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INVESTMENT IN HUMAN CAPITAL AND ECONOMIC GROWTH

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An explanation of economic growth is unsatsfactory so long as the rate of technological advance is treated as exogenous to the system. Following Schultz [4] and Becker [1] we can link technical change to investment in human capital. Economists have long since noticed that people play an important role in the process of production and in return they are rewarded by an amount which constitutes the largest fraction of income. More recent is the view that investment in human capital is a major explanation of economic growth.

In this paper we shall apply analysis of the behavior of the two basic economic units, households and firms, to a model of economic growth. We contemplate two kinds of investment: the accumulation of human capital by households, as a means of increasing their earning capacity and the accumulation of capital (in physical form as well as human form) by firms, with the purpose of extending their proudctive capacity. The rate of economic growth will be explained in terms of the rate of change in the ratio of the two forms of capital by means of these two kinds of investment.

Part I presents a theoretical model in which the interrelations between the investment in human capital and the desired path of consumption are analyzed. The model described in this part of the paper is based on a recent article by Uzawa [8] in which the behavior of an individual household concerning consumption and saving is analyzed. In the presentation here we are concerned with extending Uzawa's analysis to take into account decisions made by the household with regard to investment in human capital in addition to those regarding consumption and saving. A new concept of net real income is introduced. Consumption is taken to be a function of income net of « depreciation », which is defined as a reduction in the ratio of physical to human capital as population grows and investment in human capital takes place.

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In part II we put together the pieces that describe the behavior of individual agents in the economy into a model of growth of the economy. The principal result of this part of the paper is that the ratio of physical to human capital can explain the rates of investment made by households and firms. Thus, the rate of improvement in labor skill is explained within the system.

It turns out that the properties of the equilibrium of the economy depend crucially upon whether or not the product of the elasticity of labor-time with respect to the rate of interest and the relative share of labor income is greater, equal or smaller than the elasticity of substitution in production. By « labor-time » we mean the proportion of total time which society devotes to work.

Conditions for the existence of a regime of a complete specialization in work, and conditions for existence of a regime of mixed activities of work and education, are expressed in terms of the above magnitudes.

It is shown that under the latter regime the rate of technical progress is increasing as the economy develops.

Part III compises some remarks on the effect of fiscal policy with respect to the two forms of investment and a summary of the results.

I. *Investment in Human Capital and Consumption.*

We will assume a representative household whose life span is infinite (1), which seeks to maximize its intertemporal utility by investing in education and by saving.

We define an index of earning capacity A_t which reflects the amount of the skill possessed by labor which was acquired through education. Its relative rate of change over time is denoted by

$$\alpha_t = \dot{A}_t / A_t \, . \tag{1}$$

We assume that the rate of change in the amount of skill is related to the fraction of time, (say, nine months in a year) which is devoted to education g_t. Inverting this relationship we get

$$g_t = g\,(\alpha_t). \tag{2}$$

We assume diminishing returns in the process of education. Therefore, the function $g\,(\alpha)$ is increasing and strictly convex. The range of this function is restricted between zero and one, as g is expressed as a fraction. A negative value of α, corresponding to a situation where no time is being devoted to education (i.e., $g = 0$), implies depreciation

(1) We distinguish between an individual person whose life span is finite and a household composed of individuals with the entire distribution of ages whose life span may be infinite. We assume that the decision making unit in the economy with respect to consumption and investment in human capital is the household.

of human capital (2). Formally, the function $g(\alpha)$ is invariant with respect to time and will obey the following conditions:

$$0 \leqq g(\alpha_t) \leqq 1,\ g'(\alpha_t) > 0,\ g''(\alpha_t) > 0,\ g(0) \geqq 0, \quad \text{for all } t\,.$$

We also define the maximum and minimum rates of investment by $g(\bar{\alpha}) = 1$, $g(\underline{\alpha}) = 0$, respectively.

