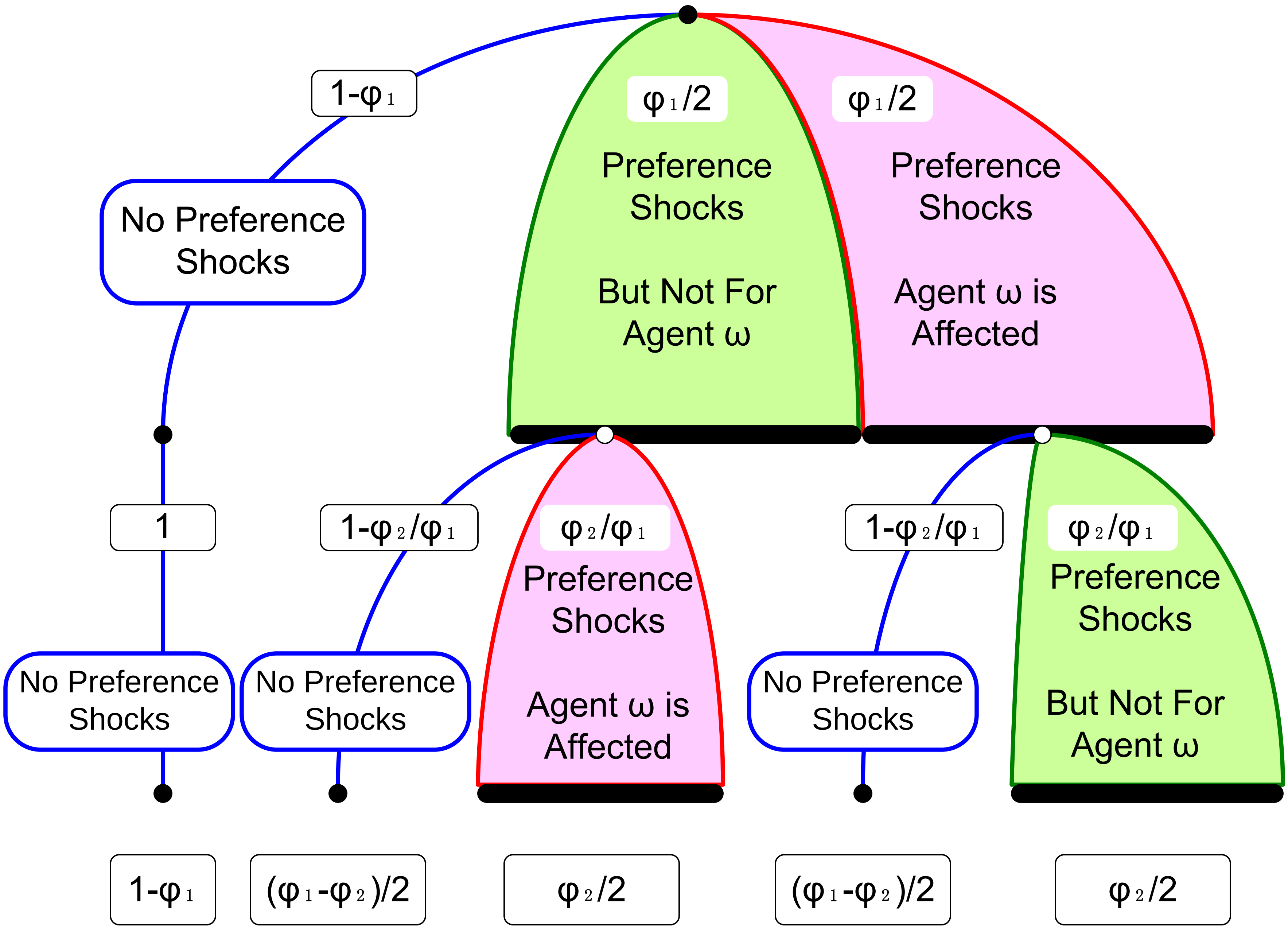
- Period 1: some agents *may* desire to consume in this period
- Period 2: some agents *may* desire to consume in this period
- Terminal period: normal period for consumption

