TOPIC 3: AGENCY MODELS OF CAPITAL STRUCTURE

1. Introduction

2. The agency cost of outside equity

The first paper that opened the black box called the firm, was the seminal paper of Jensen and Meckling (*JFE*, 1976). In this paper they look more closely at the investment decisions made by those who run the firm and show that external financing distorts these investment decisions. The cost associated with these distortions are called agency cost since the decision makers are agents for investors. The Jensen and Meckling paper is actually divided to two parts. In the first, Jensen and Meckling provide a model that shows that equity financing leads to an agency distortions and a second part provides another model that shows that debt financing also leads to agency distortions, although the nature of these distortions is different from the distortions created by outside equity.

In this section we consider the first model in Jensen and Meckling. The main idea can be presented in at least two ways. In the first way, which will be referred to as the "investment" model, an entrepreneur establishes a firm in period 0 and raises funds by issuing equity to outsiders. Then in period 1, the entrepreneur decides how much to invest in the firm. The difference between the amount of resources that the firm has and the cost of investment is spent by the entrepreneur on perks, i.e., all sorts of expenditures that benefit the entrepreneur personally but do not enhance the value of the firm. The most obvious examples of perks are maintaining an expensive office, taking expensive business trips, and receiving high wages. Other examples may include investing in pet projects that do not have a positive NPV or expanding the size of the firm in a way which is not profitable.

The second way to present the same idea is to consider the "effort" model. In this model, the entrepreneur needs in to raise in period 0 a fixed amount from external sources. That is, the amount that the entrepreneur raises from outside equityholders is now given exogenously. In period 1, the entrepreneur needs to choose a personal effort level that the determines the value of the firm.

In both the investment model and in the effort model, the agency cost arises because there is a difference between the objective function of outside equityholders, which is to maximize the total value of the firm, and the entrepreneur’s objective function that also takes into account the utility from perks in the investment model, and the disutility from effort in the effort model. This divergence of objectives means that the entrepreneur will not make optimal decisions from the point of view of outside equityholders. Outside equityholders in turn will anticipate this and will therefore not pay as much for the equity stake they buy as in the case where the entrepreneur’s decision where optimal.

The investment model

Initially in period 0 the entrepreneur has R dollars. If he invests I dollars out of this amount in the firm, then in period 1, the return on investment is V(I), where V is an increasing and concave function so that $V'(I) > 0 > V''(I)$ and $V(0) = 0$. To ensure that the entrepreneur’s problem has an interior solution we shall assume that $V'(0) = \infty$ and $V'(\infty) = 0$. The entrepreneur consumes the rest of the money which was not invested, R - I, as perks. To simplify matter we shall assume that the entrepreneur's utility is equal to the sum of his share in the firm's value plus his utility from perks which is equal to their size, i.e., given by R - I. The entrepreneur's utility is given by

$$U = V(I) + (R - I).$$

The first term is the entrepreneur's utility in period 1 while the second term is the entrepreneur's utility from perks in period 0.

Internal Financing

Absent external financing, the entrepreneur's problem in period 1 can be written as follows:

$$\max_{I} \; V(I) + (R - I) \qquad s.t. \quad I \leq R. \tag{1}$$

The constraint reflects the idea that the entrepreneur cannot investment in period 0 more money than the firm has. Ignoring the constraint for the moment, the optimal amount of investment from the entrepreneur' point of view is given by the following first order condition:

$$V'(I) = 1. \tag{2}$$

The existence of an interior solution for this condition is ensured by the assumptions that $V'(0) = \infty$ and $V'(\infty) = 0$, and we shall denote it by I^*. The resulting value of the firm $V(I^*)$. Hence the entrepreneur's payoff is:

$$U^* = V(I^*) + \boldsymbol{R} - I^*. \tag{3}$$

Now suppose that I* > R: the resource constraint is binding. Absent external financing, the entrepreneur will invest all the firm's resources and will not consume any perks. The reason for this is simple: the entrepreneur can either allocate money to investment or to perks. The marginal benefit from investment is V'(.) while the marginal benefit from perks is 1. So long as V'(.) > 1, the entrepreneur will prefer to invest. If the firm has enough money, the entrepreneur will be able to reach the optimal level of investment I* and will spend the rest of the money on perks. But if the firm has too little resources, the entrepreneur will invest all the money and will consume no perks since the marginal return from investment exceeds the marginal return from perks.

Obviously, when I* > R the entrepreneur has an incentive to raise external funds in order to invest them. The amount of money that the entrepreneur needs to raise is I* - R. The entrepreneur can raise this money by either issuing new equity or by issuing debt that becomes due at the end of period 1.

Debt Financing

Under debt financing, the entrepreneur will promise debtholders that if they lend him I* - R in period 0, he will repay them D in period 1. What should D be in that case such that debtholders will give the entrepreneur I* - R in period 0?

To answer this question, note that after raising I* - R in period 0, but before choosing I, the entrepreneur's payoff is

$$U = (V(I) - D) + R + (I^* - R) - I \\ = V(I) - I - D + I^*, \tag{4}$$

where V(I) is the return from the project minus the debt payment (i.e., the entrepreneur's payoff in period 2), R+(I*-R) is the amount of money the firm has in period 1 (R is the entrepreneur's resources and I* - R is the amount raised from debtholders), and I is the cost of investment.

It is easy to see that this expression is maximized at I*: the entrepreneur has an incentive to invest optimally. Since V(I*) > I* (if that was not the case the entrepreneur would have been better off not investing), the firm generates enough money to repay debtholders (the firm borrowed from them I* - R < I*). Hence debt is safe. Normalizing the interest rate to 0, it follows that D* = I* - R. That is, the entrepreneur will promise debtholders a payment of I* - R in period 1 provided that they give him I* - R in period 0. If the capital market is perfectly competitive, debtholders will agree.

Given that D* = I* - R, the entrepreneur's payoff in equilibrium is

$$U^* = \left(V(I^*) - D^*\right) + R + (I^* - R) - I^* \\ = V(I^*) + R - I^*, \tag{5}$$

exactly as if he had enough money to begin with.

Equity Financing with Commitment

Now suppose that the entrepreneur raises the money needed for investment by issuing equity and can commit to invest I*. In that case he can raise I*-R by promising equityholders an equity stake of 1-α (α is the equity stake that the entrepreneur retains). Since the entrepreneur commits to invest I*, the market value of equity is going to be

$$E^* = (1 - \alpha) V(I^*).$$

Hence to raise I*-R we must have

$$I^* - R = (1 - \alpha) V(I^*) \Rightarrow \alpha^* = 1 - \frac{I^* - R}{V(I^*)}.$$

Since the entrepreneur retains an equity stake α* in the firm, his expected payoff is

$$\begin{aligned} U^* &= \alpha^* V(I^*) + E^* + R - I^* \\ &= V(I^*) + R - I^*. \end{aligned}$$

This expression is exactly as in the case where the firm had enough money to invest. Hence, if the firm can either finance itself with debt or can finance itself with equity and can promise equityholders that it will take an efficient investment decision, then capital structure is irrelevant: the value of the firm is the same whether it finances itself with debt or equity.

Equity Financing without Commitment

Now let's turn to the case in which the firm does not have enough money to invest, the amount I*-R is raised by issuing equity, and the entrepreneur cannot promise to invest a particular amount. That is, let's assume that now the entrepreneur has full discretion how much to invest. A crucial assumption here is that the entrepreneur retains full control over the firm even though there are now additional equityholders. In particular, the entrepreneur alone makes decisions on how much money to invest and he alone can consume resources as perks. New equityholders are entitled to get some share, 1-α, of the net profits of the firm. The amount that can be raised by issuing such equity is

$$E = (1 - \alpha) V(I).$$

After raising E from outsiders, the total resources of the firm are R+E. Since the entrepreneur retains only a fraction α of the firm's equity, and since the amount left for perks is R+E-I, the maximization problem of the entrepreneur in period 1 becomes:

$$\max_{I} \ \alpha V(I) + (R + E - I) \qquad s.t. \quad I \leq R + E. \tag{6}$$

The constraint indicates that investment is limited by the resources that the firm has in period 1 which are equal to R+E. Ignoring the constraint for the moment, and recalling that by assumption, V'(0) = ∞ and V'(∞) = 0, the optimal amount of investment is given by the following first order condition:

$$\alpha V'(I) = 1. \tag{7}$$

We shall denote the solution to this first order condition by I**(α).

It is easy to see from (7) that since V'(.) is a decreasing function, I**(α) will be an increasing function: the higher α is, the larger is the fraction of profits that accrue to the entrepreneur, and hence the stronger is his incentive to invest. But since I* is chosen only when α = 1, i.e., I**(1) = I*, it follows immediately that so long as α < 1, we have I**(α) < I*. In other words, in the presence of outside equity, the entrepreneur will underinvest. The intuition for this is underinvestment result is straightforward: the entrepreneur has to share the net profits of the firm with other equityholders but he alone consumes perks. Hence, when α < 1, the marginal benefit from investment is lowered from the entrepreneur's point of view while the marginal benefit from perks remains 1. Consequently, outside equity distorts the entrepreneur's investment decision. This distortion represents an agency cost since the entrepreneur serves as an agent for outside equityholders. But since unlike them he also consumes perks, the entrepreneur is an imperfect agent and this results in an agency cost which is reflected in the distortion of the investment decision. Under debt financing there is no such distortion since debtholders are guaranteed a fixed sum that does not depend on the investment decision. Likewise, if the entrepreneur can commit to invest efficiently, there are no agency costs since his payoff is exactly as in the case where he had sufficient money to invest. This implies that the agency costs arise due to the inability of the entrepreneur to commit to the amount of investment that he will take.