We assume that each consumer possesses an infinitesimal fraction of the aggregate stock of human capital and does not expect to influence the market, i.e., that competitive conditions prevail in the market for the services of human capital (3). Then the expected wage W_t is a product of three components, the stock of knowledge A_t, the number of people in the household N_t and the rental on human capital w_t:

$$W_t = w_t A_t N_t\,. \tag{3}$$

The cost of further investment in human capital in terms of earnings foregone is then $(w_t N_t A_t)\, g_t$.

Let us denote the amount of non-human assets by B_t and the rate of intrest by ρ_t. Assuming that the cost of investment in human capital consists only of earnings foregone, the household's income Y_t is

$$Y_t = w_t A_t N_t [1 - g(\alpha_t)] + \rho_t B_t\,. \tag{4}$$

The rate of accumulation of the non-human assets is determined by the amount of consumption C_t:

$$\dot{B}_t = Y_t - C_t\,. \tag{5}$$

Let us assume that the individual can borrow and lend any amount (subject only to the discounted sum of consumption not exceeding the total wealth (4)) at a given rate of interest. We assume that the expectations regarding the rental on human capital and the rate of interest are stationary, i.e.,

$$w_t = w,\ \rho_t = \rho\,. \tag{6}$$

The discounted sum of returns to human capital (as seen by the individual) is

$$J_0 = \int_0^\infty w N_t A_t [1 - g(\alpha_t)]\, e^{-\rho t}\, dt\,. \tag{7}$$

(2) As we have defined the decision-making unit to be a household, the existence of a positive rate of depreciation is closely related to time needed for educating each new generation in the household.

(3) In the context of a slightly different model of investment in human capital, Ben-Porath [2] discusses some of these issues at length.

(4) Total wealth will be defined later, following Equation (7).

The quantity J_0 may be called «human wealth». Total wealth, initially, is therefore the sum $J_0 + B_0$.

We assume that the household does not enjoy either the time spent on education or the possession of a stock of human capital directly. Its utility is derived from the enlarged stream of consumption made possible by investing in education. Therefore the problem of specifying the paths of consumption and the rate of investment in human capital that yield the highest attainable (intertemporal) utility can be separated into two sub-problems. The first is to maximize human wealth by an appropriate policy of investment in human capital. Given this amount of human wealth, the second problem is to achieve the highest level of intertemporal welfare by an ontimum consumption and saving policy. The first problem is to find the optimum time path for human capital investment, that is, to achieve that highest level of discounted sum of wage earnings:

$$\text{(8)} \qquad \max J_0 = \int_0^\infty wN_t A_t [1 - g(\alpha_t)] e^{-\rho t} dt \quad \text{subject to}$$

$$\alpha_t A_t = \dot{A}_t, \qquad A(0) = A_0,$$

$$nN_t = \dot{N}_t, \; N(0) = N_0,$$

where the household expects a constant rate of increase n in its population.

In the discussion we shall assume (5) that the interest rate is higher than the sum of the maximum rate of investment and the expected population growth rate:

$$\rho > \bar{\alpha} + n$$

where $\bar{\alpha}$ is defined by $g(\bar{\alpha}) = 1$.

By the strict convexity of the function $g(\alpha)$ the desired path of investment is uniquely determined (6).

By applying the Pontryagin's Maximum Principle with λ_t as the imputed price to our problem we get the necessary conditions for equilibrium as follows.

$$\text{(9)} \qquad \begin{aligned} &\text{a)} \quad \alpha_t = \underline{\alpha} \text{ for } \lambda_t \leqq g'(\underline{\alpha}) \\ &\text{b)} \quad g'(\alpha_t) = \lambda_t \text{ for } g'(\underline{\alpha}) < \lambda_t < g'(\bar{\alpha}), \\ &\text{c)} \quad \alpha_t = \bar{\alpha} \text{ for } \lambda_t \geqq g'(\bar{\alpha}), \end{aligned}$$

(5) If this assumption is violated not only do we face the problem of the divergence of the integral in (6) for some feasible paths, but also the optimum policy defined by an alternative criterion, such as Von Weizsackers [9] overtaking criterion, does not exist. This mathematical difficulty places, of course, a severe restriction on the applicability of the objective (8) used in the model.