- Ex-post lifetime utility: $\alpha_1 u(c_1) + \alpha_2 u(c_2) + \beta c_T$

Aggregate Event Tree



Event Tree for an Individual Agent



Contingent Consumption Allocations

- c_1: consumption in pd. 1 if I am hit by a preference shock in pd. 1
- c_2: consumption in pd. 2 if I am hit by a preference shock in pd. 2
- $c_T^{n,t}$: consumption in the terminal period if there were n aggregate shocks, and I was hit by a preference shock in period t
- $c_T^{n,\text{no}}$: consumption in the terminal period if there were n aggregate shocks, and I was not hit by any preference shocks.
- Z: initial endowment, storable

Social Planner's Problem is to Maximize

$$\frac{\phi_1}{2} u\left(c_1\right) + \frac{\phi_2}{2} u(c_2) +$$

$$\beta \left[\underbrace{\left(1-\phi_1\right) c_T^{0,\text{no}}}_{\text{no shocks}} + \underbrace{\left(\phi_1 - \phi_2\right) \left(\frac{1}{2} c_T^{1,\text{no}} + \frac{1}{2} c_T^{1,1} \right)}_{\text{1 shock}} + \underbrace{\phi_2 \left(\frac{1}{2} c_T^{2,2} + \frac{1}{2} c_T^{2,1} \right)}_{\text{2 shocks}} \right]$$

subject to non-negativity constraints and

$$\text{no shocks:} \qquad c_T^{0,\text{no}} = Z.$$

$$\text{1 shock:} \qquad \frac{1}{2} c_1 + \frac{1}{2} c_T^{1,\text{no}} + \frac{1}{2} c_T^{1,1} = Z.$$

$$\text{2 shocks:} \qquad \frac{1}{2} c_1 + \frac{1}{2} c_2 + \frac{1}{2} c_T^{2,2} + \frac{1}{2} c_T^{2,1} = Z.$$

Simplifying the problem

- $u'(Z) > \beta$ ... implies that $c_T^{2,2} = c_T^{2,1} = 0$
- $c_T^{0,\text{no}} = Z$
- $\frac{1}{2}c_T^{1,\text{no}} + \frac{1}{2}c_T^{1,1} = Z - \frac{1}{2}c_1$