The question now is who bears the agency cost of outside equity? At a first glance it seems that outside equityholders bear this agency cost since the entrepreneur "wastes" their money. However a deeper look reveals that since outsiders can anticipate the entrepreneur's incentives, they will realize that after they own a fraction $1-\alpha$ of the firm's equity, the investment will be $I^{**}(\alpha)$ rather than I^* and hence, when they buy the firm's equity in period 0 they will pay a price that reflects this agency distortion. In other words, since the capital market is perfectly competitive, outsiders will just break even no matter what the investment level is and hence the entire agency cost will be borne by the entrepreneur. Thus, while in period 0 the entrepreneur will wish to promise to investors that he will invest optimally in order to induce them to pay a high price for the firm's equity, in period 1 after equity has already been issued, the entrepreneur will not be able to "resist the temptation" to consume more perks than he should.

To see the agency cost more clearly, note that since new equityholders anticipate $I^{**}(\alpha)$, the external funds that the entrepreneur will raise are equal to

$$E^{**}(\alpha) = (1 - \alpha)V(I^{*}(\alpha)).$$

Hence, the entrepreneur's payoff is

$$\begin{aligned} U^{**}(\alpha) &= \alpha V(I^{**}(\alpha)) + (R + E^{**}(\alpha) - I^{**}(\alpha)) \\ &= V(I^{**}(\alpha)) + R - I^{**}(\alpha). \end{aligned} \tag{8}$$

Comparing this expression with the entrepreneur's payoff under debt financing, we see that $V(I^{**}(\alpha)) < V(I^{*})$, but $R - I^{**}(\alpha) > R - I^{*}$: the entrepreneur owns a firm that is worth less but he consumes more perks. To determine which expression is larger, recall that when the firm has enough funds (i.e., when the resource constraint is not binding), the optimal amount of investment is I^*. Hence by revealed preferences,

$$V(I^{*}) - I^{*} > V(I^{**}(\alpha)) - I^{**}(\alpha).$$

In other words, when R is sufficiently large, the entrepreneur can choose $I^{**}(\alpha)$, but instead he chooses to invest I^*, which as we know exceeds $I^{**}(\alpha)$. Thus, the entrepreneur choice of I^* "reveals" that his payoff when he invests I^* is larger than it is when he invests $I^{**}(\alpha)$. This implies in turn that the entrepreneur is better-off under debt financing than under equity financing. The difference between his payoffs under the cases is the agency cost of outside equity.

The discussion so far is still incomplete because $I^{**}(\alpha)$ is a function of α and we still have not said anything about the amount of equity that the entrepreneur will choose to put up for sale (i.e., the fraction of equity, $1-\alpha$, that the entrepreneur will offer to outsiders). To address this issue, recall that the entrepreneur's payoff is given by $U^{**}(\alpha)$. This expression depends on α only through $I^{**}(\alpha)$ which is an increasing function of α. As we saw earlier, $V(I) + (R-I)$

attains a maximum at I*. Since $I^{**}(\alpha) \leq I^*$, it follows that the more the entrepreneur invests, the higher his payoff is. Since an increase in α leads to more investment, it follows in turn that it also increases the entrepreneur's payoff. Hence, $U^{**}(\alpha)$ increases with α. More formally, note that the derivative of the entrepreneur's payoff with respect to α is given by:

$$\frac{dU^{**}(\alpha)}{d\alpha} = \left(V'\left(I^{**}(\alpha)\right) - 1\right)\frac{dI^{**}(\alpha)}{d\alpha}. \tag{9}$$

Since $dI^{**}(\alpha)/d\alpha > 0$ ($I^{**}(\alpha)$ increases with α), the sign of the derivative depends on the sign of $V'(I^{**}(\alpha)) - 1$. However, the first order condition for $I^{**}(\alpha)$ (equation (7)) implies that $V'(I^{**}(\alpha)) = 1/\alpha > 1$. Hence, $V'(I^{**}(\alpha)) - 1 > 0$, implying that the entrepreneur's payoff indeed increases with α.

So does the previous argument imply that α should be raised as much as possible? The answer is of course no because if $\alpha = 1$, outsiders get no stake in the firm and hence E = 0: the entrepreneur will not be able to raise enough external funds to finance the investment which, when $\alpha = 1$, is equal to I* (recall that $I^{**}(1) = I^*$). Hence, the entrepreneur will wish to raise α as much as possible subject to the resource constraint:

$$I^{**}(\alpha) < R + E^{**}(\alpha),$$

where $E^{**}(\alpha) = (1-\alpha)V(I^{**}(\alpha))$

To determine the highest value of α that satisfies this constraint, let's first examine how the value of outside equity, $E^{**}(\alpha)$, varies with α. It might be thought that the larger is the outsiders' stake in the firm, i.e., the larger is $1-\alpha$, the larger $E^{**}(\alpha)$ be. This intuition however ignores the fact that the entrepreneur's incentive to invest is weakened the smaller α is. Indeed, when $1-\alpha = 1$, i.e., outsiders own the entire firm, the entrepreneur has 0 stake in the firm's equity and hence has no incentive to invest. Hence, when $\alpha = 0$, $I^{**}(0) = 0$, and hence $V(I^{**}(0)) = 0$: outside equityholders own the entire firm but the firm is worthless because the entrepreneur will not invest. This suggests that $E^{**}(\alpha)$ is an inverse U-shaped function of α: initially $E^{**}(\alpha)$ increases with α because the higher α is, the stronger is the entrepreneur's incentive to invest. The resulting increase in the firm's value, more than compensates the outside equityholders for the reduction in their equity stake which is associated with an increase in α. Put differently, outside equityholders get a smaller share in a bigger pie and overall their payoff increases. But then, as α increases beyond a certain level, the reduction in the outside equityholders' stake that results from a further increase in α outweighs the positive effect of α on the firm's value and hence for sufficiently large values of α, $E^{**}(\alpha)$ declines with α.

Given that $E^{**}(\alpha)$ is an inverse U-shaped function of α, means that the right-hand side of the resources constraint is also an inverse U-shaped function of α. On the other hand, the left-hand side is equal to $I^{**}(\alpha)$ which is an increasing function of α. Now, note that at $\alpha = 0$ the resources constraint is satisfied since $I^{**}(0) = 0$, while $R + E^{**}(0) = R$. On the other hand, at $\alpha = 1$ the resource constraint is violated since $I^{**}(1) = I^*$, while $R + E^{**}(1) = R < I^*$. Hence, there is a unique value of α at which the resource constraint is satisfied. The entrepreneur will choose this value of α. That is, the value of α that the entrepreneur will choose is defined implicitly by the equation

$$I^{**}(\alpha) = \boldsymbol{R} + (1-\alpha)V(I^{**}(\alpha)). \tag{10}$$

An example

To illustrate, suppose that $V(I) = kI^{1/2}$, where $k > 0$. Then, if the entrepreneur issues equity that gives outsiders an equity participation $1-\alpha$, he will choose I to maximize the expression

$$\alpha k I^{1/2} + R - I.$$

Maximizing this expression reveals that $I^{**}(\alpha) = \alpha^2 k^2/4$. Given this investment level, the amount that equityholders are willing to give the entrepreneur is:

$$E^{**}(\alpha) = (1-\alpha)k(I^{**}(\alpha))^{1/2} = \alpha(1-\alpha)k^2/2.$$

Together with the R dollars that the firm has to begin with, the firm can therefore invest $R + E^{**}(\alpha)$.

The entrepreneur's problem is then to maximize his own payoff which is

$$\alpha k(I^{**}(\alpha))^{1/2} + R + E^{**}(\alpha) - I^{**}(\alpha) = (2-\alpha)\alpha k^2/4 + R,$$

subject to the constraint that $I^{**}(\alpha) \leq R + E^{**}(\alpha)$. The objective function is maximized at $\alpha = 1$: the entrepreneur is as well-off as he can possibly be if he retains all the firm's equity in his hands. But then $E^{**}(1) = 0$: if outside equityholders get no equity stake they are not willing to give the firm any money. Hence at $\alpha = 1$ the constraint is violated. Thus, the entrepreneur will have to choose α such that the constraint will be just binding which occurs when

$$\alpha = (1 + (1 + 12R/k))^{1/2})/3.$$

Conclusion

The model we just saw establishes then that debt financing should be preferred to equity financing since it does not create an agency distortion like equity. However before we can conclude that firms should always prefer debt financing, we shall move to the second model that appears in Jensen and Meckling (*JFE*, 1976). This second model shows that debt financing also creates agency distortions.

The effort model

This version of the Jensen and Meckling story is very similar to the "investment" model. As before, the entrepreneur has initially R dollars. Now however he needs to invests I dollars in the firm, where I is specified exogenously. That is, investment is no longer a decision variable for the entrepreneur. Instead, the entrepreneur needs to decide how much effort to exert. Given an effort level e, the firm's value in period 1 is V(e), where, as before, V is an increasing and concave function so that $V'(e) > 0 > V''(e)$, with $V(0) = 0$, $V'(0) = \infty$, and $V'(\infty) = 0$. The entrepreneur's disutility from effort is simply equal to e.