(6) Two allegedly optimum paths can always be shown to be inferior to a path which is made by a convex combination of these two paths.

$$(10) \qquad \dot{\lambda}_t = (\rho - n - \alpha_t)\, \lambda_t - [1 - g(\alpha_t)]\,,$$

$$(11) \qquad \lim_{t \to \infty} e^{-\rho t} \lambda_t A_t N_t = 0$$

These conditions can be shown to be sufficient for optimality.

A stationary path (i.e., $\alpha_t = 0$ for all t) satisfying the equations (9) and (10) will automatically satisfy equation (11) for all initial conditions. Since the optimum is unique the desired rate of investment is given by a constant policy,

$$(12) \qquad \text{a)} \quad \underline{\alpha} = \bar{\alpha}\,, \; g'(\underline{\alpha}) \geqq \frac{1}{\rho - \underline{\alpha} - n} \quad \text{or}$$

$$\text{b)} \quad g'(\alpha) = \frac{1 - g(\alpha)}{\rho - \alpha - n} \quad \text{where } \underline{\alpha} < \alpha < \bar{\alpha}$$

depending upon the market rate of interest. In equation (12 *b*) tha familiar equality of the marginal cost and the marginal benefits of investment is reestablished [7]. To see this, we multiply both sides of Equation (12 *b*) by the rental w. On the left hand side of this expression is the marginal cost of investment (i.e., $wg'(\alpha) = \partial\, [wAg(\alpha)] / \partial\, [\dot{A}]$). On the right hand side of this expressione is the marginal benefit of an extra unit of A

$$\left[\text{i.e.,} \frac{\partial\, [wA\,(1 - g)/(\rho - \alpha - n)]}{\partial A}\right].$$

Equation (12 *a*) expresses the fact that when the marginal cost exceeds the marginal benefit of investment at the edge point and since we have increasing costs, no investment takes place. Note that if $g'(\alpha) = 0$ this is avoided.

We now define η the elasticity of the proportion of time devoted to work with respect to the interest rate.

$$(13) \qquad \eta = \frac{\partial\,(1 - g)}{\partial \rho} \; \frac{\rho}{(1 - g)}\,.$$

Differentiating Equation (12 *b*), which characterizes the desired path, with respect to ρ and using (13) we get

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(7) Alternatively equation (12 *b*) may be obtained by setting the partial derivative of $J_0 = wA_0 N_0 [1 - g(\alpha)]/[\rho - \alpha - n])$ with respect to α equal to zero. (Condition (9 *c*) cannot be followed for all t since that will violate (11)).

$$\eta = \frac{(g')^2 \rho}{g'' (\rho - \alpha - n) (1 \quad g)} . \tag{14}$$

Our assumptions imply that η is positive.

This variable η will play a crucial role in the discussion of the model of economic growth, in part II.

A change in the expected rate of return to human capital w will affect the marginal cost and the marginal benefit of investment in the same proportion. This leaves the optimum rate of investment unchanged [8].

We shall concentrate now on the derivation of the optimum consumption path for a household which owns human and non-human capital and which has a homothetic time preference ordering. The assumption that time preference ordering is homothetic, with its implication that the income elasticity of consumption is unity, was used and tested first by Friedman [3]. Our analysis is based on that of Uzawa [8] in which the bahavior of the individual unit concerning consumption and saving is more formally analyzed. Our purpose is to extend his method of analysis to include the individual's decisions regarding investment in human capital as well.

Following [8] we represent the intertemporal utility of the individual by the index U_t,

$$U_t = \int_t^{\infty} C_\tau e^{-\Delta_\tau} d_\tau \text{ where } \Delta_\tau = \int_t^{\tau} \delta_s \, ds \text{ for } 0 \leqq t \leqq \infty , \tag{15}$$

where $\delta_s = \delta (C_s/U_s)$, for $t \leqq s \leqq \infty$, is a function of the ratio of current consumption C_s to the index of future consumption U_s. The variable δ, serving as a subjective rate of discount of future consumption, is decreased as the ratio of current to future consumption is increased. The schedule $\delta (C/U)$ is assumed to be strictly convex. The index of future consumption U_t is seen in (15) to be linear homogeneous in the entire stream of consumption C_τ for $t \leqq \tau \leqq \infty$, reflecting homothetic intertemporal preference ordering.