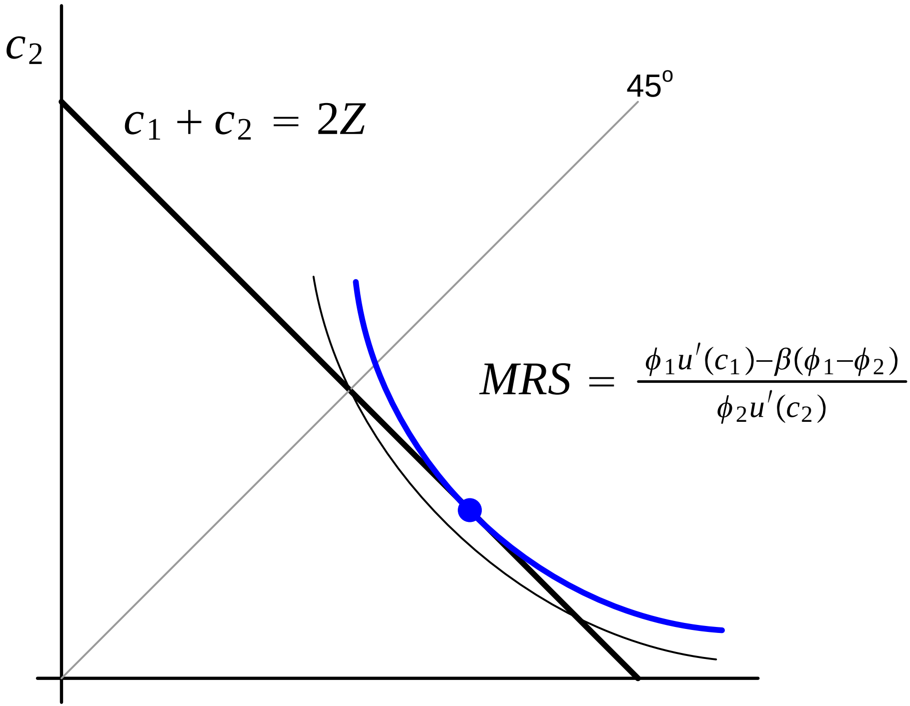
The Social Planner's Problem Simplified

Maximize

$$\frac{\phi_1}{2}u\left(c_1\right)+\frac{\phi_2}{2}u(c_2)+\beta\left[\underbrace{\left(1-\phi_1\right)Z}_{\text{no shock}}+\underbrace{\left(\phi_1-\phi_2\right)\left(Z-\frac{1}{2}c_1\right)}_{\text{1 shock}}\right]$$