Absent external financing, the entrepreneur's problem in period 1 can be written as follows:

$$\underset{e}{Max}\ V(e) - e. \tag{11}$$

The optimal effort level from the entrepreneur' point of view is given by the following first order condition:

$$V'(e) = 1. \tag{12}$$

The existence of an interior solution for this condition is ensured by the assumptions that V'(0) = ∞ and V'(∞) = 0, and we shall denote it by e*. The resulting value of the firm V(e*). Hence the entrepreneur's payoff is:

$$V(e^*) - e^*. \tag{13}$$

Of course, the above solution to the entrepreneur's problem is feasible only if R > I: the entrepreneur has enough resources to finance the required investment. If this is not the case then the entrepreneur will have to raise external funds. If the entrepreneur raises the external funds by issuing debt with face value D, then his payoff in period 0, after raising the money but before choosing his effort level, is

$$V(e) - D - e, \tag{14}$$

Since D is simply a constant that is independent of e, it clearly has no effect on the value of e. Hence the entrepreneur will exert the optimal effort level e*.

It is also easy to note that if the entrepreneur raises the money needed for investment by issuing equity but can commit to an effort level e* then we will have no agency problem. Hence we shall proceed under the assumption that the entrepreneur cannot commit to a certain effort level has in period 1 full discretion about how much effort to exert. After raising E from outsiders, the entrepreneur retains only a fraction α of the firm's equity. Hence his maximization problem in period 1 becomes:

$$\underset{e}{Max}\ \alpha V(e) - e. \tag{15}$$

Recalling that by assumption, V'(0) = ∞ and V'(∞) = 0, the optimal amount of investment is given by the following first order condition:

$$\alpha V^{/}(e) = 1. \tag{16}$$

We shall denote the solution to this first order condition by e**(α). Since V'(.) is a decreasing function, e**(α) will be an increasing function: the higher α is, the larger is the effort level that the entrepreneur will exert and hence the larger will be the firm's value. Since e* is chosen only when α = 1, it follows immediately that so long as α < 1, we have e**(α) < e*. In other words, the presence of outside equity, will induce the entrepreneur to underinvest in effort. The intuition now is that the entrepreneur has to share the net benefit from effort with other equityholders but he alone bears the disutility of effort.

The question now is how much equity will the entrepreneur will offer outsiders. To answer this question, recall that the entrepreneur has only R dollars but needs to invest I > R dollars. Hence the entrepreneur will have to choose α such that the amount he gets from outside equityholders, E**(α) = (1-α)V(e**(α)) is just enough to cover the shortfall I - R. To determine the value of α, note that as in the "investment" model, the amount that can be raised by issuing equity, E**(α) = (1-α)V(e**(α)) is an inverse U-shaped function of α: initially E**(α) increases with α because the higher α is, the stronger is the entrepreneur's incentive to invest in effort. The resulting increase in the firm's value, more than compensates the outside equityholders for the reduction in their equity stake which is associated with an increase in α. However, as α increases beyond a certain level, the reduction in the outside equityholders' stake that results from a further increase in α outweighs the positive effect of α on the firm's value and hence $E^{**}(\alpha)$ begins to decline with α. Moreover, note that $E^{**}(0) = E^{**}(1) = 0$ because when $\alpha = 0$ the entrepreneur will not exert any effort while when $\alpha = 1$, outsiders have no stake in the firm.

Given that $E^{**}(\alpha)$ is an inverse U-shaped function of α and $E^{**}(0) = E^{**}(1) = 0$, means that the equation $E^{**}(\alpha) = I - R$ has either two solutions or none (there is also a possibility that there is a single solution if the maximum value of $E^{**}(\alpha)$ happens by coincidence to be equal to $I - R$). Clearly the case where the equation has no solution is uninteresting since then there is no way in which the entrepreneur can raise enough money for investment. Hence let's consider the case where the equation $E^{**}(\alpha) = I - R$ has two solutions. Since the entrepreneur is better off the higher α is (he retains a larger fraction of the firm's equity), it follows that α will be chosen as the largest solution to the equation $E^{**}(\alpha) = I - R$.

As in the "investment" model, the entrepreneur bears the agency costs that result from his inability to commit to the optimal effort level e^*. The reason for this is that $e^{**}(\alpha)$ is less than e^*. Hence, by revealed preference,

$$V(e^{**}(\alpha)) - e^{**} < V(e^*) - e^*.$$

3. The asset substitution effect

We now turn to the second model in Jensen and Meckling that considers the agency costs of debt. Before turning into the formal model, let's review the main idea first. To this end, suppose that you were playing the following game. In a box there are two sealed envelopes. One envelop contains a $100 bill and the other one is empty. Now suppose that you are offered a choice between either picking one envelop from the box (without knowing what's inside it), or receiving an certain amount of cash instead. What would you do? Well, the answer definitely depend on your attitude towards risk. If you were risk neutral, you will be indifferent between the two options provided that the amount of cash you are offered is $50. If you are offered more than $50 you would certainly pick the certain amount.

Now suppose that the certain amount of cash that you are offered is $70. If you are not strongly risk averse you will certainly pick the cash offer. But what if according to the rules of the game you have to give a friend $50 out of your gains (of course if you end up with 0 you do not have to give him anything). At a first glance it seems that this new detail should not matter: you should still prefer the certain amount of cash over picking one of the sealed envelops at random. However, once we compare the payoffs we see immediately that this is not so. Under the cash offer, your payoff now becomes $70 - $50 = $20. On the other hand, if you pick one of the sealed envelops then there are two possibilities: either you picked the envelop with the $100 bill in which case your payoff is $100 = $50 = $50, or you pick the empty envelop in which case your payoff is 0 since you cannot pay the $50. Since either case is equally likely, the expected payoff associated with picking one of the sealed envelops is ($0+$50)/2 = $25. Since this is $5 more than your payoff if you take the cash offer, you should definitely decide to forgo the cash offer and pick one of the sealed envelops.

What was going on here? Well, if you take the cash offer, you surely need to pay your friend \$50. Your payoff is then \$70 - \$50 = \$20 for sure. If you pick one of the sealed envelops, then you take a gamble. If the gamble succeeds and you pick the envelope with \$100, you get to enjoy it since the payment to the friend is still \$50, so you get to keep all the net gain which is \$100 - \$50 = \$50 in this case. But if the gamble fails and you pick the envelope with 0, you lose only \$20. It is your friend who stands to lose \$50 from the gamble if it fails. But to the extent that you care only about your own payoff, you ignore this loss when you make your choice. In other words, once you need to pay \$50 to a friend, your incentives are no longer as they were before. In particularly, you risk-lover in the sense that you begin to like gambles that have a negative expected return (the expected return of the picking one of the envelops is merely \$50 while the cash offer is worth \$70). The reason for this is that if the gamble succeeds you enjoy the gains whereas if it fails, your friend bears most of the consequences.

This is in a nutshell the asset substitution effect of Jensen and Meckling (*JFE*, 1976). Leveraged firms who act in the interest of their shareholders, like to make risky business decisions even if they have negative expected returns because the equityholders get to keep the profits from these decisions while debtholders stand to lose in case the decisions prove to be a failure. To demonstrate the asset substitution effect, consider a firm that has a choice between either taking a safe project that yields a profit X for sure or a risky project that yields a random return distributed uniformly on the interval [0, 100]. In order to examine the firm's choice, note that the expected return from the risky project is 50 (this is just the expected value of a random variable distributed uniformly on the interval [0, 100]). Clearly then, if $X > 50$, the firm should pick the safe project. Otherwise, if X < 50 its should prefer the risky project.

Now suppose that the firm has an outstanding debt with a face value D where D ∈ (0, 100). Under the safe project, the payoff of equityholders is Y_1 = Max{X-D, 0}. Under the risky project, equityholders are going to get the difference between the return from the project and the debt payments, conditional on the return from the project being higher than D. Thus, the expected value to shareholders if the second project is adopted is

$$Y_2 = \int_D^{100} \frac{\tilde{X} - D}{100} d\tilde{X} = \frac{1}{100}\left(\frac{\tilde{X}^2}{2} - D\tilde{X}\right)\Bigg|_D^{100} = \frac{(100 - D)^2}{200}. \qquad (17)$$

Clearly, the firm should pick the safe project if and only if $Y_1 > Y_2$ or,

$$\{X - D, 0\} > \frac{(100 - D)^2}{200}. \qquad (18)$$

Since the left side of the inequality is always positive, it is obvious that the firm should always go for the risky project if the return from the safe project is less than the face value of debt. This is not very surprising since keeping the safe project leads to a sure bankruptcy while going for the risky project might yield a positive payoff to the equityholders (so long of course that D < 100; if D ≥ 100 then bankruptcy is inevitable so it does not matter what the firm does). Hence we only need to consider cases where X > D, in which case, Max{X-D, 0} = X-D. Now the inequality $Y_1 > Y_2$ becomes:

$$X > X^c(D) \equiv D + \frac{(100 - D)^2}{200} = 50 + \frac{D^2}{200}. \qquad (19)$$

Thus the firm should pick the safe project if and only if its return, X, exceeds $X^c(D)$. Note that if the firm has no debt outstanding, then $X^c(0) = 50$, so the firm is going to pick the safe project if and only if its return exceeds the expected return from the risky project. In contrast, when the firm is leveraged, $X^c(D) > 50$, so whenever $50 < X < X^c(D)$, the firm will pick the risky project despite the fact that the safe project is more profitable. One can think of the difference between $X^c(D)$ and 50 as a premium that the firm requires on the safe project in order to pick it up. This premium in turn increases as the amount of debt increases: the more leveraged the firm is, the premium is equal to 50, the more risk lover the firm becomes and hence it picks safe projects only if they are substantially more profitable then risky projects.