The problem is then to find the stream of consumption C_t (for $0 \leqq t \leqq \infty$) such that U_0 is maximized, given the initial of human and non human wealth (J_0 and B_0). The solution to this problem is a straightforward application of Uzawa's theory of homothetic preference ordering, once we transform the variables (appearing in (4) and (5)) into per human capital units. The new set of variables will be designated by lower case letters as follows

(8) If we include direct outlays in the cost of education this statement is no longer true. This case is discussed in Part III.

$$c_t = C_t/A_t N_t, u_t = U_t/A_t N_t, b_t = B_t/A_t N_t,$$

where u is expressed by $u_t = \int_t^\infty c_\tau e - \int_t^\tau (\delta_s - \alpha - n)\, ds\, d\tau$, $\delta_s = \delta\,(c_s/u_s)$,

with a new rate of discount (i.e., $\delta - \alpha - n$) and where α is determined in (12). Since A_t and N_t are given (as of time t_t) the problem of maximizing the quantity U_t at time t is equivalent to that of maximizing u_t at time t.

The rate of increase in the ratio of physical to human capital b_t is given by

$$(16) \qquad \dot{b}_t = \frac{\dot{B}_t}{A_t N_t} - (\alpha + n)\, b_t$$
$$= w\,(1 - g\,(\alpha)) + (\rho - \alpha - n)\, b_t - c_t \text{ for } 0 \leqq t \leqq \infty$$

where use was made of (5).

The last expression in (16) suggests a new definition of net income y_t (in units of human capital),

$$(17) \qquad y_t = w\,(1 - g\,(\alpha)) + (\rho - \alpha - n)\, b_t$$

where total wealth (in units of human capital) is given by $y_t/(\rho - \alpha - n)$.

Comparing (17) with (4), we see that net income is equal to total income minus the « depreciation » of the ratio of phycisal to human capital, stemming from investment in human capital and population growth (i.e., the « depreciation » of b_t is given by $(\alpha + b)\, b_t$).

We can transform the differential equation of b (16) into one involving net income y (using (16) and (17)):

$$(18) \qquad \dot{y}_t = (\rho - \alpha - n)\,(y_t - c_t)\,.$$

The problem is now restated as the one of maximizing u_0 subject to (18) and the initial value of net income y_0.

Along the desired path, as planned initially, the consumption-net income ratio c_t/y_t must remain constant (9); therefore we can write

$$(19) \qquad c_t = (1 - s)\, y_t$$

(9) See [8], page 635.

where s is a function of ρ (to be specified in (20)). In view of (19), the relative rate of change of consumption (per human capital) $\dot{c}_t/c_t$ must equal that of net income (per human capital) $\dot{y}_t/y_t$. Let us denote the rate of change in consumption (in original units) by β_t and that of consumption per human capital by $\beta_t - \alpha - n$.

We make use of (18) and (19) to get

$$(20) \qquad s = \frac{\beta - \alpha - n}{\rho - \alpha - n}$$

where $\beta = \frac{\dot{C}_t}{C_t}$ is a function of the rate of interest ρ (10).

Of particular interest is the rate of interest ρ^* for which the consumption-net income ratio is unity. From the definition in (16), the ratio of physical to human capital in the households possession will be maintained constant as the rate of interest is equal to ρ^* (11), Moreover, although s is not related monotonically to ρ everywhere. it is positively related to ρ in the vicinity of ρ^*. Therefore the ratio of physical to human capital in the households possession will be increasing (decreasing) as the rate of interest ρ is higher (lower) than ρ^*. This rate of interest ρ^* was dubbed (by Uzawa) the natural rate of interest.