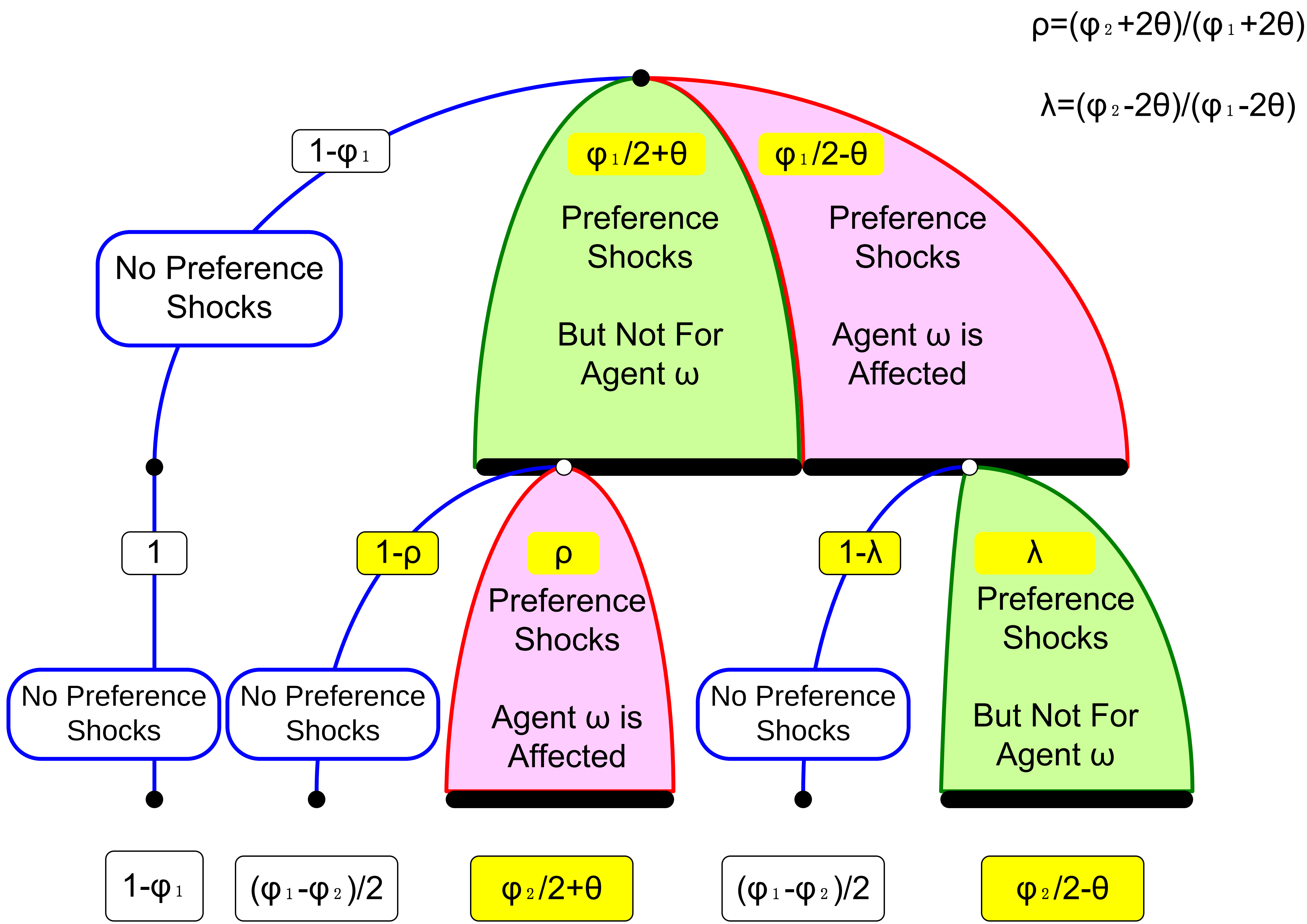
subject to

$$c_1+c_2=2Z.$$



c_2
$c_1 + c_2 = 2Z$
45^o
$MRS = \frac{\phi_1 u'(c_1) - \beta(\phi_1 - \phi_2)}{\phi_2 u'(c_2)}$
c_1

Event Tree for Individual Agent, Knightian Uncertainty Case



Equilibrium with Knightian Uncertainty

Agents maximize

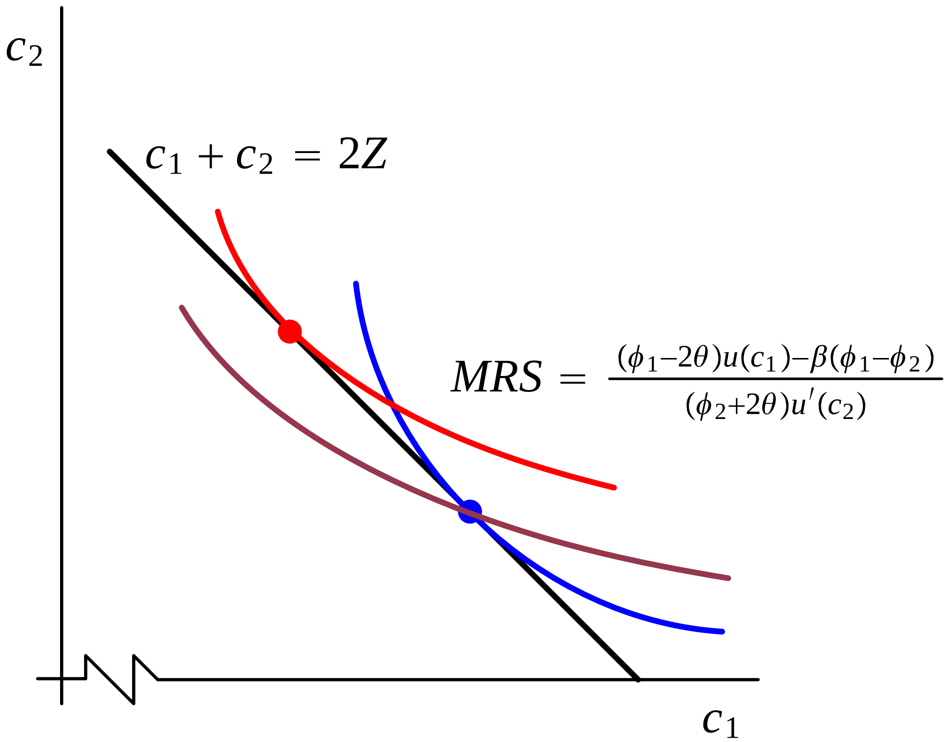
$$\left(\frac{\phi_1}{2} - \theta\right) u\left(c_1\right) + \left(\frac{\phi_2}{2} + \theta\right) u(c_2) +$$

$$\beta \left[\underbrace{\left(1 - \phi_1\right) Z}_{\text{no shock}} + \underbrace{\left(\phi_1 - \phi_2\right) \left(Z - \frac{1}{2} c_1\right)}_{\text{1 shock}} \right]$$

subject to

$$c_1 + c_2 = 2Z.$$

- Nature always chooses the biggest θ if agent's optimal choice is $c_1 > c_2$.