The next step is to generalize this model a bit further. Now suppose that the returns from the risky project, $\tilde{X}$, are distributed on the interval $[0, \bar{X}]$ according to a cumulative distribution function F(X) and let $\hat{X}$ denote the mean return. As before, the payoff to shareholders from the safe project is $Y_1 = \text{Max}\{X\text{-}D, 0\}$. Under the risky project, their payoff is

$$Y_2 = \int_D^{\bar{X}} (\tilde{X} - D) dF(\tilde{X}), \tag{20}$$

where $dF(\tilde{X}) \equiv f(\tilde{X})d\tilde{X}$. Again, since $Y_2 > 0$, the firm should always pick the risky project if $X < D$ since keeping it would lead to a sure bankruptcy. If $X > D$, then comparing Y_1 and Y_2 reveals that $Y_1 \geq Y_2$, i.e., the firm should pick the safe project if and only if

$$\begin{aligned} X > X^c(D) &\equiv D + \int_D^{\bar{X}} (\tilde{X} - D) dF(\tilde{X}) \\ &= \hat{X} + \int_0^D (D - \tilde{X}) dF(\tilde{X}), \end{aligned} \tag{21}$$

where $\hat{X}$ is the mean returns from the risky project. To study the firm's decision note that $X^c(0) = \hat{X}$, so the firm uses an efficient decision rule and picks the safe project if and only if it is more profitable. Thus the second term on the right side of equation (21) represents the premium that the firm requires in order to pick the safe project. Straightforward differentiation reveals that

$$\frac{dX^c(D)}{dD} = F(D) > 0. \tag{22}$$

Thus, as the firm becomes more leveraged it uses more and more risk-lover in sense that it requires an ever larger premiums on safe projects in order to keep them.

So far we just demonstrated that in the presence of debt the firm may like to take gambles that it would not take if it were all-equity. A natural question at this point is why should we care about whether the firm take or does not take gambles? Presumably, the management of the firm makes optimal decisions from the point of view of the equityholders so the value of the firm must be the highest possible irrespective of its capital structure. Well, as we shall see below, this argument is only partly true. It is true that when the firm actually makes the investment decision it behaves optimally as far as the equityholders are concerned. But the behavior of the firm is not at all optimal for debtholders. Hence debtholders lose value due to the inefficient decision that the firm makes and this is going to be reflected in their willingness to pay for the firm's debt when it is just issued. Since the overall payoff of equityholders include the value of debt, the inefficient decisions that the firm makes are going to hurt the equityholders. Of course after debt has been issued it is in their interest to replace projects too often and take gambles. However before they issue debt this is inefficient and hence it lowers their payoff.

To see this point, let us compute the total value of the firm from an ex ante point of view. Let us assume that ex ante, the value of the safe project is yet unknown. What is known is that the value of the safe project will be drawn from the distribution F(.). It is also known that when the firm will have to make a decision, it will know the realization of the safe project X and will be able to either pick this project or the risky one. If the firm is all-equity, then it takes the safe project if and only if $X > \hat{X}$. Ex ante before X is yet known, equityholders anticipate that with probability $1-F(\hat{X})$, X will exceed $\hat{X}$ and the firm will pick the safe project, whose value will be X, and with probability $F(\hat{X})$, X will fall short of $\hat{X}$ and the firm will pick the risky project and will get in expectation, $\hat{X}$. Hence the total value of equity and hence the value of the firm is given by

$$V(0) = F(\hat{X})\hat{X} + \int_{\hat{X}}^{\bar{X}} \tilde{X} dF(\tilde{X}). \tag{23}$$

The first expression here represents the probability that the firm picks the risky project times the expected return in this case and the second expression represents the expected return if the firm picks the safe project. We can also express the value of the firm as follows:

$$V(0) = \hat{X} + \int_{\hat{X}}^{\bar{X}} (\tilde{X} - \hat{X}) dF(\tilde{X}). \tag{24}$$

That is the value of the firm is equal to the expected return from the project plus the value of the option to pick the safe project if it turns out to be successful and yields an above average return. In other words, the firm can always guarantee itself a return of $\hat{X}$ by always picking the risky project. But when the return of the safe project exceeds $\hat{X}$, the firm gets an extra return of $\tilde{X}-\hat{X}$. The second term on the right side of equation (24) is the expected value of this excess return over all states of nature in which the firm realizes it by keeping the first project.

Now suppose that the firm is leveraged. We already know that in that case, the firm is going to pick the risky project whose return is less than $X^c(D)$. Hence the probability of picking the risky project is $F(X^c(D))$. The value of equity now is given by

$$E(D) = F(X^c(D))\int_{D}^{\bar{X}}(\tilde{X} - D)dF(\tilde{X}) + \int_{X^c(D)}^{\bar{X}}(\tilde{X} - D)dF(\tilde{X}). \tag{25}$$

Similarly, noting that $X^c(D) > D$ (otherwise the firm surely picks the risky project), the value of debt is given by

$$B(D) = F(X^c(D))\left[\int_{0}^{D}\tilde{X}dF(\tilde{X}) + \int_{D}^{\bar{X}}DdF(\tilde{X})\right] + \int_{X^c(D)}^{\bar{X}}DdF(\tilde{X}). \tag{26}$$

When the firm is just established, the entrepreneur's wealth consists of the combined value of equity and debt, or the total value of the firm. Using equations (25) and (26) we therefore have:

$$V(D) = E(D) + B(D) = F(X^c(D))\hat{X} + \int_{X^c(D)}^{\bar{X}}\tilde{X}dF(\tilde{X}). \tag{27}$$

After rearranging terms, we can express the value of the firm as follows:

$$V(D) = \hat{X} + \int_{X^c(D)}^{\bar{X}}(\tilde{X} - \hat{X})dF(\tilde{X}). \tag{28}$$

Equation (28) is similar to equation (24). It shows that the value of the firm is equal to the expected return from the project plus the value of the option to adopt the safe project if it is successful and yields a higher payoff to equityholders than what they can get by picking the risky project. Using equation (24) we can now rewrite the value of the firm as:

$$V(D) = V(0) - \int_{\hat{X}}^{X^c(D)} (\tilde{X} - \hat{X})\, dF(\tilde{X}). \tag{29}$$

This equation shows that a leveraged firm is worth less than an all-equity firm. The reason for this is that debtholders anticipate that the firm will take excessive risks at their expense and hence they do not pay for the firm debt as much as they would have if the firm were to adopt efficient replacement rules. Formally, whenever the return on the safe project is such that $X \in [\hat{X}, X^c(D)]$, the leveraged firm selects the risky project that yields a return which in expectation is equal to $\hat{X}$ even though the safe project yields a higher return.

4. The debt overhang problem

Another important paper that examines the implications of agency frictions for the choice of capital structure is "Determinants of Corporate Borrowing" by Myers (*JFE*, 1977). This paper shows that risky debt creates a special kind of agency problem, often referred to as the debt overhang problem, or the underinvestment problem. The latter name points to the heart of the problem: a firm that issues risky debt to outsiders will have an incentive to underinvest in projects and will sometime forgo positive NPV projects.

Myers considers the following two-period model: in period 1, the firm is just established by entrepreneur. At this point, the firm has no assets and no debt (i.e., the firm is all-equity), and has only one future investment opportunity. In period 2, the firm must decide whether or not to invest I dollars in a project that generates earnings equal to X.

A key assumption in the model is that when the firm chooses whether or not to invest in the project, X is <u>already known</u>. However, when the firm is just established, only I is known for sure. The firm still does not know what the realization of X is going to be. Hence from the view point of period 1, the earnings of the project are still a random variable and are represented by $\tilde{X}$ which is distributed over the interval $[0, \bar{X}]$ (where $\bar{X}$ could be ∞) according to a cumulative distribution function $F(\tilde{X})$. Let $\hat{X}$ denote the mean earnings of the firm. That is,

$$\hat{X} = \int_0^{\bar{X}} \tilde{X}\, dF(\tilde{X}). \tag{30}$$

Another important assumption is that the objective of the firm is to maximize the entrepreneur's wealth. This is either because the entrepreneur is also the manager of the firm or because the firm's management is appointed by the entrepreneur and is completely loyal to him. Of course in reality managements may have their own goals and these need not necessarily coincide with those of the owners. Nonetheless at this point we want to focus on the frictions that arise between the entrepreneur (i.e., shareholders) and debtholders so we eliminate all other potential frictions by assuming that the management and the entrepreneur have the same objectives. If the firm forgoes the project, its value, and hence the entrepreneur's wealth is 0. As usual, we simplify matters by assuming that all agents are risk neutral and we normalize the discount rate to 0.

To solve the model, note that it is worthwhile to invest in the project if and only if $\tilde{X} \geq I$, i.e., the project has a positive NPV. Hence, the expected value of the firm in period 1 if it has no debt (so that D = 0) is given by:

$$V(0) = \int_{I}^{\bar{X}} \left[\tilde{X} - I\right] dF(\tilde{X}). \tag{31}$$

The right side of equation (31) represents the firm's expected earnings, net of the cost of the project, over states of nature in which the firm invests in the project. Since the entrepreneur is the sole owner of the firm, his expected wealth is simply V(0).