We are able now to use the foregoing discussion to derive explicitly the consumption function by transforming the variables back into the original variables (in Equations (4) and (5)). The consumption function, in view of (19) and (20), is a constant fraction of net income.

$$(21) \qquad C_t = \frac{\rho - \beta - n}{\rho - \alpha - n} \left[Y_t - (\alpha + n) B_t \right]$$

Multiplying (21) through by $(1/\rho - \beta - n)$ and using (4), the above expression can be transformed into

$$(22) \quad C_t/(\rho - \beta - n) = wA_t N_t [1 - g(\alpha)]/(\rho - \alpha - n) + B_t ,$$

which is a restatement of the budget constraint for a constant rate of increase in consumption β. On the left hand side of (22) is the present value of life time consumption (the discount factor being the rate of interest minus the sum of the rate of increase in consumption and the population growth rate). On the right side of (22) is the

(10) The rate β is, therefore, not a function of time since ρ is stationary. This rate β is determined by the schedule $\delta(c/u)$ and by the rate of interest as shown in [8] page 636.

(11) This rate of interest ρ^* can be shown to be uniquely determined by the schedule $\delta(c/u)$ and by $g(\alpha)$. See [8] page 637 and use (20).

total wealth in human and physical forms. Therefore, *for a consumer with an infinite horizon and homothetic intertemporal preference ordering, the desired consumption can be expressed as a multiple of total wealth.*

Some remarks on the consumption function, just derived, are now in order.

First, we note that a subsidy given to the investor in human capital, under which total wealth remains constant, will leave consumption intact. Therefore the induced change in the rate of investment in human capital will be offset by opposite change in the rate of saving. This is due to the fact that the multiple which relates consumption to total wealth is a function only of the rate of interest.

Secondly, Equation (21) implies that the ratio of consumption to current income, C_t/Y_t, is related negatively to the relative share of income from non-human assets $\rho B_t/Y_t$ (if the rate of change in human capital $\alpha + n$ is positive). Therefore, this consumption function Equation (21), is consistent with the observed difference between the measures of the average propensity to consume from time series and cross-sectional data (12). In the first case the rate of interest and the relative share of income from physical capital are relatively constant. In the latter, there might be a positive correlation between the level of income and the share of income from physical capital in total income. This might explain the findings of zero correlation between the average propensity to consume and the level of income in time series studies, and the negative correlation between those variables in cross-sectional data.

II. *A Model of Economic Growth.*

In this section we present a simple aggregative growth model. The purpose is to describe the interrelationship existing between investment by households, and investment made by firms, along the growth path of the economy (13).

We assume an aggregative production function, in which labor is measured in efficiency units, exhibiting diminishing marginal rates of substitution and constant returns to scale:

$$Y_t = F(K_t, A_t L_t) = A_t L_t f(K_t/A_t L_t) \tag{23}$$

where Y: total output
A: the amount of human capital
K: the amount of physical capital
L: labor engaged in production,

where $f' > 0$, $f'' < 0$ and $f'(\infty) > \bar{\alpha} + n$ (14).

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(12) I owe this observation to Frank Mills.

(13) In our simplified model the aggregation is carried out over identical individual units. The aggregate amount is simply the number of individuals times the individual amount.

The supply of labor available in production is related to the total population N_t by

$$L_t = (1 - g_t)\, N_t \tag{24}$$

where g is the proportion of the population engaged in education ([15]). Aggregate consumption is given by the fraction of income to be consumed, $1 - s$, times the income concept appropriate for consumption. (We assume, as in Part. I, that the time preference ordering of the representative consumer is homothetic). In equilibrium, where the value of assets holdings B equals the capital value K, the consumption function (derived from (21)) is given by

$$C_t = [1 - s_t]\,[Y_t - (\alpha_t + n)\, K_t] \tag{25}$$

where s is the saving net income ratio.

Investment in human capital will follow the rule given by Equation (12).

Investment by firms I is written for simplicity as the change in the capital stock ([16]),

$$I_t = \dot{K}_t\,. \tag{26}$$

The investment by firms is equal to the saving by households S at any point of time,

$$I_t = S_t \tag{27}$$

$$\text{where } S_t = Y_t - C_t\,.$$

We assume that competition prevails in the market, so that the wage rate is equal to the marginal productivity of labor and the interest rate is equal to the marginal productivity of capital

$$w_t = f - \left(\frac{K_t}{A_t\, L_t}\right) f',\ \rho_t = f'\,. \tag{28}$$

([14]) Note that this would guarantee, in term of the market rate of interest given by (28) that the discounted sum of human earnings is always finite. See footnote (5).