c_2
$c_1 + c_2 = 2Z$
$MRS = \frac{(\phi_1 - 2\theta)u(c_1) - \beta(\phi_1 - \phi_2)}{(\phi_2 + 2\theta)u'(c_2)}$
c_1

Key Insight of the Paper

- In the original planning problem, only the aggregate probabilities appear
  - There is no uncertainty over aggregate probabilities only over idiosyncratic risk of being first conditional on two shocks being realized.
- The aggregate probabilities are common knowledge
- So, a central bank that implements the social planning problem does not have an information advantage

Key Insight of the Paper

- In the original planning problem, only the aggregate probabilities appear
  - There is no uncertainty over aggregate probabilities only over idiosyncratic risk of being first conditional on two shocks being realized.
- The aggregate probabilities are common knowledge
- So, a central bank that implements the social planning problem does not have an information advantage
- The paper does not really discuss direct implementation of the social planner's solution.

Lender of Last Resort Policy

- Let the central bank commit to giving Z^G extra units of consumption to any agent affected by a second period shock.
- Agents maximize

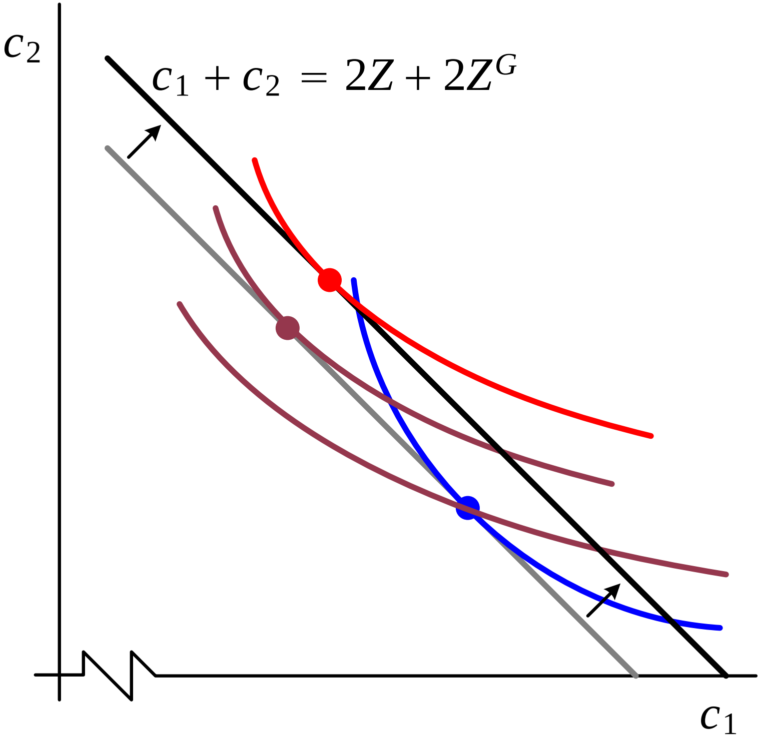
$$\left(\frac{\phi_1}{2}-\theta\right)u\left(c_1\right)+\left(\frac{\phi_2}{2}+\theta\right)u(c_2)+$$

$$\beta\left[\underbrace{\left(1-\phi_1\right)Z}_{\text{no shock}}+\underbrace{\left(\phi_1-\phi_2\right)\left(Z-\frac{1}{2}c_1\right)}_{\text{1 shock}}\right]$$