Now suppose that in period 1 the firm has outstanding debt with face value D. For instance, the firm borrowed money in order to begin its operations, or to fund an R&D project, the result of which is the investment project that the firm can undertake in period 2. Alternatively, the firm takes the proceeds of the debt issue and pays the money to the entrepreneur as a dividend. As before, the firm needs to decide whether or not to invest in period 2. This time however, the firm needs to pay D to outsiders out of its earnings. As it turns out, the firm's decision on whether to invest depends on the maturity of debt. We will therefore consider two situations: in the first situation, debt is short-term and it becomes due before the investment decision is made. In the second situation, debt is long-term and becomes due after the investment decision is made. As we shall see, it is the second situation which is problematic. That is, the debt overhang problem arises only when debt is long-term.

Short-term debt

When debt is short-term, it becomes due before the investment decision is made. Now, there are two decisions that have to be made in period 2: first, the entrepreneur needs to decide whether to pay debtholders their debt or not. If he decides to pay them back, the firm takes a second, short-term loan equal to D^{st} (the short-term loan is needed because the firm still does not have any money as investment has not been even made). The money is then used to pay debtholders their debt, and the entrepreneur retains control over the firm. If the entrepreneur decided to default on the debt payment, the firm goes bankrupt and debtholders become the new owners. Since the entrepreneur is protected by limited liability, his payoff in this case is 0. The second decision that has to be made is whether or not to invest in the project. If debt was paid in full, the investment decision is made by the entrepreneur; otherwise it is made by debtholders.

To analyze the two decisions made in period 2, note that if $\tilde{X}-I \geq D^{st}$, the entrepreneur realizes that if the firm undertakes the project, there will not only be enough money to pay debtholders in full, but there will also remain a profit of $\tilde{X}-I-D^{st}$. This situation is clearly better than defaulting on the debt payment and receiving a payoff of 0. Hence, it is clearly worthwhile for the entrepreneur to take a short-term loan of D^{st}, pay debt in full, and invest I dollars in the project. If on the other hand $\tilde{X}$-I < D^{st}, the entrepreneur realizes that even if the firm undertakes the project, there will not be enough money to cover the debt payment in full. Moreover, in this case the entrepreneur will not be able to get a short-term loan of D^{st} since lenders will realize that the firm will not be able to pay them back this amount even if the project is undertaken. Therefore, the firm declares bankruptcy, the entrepreneur walks away with a payoff of 0, and debtholders gain control over the firm. Once in control, debtholders will decide to invest in the project if and only if $\tilde{X} \geq I$ since their payoff is $\tilde{X}$-I. Overall, the project will be therefore taken again if and only if $\tilde{X} \geq I$, exactly as in the case where the firm was all-equity. When $\tilde{X}-I \geq D^{st}$, the entrepreneur retains control over the firm and invests in the project, while when $D^{st} \geq \tilde{X}-I \geq 0$, debtholders gain control and invest in the project. Hence, the presence of short-term debt does not alters the investment decision of the firm.

Now we shall see that short-term debt also does not affect the market value of the firm. To this end, note that given the above analysis, the market value of debt as a function of its face value, D, is

$$\boldsymbol{B}(\boldsymbol{D}^{st}) \;=\; \int_{I}^{D^{st}+I} \left[\tilde{X} - I\right] dF(\tilde{X}) \;+\; \int_{D^{st}+I}^{\bar{X}} \boldsymbol{D}^{st} dF(\tilde{X}). \qquad (32)$$

The first term on the right side of the equation represents the expected return to debtholders over states of nature in which the firm goes bankrupt and debtholders gain control over the firm. The second term represents the expected return to debtholders over states of nature in which they are paid in full.

Next consider the entrepreneur’s wealth in period 1. It consists of two arguments: the expected value of equity and the expected value of debt (recall that the proceeds from selling debt go to the firm and hence become part of the entrepreneur's wealth). The market value of equity as a function of the face value of debt is given by

$$E(D^{st}) = \int_{D^{st}+I}^{\bar{X}} \left[\tilde{X} - I - D^{st}\right] dF(\tilde{X}). \qquad (33)$$

The market value of equity is the expected earnings of the firm net of the cost of investment and net of debt payments over states of nature in which the entrepreneur remains in control. Adding equations (32) and (33) reveals that the entrepreneur's wealth is

$$E(D^{st}) + B(D^{st}) = \int_{I}^{\bar{X}} \left[\tilde{X} - I\right] dF(\tilde{X}) = V(0). \qquad (34)$$

This equation shows that short-term debt has no effect on the entrepreneur's expected wealth. Hence, the M&M irrelevance result holds in this case.

Long-term debt

When debt is long-term, it becomes due <u>after</u> the investment decision is made. Let's denote the fact value of long-term debt by D^{lt}. If $\tilde{X}$-I $\geq$ D^{lt}, the entrepreneur realizes that if the firm undertakes the project, there will not only be enough money to pay debtholders in full, but there will also remain a profit of

$\tilde{X}$-I-D^{lt}. Hence, the firm will invest in the project and will earn enough money to pay debtholders in full. The resulting wealth of the entrepreneur is $\tilde{X}$-I-D^{lt}. If on the other hand $\tilde{X}$-I < D^{lt}, the entrepreneur realizes that even if the firm undertakes the project, there will not be enough money to cover the debt payment in full. Hence, whether the project is taken or not, the entrepreneur's wealth is 0. Clearly then, the firm whose objective is to maximize the entrepreneur's wealth, has no reason to invest in the project. Consequently, when $\tilde{X}$-I < D^{lt}, the firm does not invest and its earnings are 0. Since the firm earns no money, it cannot pay its debt and goes bankrupt. The entrepreneur who is protected by limited liability walks away with a payoff of 0. As for debtholders, they gain control over the firm, but since the firm already decided to forgo the project, the firm is worthless and debtholders receive a 0 payoff as well.

In sum, a firm with long-term debt invests in the project if and only if $\tilde{X} \geq I+D^{lt}$. Thus, when $I+D^{st} \geq \tilde{X} \geq I$, the firm does not invest even though the project has a positive NPV. This inefficiency is due to the fact that the firm does not take into account the gain to debtholders in all states of nature in which the project is profitable but not profitable enough to ensure that the firm will remain solvent. Put differently, the entrepreneur bears alone the cost of investing in the project but whenever the firm goes bankrupt, it is the debtholders who enjoy the return on this investment. Hence, the firm underinvests. The resulting inefficiency is the debt overhang problem or the underinvestment problem.

To see the impact of the debt overhang problem on the entrepreneur's wealth in period 1, recall that the entrepreneur's wealth consists of the expected value of equity and the expected value of debt. The market value of equity, $E(D^{lt})$ is equal to the expected earnings of the firm, net of the cost of investment and net of debt payments, over all states of nature in which the entrepreneur remains in control. As before, $E(D^{lt})$ is given by equation (33). Given that debt is paid in full only if the firm invests, the market value of debt as a function of D^{lt} is given by

$$B(D^{lt}) = \int_{D^{lt}+I}^{\bar{X}} D^{lt} dF(\tilde{X}). \tag{35}$$

The equation shows that the firm faces a tradeoff when it issues debt: the more the firm promises to pay outsiders, the smaller is the likelihood that it will be able to keep its promise. At the extremes, when $D^{lt} = 0$ or $D^{lt} = \infty$, $B = 0$. Since $B(D^{lt}) > 0$ for $0 < D^{st} < \infty$, it is clear that $B(D^{lt})$ attains a maximum at some positive level of D^{lt}. This debt level is referred to as the debt capacity of the firm since this is the maximum amount that the firm can borrow in the capital market. Note that the debt capacity of the firm is decreasing with I: the more costly is investment, the less likely it is that the firm will invest and be able to pay back it debt.

Comparing equations (35) and (32) reveals that the market value of long-term debt is less than the market value of short-term debt. This is because debtholders know that when debt is long-term, it is too late for them to invest in the project if the firm defaults on the debt payment. The difference between the two equations, which is equal to the first term on the right side of equation (32) represents the net expected proceeds from the project when debtholders are in control.

Adding equations (35) and (33), the entrepreneur's wealth is given by

$$E(D^{lt}) + B(D^{lt}) = \int_{I+D^{lt}}^{\bar{X}} \left[\tilde{X} - I\right] dF(\tilde{X}) < V(0). \tag{36}$$

Equation (36) shows that long-term debt leads to a reduction in the entrepreneur's expected wealth; the extent of this reduction is increasing as D^{lt} grows larger: the more long-term debt the firm has, the less is the entrepreneur's wealth. This intuition for this result is as follows. The capital market is perfectly competitive, so the inefficiency associated with the debt overhang problem is borne completely by the entrepreneur. This suggest that if the firm could have promised in advance not to ignore the payoff of debtholders it would have done so as this would have boosted the market value debt. However, ex post the firm looses interest in debtholders despite making promises ex ante. Debtholders anticipate this and are therefore paying less for debt when the buy it.

Nonverifiability of investments is a crucial element of the story since had it been possible to write a contract that specifies that the entrepreneur should invest whenever the project has positive NPV, i.e. whenever $\tilde{X} \geq I$, or else be severely punished, then there would have been no agency problem. The agency problem is therefore a consequence of the entrepreneur's inability to commit to invest optimally: whatever the entrepreneur promises ex ante, debtholders will correctly anticipate that ex post he has an incentive to invest only if $\tilde{X} \geq I + D^{lt}$, which is inefficient because some positive NPV projects are foregone whenever $I < \tilde{X} \leq I + D^{lt}$.