([15]) This is true since households are assumed to be identical and g is the proportion of time each one of them allocates for education.

([16]) Exponential depreciation of K can be easily incorporated into the model without any change in its qualitative results. In [8], a more general approach is adapted. The investment is the amount of current output required for the expansion of the capital stock.

Using this approach we cannot ascertain the uniqueness of the monetary equilibrium of the model, and there are other analytical difficulties.

In order to focus our attention on the role of the investment in human capital in the growth process, we preferred to adapt the simpler approach with respect to the firms' investment.

The system of Equations (12) and (23)-(28) describes fully the momentary equilibrium for the economy, while the amounts of A_t, N_t, and K_t at any istant of time are inherited from the past hystory of the economy.

The motion of the system from one instant of time to the next one is specified as follows:

$$(29)\quad \begin{array}{lll} \dot{A}_t = \alpha_t A_t, & A(0) = A_0 & \text{(change in human capital)},\\ \dot{N}_t = nN_t, & N(0) = N_0 & \text{(population growth)},\\ \dot{K}_t = I_t, & K(0) = K_0 & \text{(change in firms' capital)}. \end{array}$$

We turn now into the analysis of the working of the above model.

We can first, reduce the model into the one expressed in terms of the ratio of physical to human capital

$$(30)\qquad k_t = K_t/A_t N_t.$$

This will be called simply the Capital Ratio.

The momentary equilibrium values of the rate of interest and the rate of investment in human capital are determined by the system of the two equations (using (12) and (28)):

$$(31)\qquad \rho_t = f'\left(\frac{k_t}{1 - g(\alpha_t)}\right), \text{ and } (\rho_t - \alpha_t - n)\, g'(\alpha_t) = 1 - g(\alpha_t)$$

for $\alpha > \underline{\alpha}$, or $(\rho_t - \underline{\alpha} - n)\, g'(\underline{\alpha}) \geq 1$ for $\alpha = \underline{\alpha}$, given the value of k_t.

We wish to find conditions under which a momentary equilibrium exists. Let us define a differentiable function of α

$$(32)\quad \Lambda(\alpha) = [f'\left(\frac{k}{1-g}\right) - \alpha - n]\, g' - 1 + g \text{ for } \underline{\alpha} \leqq \alpha \leqq \overline{\alpha},$$

where the values of the function at the two edges are

$$\Lambda(\overline{\alpha}) = [f'(\infty) - (\overline{\alpha} + n)]\, g'(\overline{\alpha}),$$
$$\Lambda(\underline{\alpha}) = [f'(k) - \underline{\alpha} - n]\, g'(\underline{\alpha}) - 1.$$

In view of (23) $\Lambda(\overline{\alpha})$ is positive.

The sign of the derivative of the function is

$$(33)\qquad \text{Sign } \Lambda'(\alpha) = \text{Sign } (\sigma - \eta S_L)$$

where S_L is the relative share of labor income in total income, σ is the elasticity of substitution in production, and η is the elasticity of the proportion of time devoted to work with respect to the rate of interest [17].

In terms of (31)-(33) we can state that [18] *if* $g'(\underline{\alpha}) \leqq 1/[f'(0) - \underline{\alpha} - n]$ *and* $\sigma > \eta S_L$ *then at every point of time there exists a unique equilibrium with some positive fraction of the labor force engaged in education; whereas if* $g'(\underline{\alpha}) > 0$ *and* $\sigma < \eta S_L$ *then at every point of time there exists a unique equilibrium at which there is no educational activity.*

This is a strong result. It says that the magnitudes of few characteristics of the production process and the educational technology might determine whether or not the economy specializes in work.

(All other cases, not covered under the above statement, leave open the possibilities of either no equilibrium or of several solutions).