subject to

$$c_1+c_2=2Z+2Z^G.$$

- Agents increase c_1 and decrease *privately funded* c_2



c_2
$c_1 + c_2 = 2Z + 2Z^G$
c_1

An Alternative Policy with the Opposite Flavor

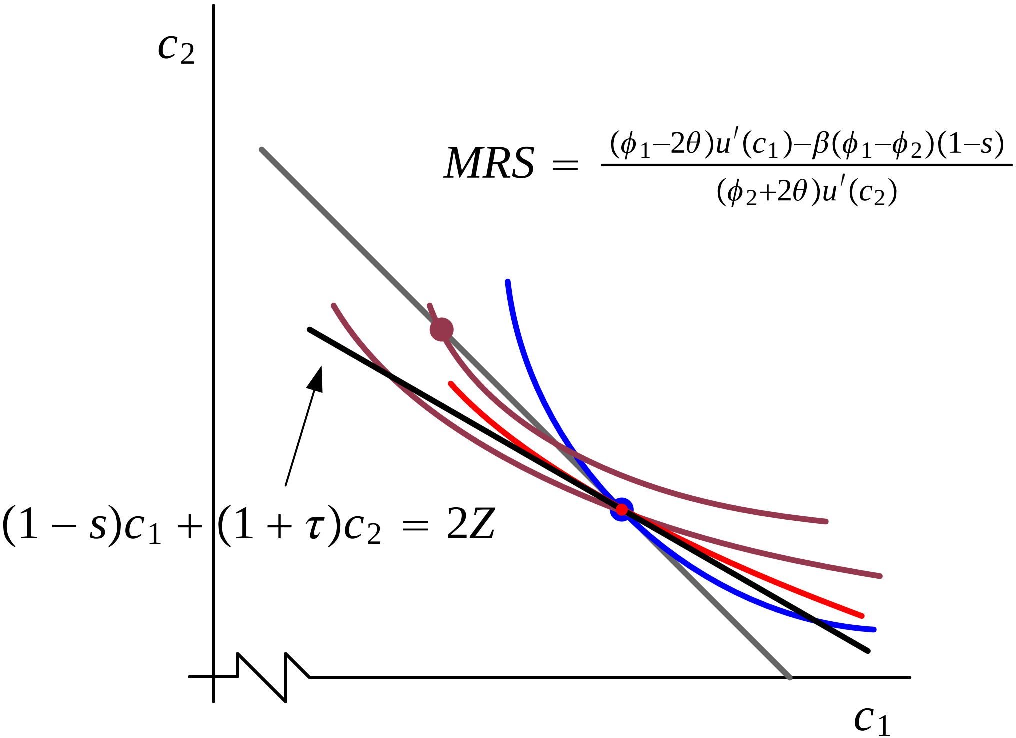
- As described the LLR policy is easy on liquidity when period 2 shocks occur
- But a policy designed specifically to tax period 2 liquidity and subsidize period 1 liquidity appears to be move the economy towards the original social optimum

$$\left(\frac{\phi_1}{2}-\theta\right)u\left(c_1\right)+\left(\frac{\phi_2}{2}+\theta\right)u(c_2)+$$

$$\beta\left[\underbrace{\left(1-\phi_1\right)Z}_{\text{no shock}}+\underbrace{\left(\phi_1-\phi_2\right)\left(Z-\frac{1}{2}(1-s)c_1\right)}_{\text{1 shock}}\right]$$

subject to

$$(1-s)c_1+(1+\tau)c_2=2Z.$$



c_2
$MRS = \frac{(\phi_1 - 2\theta)u'(c_1) - \beta(\phi_1 - \phi_2)(1-s)}{(\phi_2 + 2\theta)u'(c_2)}$
$(1-s)c_1 + (1+\tau)c_2 = 2Z$
c_1

Do Central Bankers have Sensitive Olfactory Glands?

- A promise to make credit easy in "two wave" crises requires that the central bank knows the "over"-insurance is a reaction to Knightian uncertainty
- In a world with shocks to uncertainty AND to the true aggregate probabilities, the central bank faces a signal extraction problem
- Some crises smell like Knightian uncertainty, but an unconditional promise to provide liquidity could lead to "under"-insurance against aggregate events

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- But ... the paper rightly focuses on what has been under-emphasized in the literature, and does so elegantly!