An Example

To illustrate the debt overhang problem, assume that $F(\tilde{X})$ is uniform on the interval [0, 1], so $F(\tilde{X}) = \tilde{X}$ and $f(\tilde{X}) = 1$. Assume further that $0 < I < 1$. Absent debt, the firm invests whenever $\tilde{X} \geq I$, so its expected value in period 1 is

$$V(0) = \int_I^1 \left[\tilde{X} - I\right] d\tilde{X} = \left[\frac{\tilde{X}^2}{2} - I\tilde{X}\right]_I^1 = \frac{1}{2} - I - \frac{I^2}{2} + I^2 = \frac{(1-I)^2}{2}. \qquad (37)$$

As we saw earlier, if the firm had only short-term debt its value will remain the same so let's consider long-term debt. To make the example interesting, assume that $D^{lt}+I < 1$ otherwise the firm goes bankrupt for sure and its value is 0. (Moreover, nobody will give the firm money in this case, so there is simply no point in issuing debt with face value such that $D^{lt}+I > 1$. In other words, issuing such debt is a dominated strategy for the firm and will never be observed in equilibrium.)

Given D, the firm invests in period 2 if and only if $\tilde{X} \geq D^{lt}+I$. Anticipating this decision rule, the market value of debt is determined in the capital market such that debtholders break even in expectation:

$$B(D^{lt}) = \int_{D^{lt}+I}^{1} D^{lt} d\tilde{X} = D^{lt}\left[1 - D^{lt} - I\right]. \tag{38}$$

The debt capacity of the firm is the debt level that maximizes $B(D^{lt})$. From equation (38) it is easy to see that the debt capacity of the firm is equal to (1-I)/2. Given $B(D^{lt})$, the entrepreneur's expected wealth in period 1 is

$$\begin{aligned} V(D^{lt}) &= \int_{D^{lt}+I}^{1} \left[\tilde{X} - I\right] d\tilde{X} = \left[\frac{\tilde{X}^2}{2} - I\tilde{X}\right]_{D^{lt}+I}^{1} \\ &= \frac{1}{2} - \frac{(D^{lt}+I)^2}{2} - I + I(D^{lt}+I) \\ &= \frac{(1-I)^2 - D^2}{2}. \end{aligned} \tag{39}$$

Comparison of equations (37) and (39) shows that D reduces the entrepreneur's wealth thus illustrating the debt overhang problem.

5. The overinvestment and underinvestment problems

In the two previous sections we saw that leveraged firms may either invest in risky projects that have a negative expected return or may fail to invest in projects that yields a positive expected return. These problems are often referred to as the overinvestment and underinvestment problems, respectively. The question is how these problem are related to one another. An answer to this question is provided by Berkovitch and Kim in the paper, "Financial contracting and Leverage Induced Over- and Under-Investment Incentives," (*JF*, 1990) that presents the two problems in a unified model. In this section I will present a similar model that can give rise to either overinvestment and underinvestment depending on the current cash flow of the firm.

The model works as follows: a firm is established by entrepreneur in period 1 and operates for two periods. When the firm is just established it issues debt with face value D that is due at the end of period 2. After its cash flow in period 1 is realized, the firm has an opportunity to invest. If the firm invests I dollars, its earnings in period 2 are equal to zg(I), where $g'(I) > 0 > g''(I)$, and z is a random variable distributed over the interval $[0, \bar{z}]$ (where $\bar{z}$ could be ∞) according to a cumulative distribution function F(z). That is, investment increases the cash flow of the firm in period 2 but at a decreasing rate, but the return is stochastic.

Given the period 1 cash flow of the firm, X, and the level of investment, I, the firm has in period 1 excess funds if $X > I$ and a cash deficit if $X < I$. If the firm has excess funds, it simply keeps the money and pays it at the end of period 2 to securityholders along with the proceeds from the investment project. If the firm has a cash deficit we assume that it makes up for this deficit by <u>issuing additional equity</u>. Given these assumptions, the cash flow of the firm in period 2 is either zg(I)+(X-I) if the firm had excess funds at the end of period 1 or zg(I) if the firm had a cash deficit.

Given a debt obligation, D, and a period 1 cash flow X, the firm remains solvent at the end of period 2 provided that its period 2 cash flow exceeds D. If the firm has excess funds at the end of period 1, then its cash flow at the end of period 2 is zg(I)+X-I. Consequently, the firm remains solvent only if zg(I)+X-I > D, or z > z_1*(D, X) ≡ (D+I-X)/g(I). Given this expression, the expected payoff of equityholders as a function of D and X is given by

$$E_1(D,X) = \int_{z_1^*}^{\bar{z}} [zg(I) + X - I - D]\, dF(z). \tag{40}$$

This expression represents the expected cash flow of the firm net of debt payments over all states of natures such that the firm remains solvent.

In contrast, if the firm has a cash deficit at the end of period 1, then its cash flow at the end of period 2 is zg(I), so the firm remains solvent only if zg(I) > D, or z > z_2*(D, X) ≡ D/g(I). Given this expression, the expected payoff of equityholders as a function of D and X is given by

$$E_2(D,X) = \int_{z_2^*}^{\bar{z}} [zg(I) - D]\, dF(z) - (I - X). \tag{41}$$

The first term on the right side of the equation the expected cash flow of the firm net of debt payments over all states of natures such that the firm remains solvent while the second term is the extra money that equityholders need to inject into the firm to enable it to invest at the end of period 1.

Now, let us examine the investment levels of the firm in these two situations. To this end, let I_1* be the investment level that equityholders choose if the firm has excess funds at the end of period 1 and let I_2* be the investment level that equityholders choose if the firm has a cash deficit. Using the definition of z_1*, the first order condition for I_1* is given by:

$$\frac{\partial E_1(D,X)}{\partial I} = \int_{z_1^*}^{\bar{z}} \left[z g'(I) - 1 \right] dF(z) = 0, \tag{42}$$

or,

$$g'(I) = \frac{1}{\dfrac{\int_{z_1^*}^{\bar{z}} z \, dF(z)}{1 - F(z_1^*)}}. \tag{43}$$

The right side of the equation is the inverse of expectation of z conditional on z being larger than z_1* (i.e., its the inverse of the "average" value of z among all values of z between z_1* and $\bar{z}$). The equation then says that the firm invests up to the point where the marginal return from the investment is equal to the inverse of expectation of z conditional on z being larger than z_1*.

Similarly, given the definition of z_2*, the first order condition for I_2* is given by:

$$\frac{\partial E_2(D,X)}{\partial I} = \int_{z_2^*}^{\bar{z}} z g'(I) \, dF(z) - 1 = 0, \tag{44}$$

or

$$g^{/}(I) \;=\; \frac{1}{\int_{z_2^*}^{\bar{z}} z\, dF(z)}. \tag{45}$$

To determine whether the firm overinvests or underinvests, let's examine the first-best level of investment, I^{fb}. This level of investment maximizes the total expected value of the firm in period 2 (as opposed to only the equityholders payoff as we assumed earlier). When the firm has excess funds at the end of period 1, its total expected value, as a function of D and X is given by the following expression:

$$V_1(D,X) \;=\; \int_0^{z_1^*}[zg(I) + X - I]\, dF(z) \;+\; \int_{z_1^{\hat{}}}^{\bar{z}} D\, dF(z) \;+\; \int_{z_1^*}^{\bar{z}}[zg(I) - D + X - I]\, dF(z). \tag{46}$$

The first two integrals represent the expected payoff of debtholders. The third integral represents the expected payoff of equityholders. When the firm has a cash deficit at the end of period 1, its total expected value, as a function of D and X is given by the following expression:

$$V(D,X) \;=\; \int_0^{z_2^*} zg(I)\, dF(z) \;+\; \int_{z_2^*}^{\bar{z}} D\, dF(z) \;+\; \int_{z_2^{\hat{}}}^{\bar{z}}[zf(I) - D]\, dF(z) \;-\; (I - X). \tag{47}$$

The first two expressions on the equation represent the expected payoff of debtholders, and the third expression represents the expected payoff of equityholders.

If we simplify equations (46) and (47) then we get that in both cases, the expected value of the firm is equal to

$$V(D,X) = \hat{z}\,g(I) + (X - I), \tag{48}$$

where

$$\hat{z} = \int_0^{\bar{z}} z\,dF(z), \tag{49}$$

is the mean value of z. In other words, the expected value of the firm is simply equal to the expected return from the project plus an amount equal to the excess funds retained in the firm in period 1 or the extra money that equityholders inject into the firm at the end of period 1.

The first order condition for I^{fb} is given by the following equation:

$$g'(I) = \frac{1}{\hat{z}}. \tag{50}$$

In order to compare the investment levels I_1* and I_2* with the first-best level of investment, I^{fb}, note that the left side of equations (43), (45), and (50) is the same. Hence things depend only on the right side of the equations. Now, it is easy to note that

$$\int_{z_2^*}^{\bar{z}} z\,dF(z) < \hat{z} < \frac{\int_{z_1^*}^{\bar{z}} z\,dF(z)}{1 - F(z_1^*)}. \tag{51}$$

These inequalities follow because the integral on the left is taken from z_2* to $\bar{z}$ while in $\hat{z}$ it is taken from 0 to $\bar{z}$ and since the expectation of z conditional on z being larger than z_1* is larger than $\hat{z}$ which is the unconditional expectation of z (in other words, the average of z over all possible values of z is less than the average of z when only high values of z are being taken into account). Recalling that $g''(I) < 0$, it follows that $I_2^* < I^{fb} < I_1^*$. That is, the firm underinvests when it has a cash deficit but it overinvests when it has excess funds.