In the rest of the paper we shall restrict our attention to the case,

$$g'(\underline{\alpha}) \leqq \frac{1}{f'(0) - \underline{\alpha} - n}\,, \quad \sigma > \eta S_L\,. \tag{34}$$

Let us now denote the amount of investment undertaken by firms, per unit of physical capital by z_t (i.e., $z_t = I_t/K_t$). Equations (25)-(27) then reduce to

$$z_t = s_t\, a_t + [1 - s_t]\,[\alpha_t + n] \tag{35}$$

where a is the average productivity of physical capital.

Given the equilibrium values of ρ and α in (31) the amount of investment z is solved from (35). This completes our discussion of the momentary equilibrium of the system.

The motion of the system expressed by (29) can be simply restated in terms of the Capital Ratio as

$$\dot{k}_t = (z_t - \alpha_t - n)\, k_t\,, \qquad k(0) = k_0\,. \tag{36}$$

Consider first the steady state of the economy, in which the Capital Ratio is maintained constant at k^*. In view of our discussion in part I, there exists only one rate of interest ρ^* which will induce households to maintain the Capital Ratio constant (where the ratio of consumption to net income is one). Therefore we can characterize the steady state of the economy by

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[17] The explicit expressions for S_L and σ are given by:

$$S_L = 1 - \frac{f'k}{(1-g)\,f} \qquad \sigma = -\frac{f'\,(f - (k/(1-g))\,f')}{ff''\,k/(1-g)}\,.$$

The concept of η is defined in (13).

[18] The inequalities should hold for $0 < k < \infty$ and for $\underline{\alpha} < \alpha < \bar{\alpha}$. The above statement is an application of Weierstrass Intermediate Value Theorem for continuous functions defined on a closed interval.

(37)
$$\rho^* = f'(k^*/(1 - g(\alpha^*)) ,$$
$$(\rho^* - \alpha^* - n)\, g'(\alpha^*) = 1 - g(\alpha^*),$$
$$z^* = \alpha^* + n .$$

The steady state values of α, z, k are uniquely solved from (37). (First we solve for α^* from the second Equation in (37). Then, the value of z^* is determined in the last equation of (37) and the value of k^* from the first one.) To discuss the stability of the long run equilibrium (the steady state) of the economy we differentiate the tight hand side of (36) with respect to k where this expression is evaluated at the steady state.

(38)
$$\frac{d\,(z - \alpha - n)\,k}{dk} =$$
$$= \frac{\rho s S_L}{\eta S_L - \sigma}\left[\frac{s'}{s}(a - \alpha - n) + \frac{(1 - g)}{g'}\eta + \frac{\sigma}{(1 - S_L)}\right].$$

According to our assumptions the term in the brackets on the right hand side of (38) is positive. Therefore, *the long run equilibrium of the economy is stable.* (Note that in the case of a complete specia-lization in work the stability of a long run equilibrium is also gua-ranteed).

We can say even more about the rate of technical progress as the economy develops. Under conditions (34) we differentiate totally Equation (31) and solve for $d\rho/dk$ to get

$$\frac{k}{\rho}\,\frac{d\rho}{dk} = \frac{S_L}{\eta S_L - \sigma}.$$

Therefore, the rate of interest is negatively related to the Capital Ratio. This implies that *the rate of technical progress due to investment in human capital is an increasing function of the Capital Ratio.*

We might say that the higher the stage of development the higher the rate of technical progress.

We describe in the following diagram, in Figure 1, the market paths for the rates of investment in the two forms of capital, for our economy. This picture summarizes our view of how technical progress is related to the other *economic* variables. In particular, as our model of technical progress implies, the factor which determines the rate of augmentation of the labor productivity, is the Capital Ratio.

