9.5.2016

14. The Ricci tensor is defined by contracting one index of the curvature tensor:

$$R_{\mu\nu} = R^{\alpha}_{\mu\nu\alpha}$$

"Count" the number of independent DOF of the Ricci Tensor, and curvature tensor for $d = 2, 3, 4$. (2=1+1 etc.).

What could be the physical implications for Einstein's theory in low dimensions?

15. The Einstein equation is given by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

where $R = R^{\alpha}_{\alpha}$ is the curvature scalar, and G Newton's constant.

Assume that $d = 2 + 1$ and that:

a. $T^{00} = M\delta(r)$.
(i.e. matter is static and matter carries only energy).
b. $g_{00} = N^2(r)$ and $g_{ij} = -e^{\phi(r)}\delta_{ij}$.
(the metric has spherical symmetry).

Write down the resulting differential equations for $\phi(r)$ and $N(r)$,

16. Show that the solution is

$$(ds)^2 = (dt)^2 - r^{-8GM}((dr)^2 + r^2(d\theta)^2)$$

** Generalize to the case of several point-particle sources, at $r = r_i$.

17. Show that spacetime is flat everywhere for $r \neq 0$: find new coordinates ρ and Θ and a transformations that brings the metric to the form:

$$(ds)^2 = (dt)^2 - ((d\rho)^2 + \rho^2(d\Theta)^2)$$

with $0 < \rho < \infty$, and $0 < \Theta < 2\pi\alpha$, $\alpha \equiv 1 - 4GM$.

Therefore space is equivalent to a cone (embeded in 3+1), and the matter is located at the wedge of the cone.

** What happens in the case of several point-particle sources?