The intuition for this result is the same as in the asset substitution effect (overinvestment) and in the debt overhang problem (underinvestment): when the firm has excess funds, equityholders realizes that with some probability, the excess funds "belong" to debtholders. Therefore equityholders do not stand to lose much if they take a gamble and invest the money in the project. In other words, the firm substitutes safe cash for a risky return from a project in the hope that the project will turn out to be a success; if it turns out to be a failure, then it is the debtholders who lose the money. Consequently, equityholders have an incentive to invest "too much". Things are very different however when the firm has a cash deficit. Then equityholders need to invest their own money in the project. But since they realize that with some probability the project will not yield enough cash flows and the firm will go bankrupt (in which case only debtholders will capture the returns from the project), they have an incentive to invest "too little."

Berkovitch and Kim (*JF*, 1990) were therefore able to put the asset substitution effect and the debt overhang problem in a unified framework and show that either problem may occur depending on whether the firm has excess funds or a cash deficit. It is reasonable to believe that firms in mature industries are likely to have a relatively large amounts of funds and little profitable investment opportunities while firms in high growth industries will have little available funds and many attractive investment opportunities. Hence one should expect that the overinvestment problem would be more severe in mature industries (the firm has a high cash flow that belongs to debtholders) whereas the underinvestment problem would be more severe in high growth industries (where equityholders need to invest more money in the firm). One empirical evidence for this hypothesis is provided by Maksimovic and Phillips. They examine data from ... plants in the US that were in financial distress....

The result that the firm tends to overinvest when it has excess funds is reminiscent of the Jensen's (1986) free cash hypothesis. Jensen argued that when firms have free cash flows they tend to waste it and that therefore it is optimal to force managers to pay out the free cash flows either as dividends or as debt payments (by issuing large amounts of debt before the cash flows are generated). Here, the free cash flows arises because whenever the firm has free cash flows it wastes them by overinvesting. To eliminate this problem, the firm should have a rule that says that all free cash flows have to be paid out as dividends. Such a rule will correct the firm's incentive to overinvest and will make equityholders better off. Note that although debtholders will be less likely to get repaid, the firm will not lose since the debtholders will take this into account and will simply pay less for the firm's debt. But then equityholders who anticipate higher dividends will pay more and the total value of the firm will reflect the optimal investment decision.

6. Assets destruction and debt renegotiation

Similarly to the debt overhang problem, the possibility of debt renegotiation coupled with the ability of equityholders to destroy assets may impose another cost on the firm when it issues debt. This idea is developed in the paper "Opportunistic Underinvestment in Debt Renegotiations and Capital Structure," by Bergman and Callen (*JFE*, 1991). To illustrate the main idea of the paper let us consider a simplified version of the model: A firm is established by an entrepreneur in period 1 and it operates only in period 2. The earnings of the firm in period 2 are represented by a random variable $\tilde{X}$, distributed over the interval $[X_0, X_1]$ (where X_1 could be ∞) according to a cumulative distribution function $F(\tilde{X})$. The mean earnings of the firm are given by

$$\hat{X} = \int_{X_0}^{X_1} \tilde{X}\, dF(\tilde{X}). \tag{52}$$

Now suppose that in period 1 the firm issues debt with face value D which is due at the end of period 2 after the cash flows of the firm have been realized. The proceeds from issuing debt are used to finance investments in physical capital, or to fund an R&D project, or simply to pay dividends to the entrepreneur. A key assumption in the model is that before debt is due at the end of period 2, the entrepreneur can approach the debtholders and ask them to renegotiate the debt payments. Moreover, at any point in time after the realization of earnings and before debt is due, the entrepreneur can waste the firm's resources (some or even all of them), say by spending money or by destroying its assets if they are still not liquid.[1] This may look like a

[1] An implicit assumption here is that debtholders cannot stop the entrepreneur from doing that since debt is still not due at that point and the entrepreneur can always masquerade the waste of resources as (unlucky) business decisions.

strange assumption because at a first glance it is not at all clear why the owner of a firm would like to waste valuable resources. However as we shall see this mode of behavior may confer a strategic advantage on the entrepreneur in its dealings with debtholders.

Analysis

To analyze the model, note first that if the entrepreneur does nothing at the end of period 2 (does not waste resources nor asks the debtholders to renegotiate the debt payments) his payoff is $Y_E = Max\{\tilde{X}-D, 0\}$ and the payoff of the debtholders is $Y_D = Min\{\tilde{X}, D\}$. If the entrepreneur approaches the debtholders and ask them to renegotiate, they can either agree or disagree. If they disagree, the payoffs of the parties are Y_E and Y_D if the entrepreneur does not waste resources, and $Max\{\tilde{X}-D-R, 0\}$ and $Min\{\tilde{X}-R, D\}$, if the entrepreneur destroys assets that are worth R. Obviously, so long as $\tilde{X} > D$, $Y_E > 0$, so the entrepreneur's threat to destroy assets is not credible: if the debtholders ignore his request to renegotiate the debt payments the entrepreneur will only loose money by wasting resources as his payoff will be $Max\{\tilde{X}-D-R, 0\}$ instead of $Max\{\tilde{X}-D, 0\}$. Hence whenever $\tilde{X} > D$, the entrepreneur can rationally anticipate that if he approaches the debtholders with a request to renegotiate the debt payments they will refuse knowing that he will not find it worthwhile to destroy assets. Consequently, the entrepreneur's payoff when $\tilde{X} > D$ is simply $\tilde{X} - D$.

Things look different however when $\tilde{X} \leq D$. Then $Y_E = 0$ and the entrepreneur's threat to destroy assets is credible as he does not stand to lose anything if he wastes resources: everything goes to the debtholders anyway so he does not waste any of his own money. Clearly then, the debtholders can no longer ignore the entrepreneur's request to renegotiate the debt

payments since they realize that if they do, the entrepreneur will waste their money. Instead of modelling the renegotiation process in great detail, let us simply assume that the two parties decide to split the surplus from renegotiation between them such that the entrepreneur gets a fraction γ of the surplus and debtholders get the rest. In our case, the surplus from renegotiation is the difference between the firm's resources, $\tilde{X}$, and what will be left after the waste of resources, $\tilde{X}$-R. Since the surplus from renegotiation is $\tilde{X}-(\tilde{X}-R) = R$, the entrepreneur's payoff from renegotiation is γR and the payoff of debtholders is
$(1-\gamma)R$.

Note that since the entrepreneur's payoff from renegotiation increases with the amount that he threatens to waste, it is best for him to threaten to waste as much as possible. Thus so long as $\tilde{X} \leq D$, the entrepreneur will threaten debtholders that if they will refuse to renegotiate the debt payments he will waste all of the firm's resources before the date at which debt is due so there will be nothing left in the firm at that point. Debtholders will realize that this threat is credible as the entrepreneur has nothing to loose from carrying out the threat if they refuse to renegotiate, and so the entrepreneur will walk out of the renegotiations with a payoff of $\gamma\tilde{X}$, leaving a payoff of $(1-\gamma)\tilde{X}$ to the debtholders.

So far we saw that when $\tilde{X} > D$, no renegotiation takes place since the entrepreneur cannot credibly threaten the debtholders that he will destroy assets, and the entrepreneur's payoff is $\tilde{X}$-D. On the other hand, when $\tilde{X} \leq D$, the entrepreneur has nothing to lose if he destroys assets so renegotiation will take place and the entrepreneur will end up with a payoff of $\gamma\tilde{X}$. Thus for example, if D = 1000 and $\gamma = 1/2$, the entrepreneur's payoff when $\tilde{X} = 999$ is 999/2 = 499.5, but when $\tilde{X} = 1001$, the entrepreneur's payoff is 1001-1000 = 1. This result seems absurd since it is hard to believe that two very similar situations, $\tilde{X}$ = 999 and $\tilde{X}$ = 1001 will produce such different results. Moreover, if indeed the entrepreneur's payoff increases by so much when earnings are lowered from 1001 to 999, the entrepreneur would surely destroy assets worth 2 even before he approaches the debtholders to convince them that he has nothing to lose by destroying assets even further, and hence force debtholders to renegotiation the debt contract and thereby increase his payoff from 1 to 499.5. Thus, whenever $\tilde{X} > D$, the entrepreneur can destroy assets worth $\tilde{X}$-D to make sure that the firm's cash flow is in the range where renegotiation takes place. If the entrepreneur acts in this way, his payoff ends up being γD. Since if the entrepreneur does nothing his payoff is $\tilde{X}$-D, it is clear that assets destruction is not worthwhile if $\tilde{X} \geq (1+\gamma)D$ and is worthwhile otherwise. To sum up, we just showed that the payoffs are as in the following table:

State of nature	The entrepreneur's payoff	The debtholders' payoff
$\tilde{X} > (1+\gamma)D$	$\tilde{X}$-D	D
$D < \tilde{X} \leq (1+\gamma)D$	γD	$(1-\gamma)D$
$\tilde{X} \leq D$	$\gamma\tilde{X}$	$(1-\gamma)\tilde{X}$

An alternative scenario is that the entrepreneur approaches the debtholders just before destroying the assets for the first time. The debtholders understand that it is in the entrepreneur's interest to destroy assets worth $\tilde{X}$-D so that when they meet next time his threat to destroy the remaining assets will be credible. Since the assets destruction is costly for the debtholders they may wish to begin negotiations right away realizing that if they wait, the entrepreneur will destroy assets worth $\tilde{X}$-D. Under this scenario, renegotiation will take place whenever $\tilde{X} \leq (1+\gamma)D$ without the need to resort to assets destruction. The payoffs are such that if $\tilde{X} > (1+\gamma)D$, the entrepreneur gets $\tilde{X}$-D and debtholders get D, and if $\tilde{X} \leq (1+\gamma)D$, the entrepreneur gets $\gamma\tilde{X}$ and the debtholders get $(1-\gamma)\tilde{X}$.