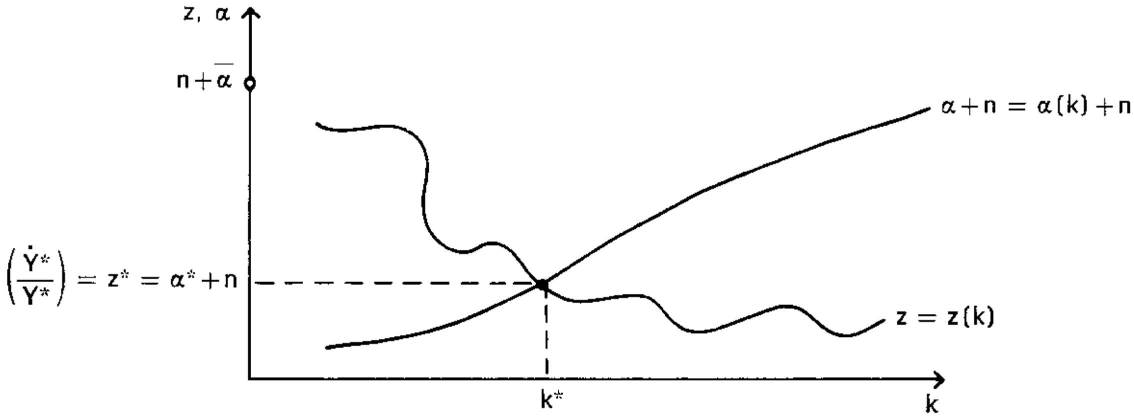

Figure 1. - The Market Paths of Investment in Two Forms of Capital.

III. *Policy Implications and a Summary.*

Let us demonstrate how to analyze some impacts of fiscal policy on the rate of growth of the economy. Consider the example of a direct subsidy to education.

Suppose a direct subsidy p_t (per unit of education) is given to the household persuing education, at the rate $\dot{A}_t$. The total increase in the individual's income is therefore $p_t \dot{A}_t$ (19). We assume stationary expectations regarding the level of p_t in the future, thus $p_t = p$. We proceed, in solving this problem, as in Part. I. The desired rate of investment is determined by equality of marginal cost of investment $wg'(\alpha) - p$, with the present value of increments to income resulting from the enlarged human capital stock, $[w(1 - g(\alpha) + p\alpha]/[\rho - \alpha - n]$,

$$wg'(\alpha) - p = \frac{w(1 - g(\alpha) + p\,\alpha}{\rho - \alpha - n} \tag{40}$$

The rate of investment in human capital (solved implicitly from Equation (40)) now depends on the rate of interest and the subsidy rate, relative to the rate of return to human capital, namely $g = g(\rho, w/p)$.

Let us assume that the government raises the funds required for this policy from a tax on income. Without going into the details, wa can demonstrate some implications of such policy. At any moment of time the ratios of capital to effective labor $(k/(1 - g))$ would be higher than otherwise; therefore the wage rate would increase and

(19) This might also be the simplest way to include, in the analysis in Part I, the case of direct cost, additional to foregone earnings, in the process of education (p_t is then negative).

the interest rate decrease. As a result of this policy, the rate of investment made by households would increase and the rate of investment of the firms would most likely decrease.

The implications of direct subsidies to education in terms of the diagram in Figure 1 is a downward shift in the z-curve and upward shift in the α-curve, thus, the ultimate Capital Ratio is lower than otherwise and so is the per capita income in the long run. The rate of accumulation of the Capital Ratio at each instant of time is decreased.

We have analyzed the pattern of economic growth for an economy with two forms of capital stock, human capital accumulated by the household sector and the capital accumulated by firms. The latter capital, of material and also of human type was treated as a specific factor of production to the firm, that is, once invested it yields productive services only to that particular firm. Both forms of capital are therefore non-marketable; only the rents to their services are determined in the market. We have assumed the existence of a dichotomy between two types of decision-making units — households and firms. Assuming that expectations with respect to the rents on the two forms of capital are stationary, we analyzed a model of economic growth, paying special attention to the implied interrelations between the rate of investment in human capital and the rate of investment in the other type of capital along the growth path of the economy. It is possible to explain these interrelations, as well as the movement of the other variables in the system (such as total output, wage rate, and the rate of interest) over time by the Capital Ratio — the ratio of capital accumulated by firms to capital accumulated by households. This theoretical finding is in accord with Uzawa [6] and also with some empirical results in the literature, such as Schultz [5].

It might prove fruitful to analyze a more complex structure of expectations compared with that being used in the present paper. The omission of the supply of education, by the educational institutions, from the above model, limits its applicability.

The above study may therefore be viewed as a limited attempt to explain the economic role of investment in human capital in the general pattern of economic growth.

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