Given the two scenarios just described, we can now compute the market values of equity and debt and the entrepreneur's wealth in period 1. We begin with the first scenario in which asset destruction is necessary in states of nature in which $D < \tilde{X} \leq (1+\gamma)D$. Then, the market value of equity as a function of the face value of debt is given by

$$\boldsymbol{E(D)} = \int_0^D \gamma \tilde{X} dF(\tilde{X}) + \int_D^{(1+\gamma)D} \gamma \boldsymbol{D} dF(\tilde{X}) + \int_{(1+\gamma)D}^{X_1} \left[\tilde{X} - \boldsymbol{D}\right] dF(\tilde{X}). \tag{53}$$

The first expression represents the expected payoff of the entrepreneur over states of nature in which renegotiation takes place and no waste of resources is necessary to force the debtholders to agree to renegotiate. The second expression represents the entrepreneur's payoff in states of nature in which he forces the debtholders to agree to renegotiate their debt payments by wasting resources. The third expression represents the entrepreneur's payoff when his residual claim is so high that he does not find it in his interest to destroy assets so he cannot force the debtholders to renegotiate. Analogously, the market value of debt as a function of the face value of debt is given by

$$\boldsymbol{B(D)} = \int_0^D (1-\gamma) \tilde{X} dF(\tilde{X}) + \int_D^{(1+\gamma)D} (1-\gamma) \boldsymbol{D} dF(\tilde{X}) + \int_{(1+\gamma)D}^{X_1} D dF(\tilde{X}). \tag{54}$$

Adding equations (53) and (54) reveals that the entrepreneur's expected wealth in period 1 is

$$V(D) \; = \; E(D) + B(D) \; = \int_{0}^{D} \tilde{X} dF(\tilde{X}) \; + \int_{D}^{(1+\gamma)D} D dF(\tilde{X}) \; + \int_{(1+\gamma)D}^{X_1} \tilde{X} dF(\tilde{X})$$
$$= \hat{X} \; - \int_{D}^{(1+\gamma)D} \left[\tilde{X} - D\right] dF(\tilde{X}). \tag{55}$$

Equation (55) shows that issuing debt is costly to the entrepreneur because it gives him an incentive to destroy assets in some states of nature in order to make it credible for to threaten the debtholders that if they will not agree to renegotiate the debt payments that they are entitled to he will destroy assets.

Under the second scenario where no assets are being destroyed, the market values of equity and debt, as a function of the face value of debt, are given by

$$E(D) \; = \int_{0}^{(1+\gamma)D} \gamma \tilde{X} dF(\tilde{X}) \; + \int_{(1+\gamma)D}^{X_1} \left[\tilde{X} - D\right] dF(\tilde{X}), \tag{56}$$

and

$$B(D) \; = \int_{0}^{(1+\gamma)D} (1-\gamma) \tilde{X} dF(\tilde{X}) \; + \int_{(1+\gamma)D}^{X_1} D dF(\tilde{X}). \tag{57}$$

Adding the two equations reveals that the entrepreneur's expected wealth in period 1 is

$$V(D) \; = \; E(D) + B(D) \; = \int_{0}^{(1+\gamma)D} \tilde{X} dF(\tilde{X}) \; + \int_{(1+\gamma)D}^{X_1} \tilde{X} dF(\tilde{X}) \; = \; \hat{X}. \tag{58}$$

Under the second scenario then, debt is completely irrelevant as in the M&M model. Still as we shall see in a moment, this is not the end of the story.

To complete their model and explain why firms issue debt if it is costly for them to do so, Bergman and Callen assume that debt provides the firm with a tax shield. Hence under the first scenario, the firm chooses an optimal level of debt by trading-off the tax benefits of debt against the costs associated with the waste of resources. Under the second scenario it might be thought that since debt generates tax benefits but it is not costly, the firm becomes an all-debt firm. Bergman and Callen show however that while it is certainly true that the firm may wish to issue as much debt as it possibly can, there is an upper bound on the amount of debt the firm can raise. This upper bound is called the "debt capacity" of the firm and we will denote it by D^c. The firm has a finite debt capacity because debtholder anticipate that in period 2 the firm would like to exploit them by renegotiating the debt obligation. Moreover since this incentive is stronger when D is larger there is an upper bound on the face value of debt. To see this point, let us differentiate B(D). Then,

$$\begin{aligned} B'(D) &= [(1-\gamma)(1+\gamma)Df((1+\gamma)D) - (1+\gamma)Df(((1+\gamma)D)]\,(1+\gamma) \\ &\quad + 1 - F((1+\gamma)D) \\ &= -\gamma^2 Df((1+\gamma)D)(1+\gamma) + 1 - F((1+\gamma)D) = 0. \end{aligned} \tag{59}$$

We can now rewrite this equation as follows:

$$(1+\gamma)DH((1+\gamma)D) = \frac{1}{\gamma^2}, \tag{60}$$

where H(.) ≡ f(.)/(1-F(.) is the "hazard rate" of the distribution of cash flows. The hazard rate at a certain point x, is equal to the density of the cash flow at x, conditional on the fact that the firm's cash flow is at least as large as x. In most commonly used distribution functions (including normal, log-normal, uniform, and exponential), H'(.) ≥ 0, so the left hand of side (60) is an increasing function of D. Moreover, at $(1+\gamma)D = X_1$, $F((1+\gamma)D) = 1$, implying that $H'((1+\gamma)D)$ tends to infinity. Noting that the right hand side of (60) is just a constant, it follows from (60) that the debt capacity of the firm, D^c, is implicitly defined by the intersection of the increasing left hand side of (60) with the constant right hand side of (60). Notice that $H'((1+\gamma)D)$ tends to infinity as $(1+\gamma)D = X_1$, it must be the case that $(1+\gamma)D^c < X_1$, or $D^c < X_1/(1+\gamma)$. Given D^c, the largest amount of money that the firm can raise by issuing debt is

$$\begin{aligned} B(D^c) &= \int_0^{(1+\gamma)D^c} (1-\gamma)\tilde{X}dF(\tilde{X}) + \int_{(1+\gamma)D^c}^{X_1} D^c dF(\tilde{X}) \\ &= \hat{X} - \gamma \int_0^{(1+\gamma)D^c} \tilde{X}dF(\tilde{X}) - \int_{(1+\gamma)D^c}^{X_1} \left(\tilde{X} - D^c\right)dF(\tilde{X}) < \hat{X}. \end{aligned} \tag{61}$$

In contrast, if the entrepreneur had a way to commit to debtholders that he'll never renegotiate debt (this is equivalent to a case where $\gamma = 0$), the debt capacity of the firm would have been X_1 and the largest amount that the firm would have been able to raise by issuing debt would have been $\hat{X}$. Hence, the ability to destroy assets limits the ability of the entrepreneur to raise external funds even in the case where eventually he does not destroy any assets.

7. Financial structure and managerial incentives

Another type of agency problem that can arise in firms is between shareholders and managers. Capital structure can then be used to discipline managers who would otherwise waste firm resources on perks. This idea was first developed in the paper "Corporate Financial Structure and Managerial Incentives," by Grossman and Hart (1982). To illustrate the main idea of the paper let us examine the following story. A captain of a ship is going to sail soon to a foreign port and is trying to convince potential clients to let him deliver cargo for them. But since the captain requires a full payment once the cargo is loaded on the ship, the clients suspect that once they pay him the money he'll sail leisurely and will not have an incentive to deliver the cargo rapidly. Of course the captain can assure the clients that he takes his job seriously and is going to sail as quickly as possible but unless there is a way to make this promise credible, the clients will not trust his intentions. One way to make the promise to sail as quickly as possible credible is to draw a small hole is the ship. Since the ship is being constantly filled up with sea water, it is going to sink unless it reaches the foreign port quickly. Thus, by drawing a hole in the ship, the captain makes his promise to sail quickly credible. The cost of this commitment however is that the ship may actually sink before reaching its destination even if the captain did his best effort to reach the port as quickly as possible. The Grossman and Hart model is similar: the firm takes on debt to make it susceptible to bankruptcy. This provides the manager with an incentive to invest and boost the cash flows of the firm to minimize the likelihood that the firm goes bankrupt.

The basic model

The following is a simplified version of the Grossman and Hart model. A firm is established in period 0 and it operates only in period 1. In period 0 the firm raises R dollars from external investors by issuing debt and equity. Debt is due at the end of period 1 and its face value is D. After the firm raises the funds, the manager decides how much money to invest in a project. If the manager invests I dollars in the project then the return in period 1 is given by zg(I), where z is a random shock that is realized only in period 1 after the level of investment I has already been chosen by the manager. The random shock z is distributed over the interval [0, 1] according to a distribution function f(z) and a cumulative distribution function F(z), and g(I) is an increasing and concave function (i.e., $g'(I) > 0 > g''(I)$). One way to think about this is as follows: g(.) is a production function and g(I) is the firm's output. The firm sells its output in a competitive market where the price is random and equal to z. When the manager decides how much to produce he only knows the distribution of z but not its actual realization. The money not invested in the project, R-I, is spent by the manager on perks, which the manager can consume in period 1 provided that the firm remains solvent.[2]

Next, we need to define the probability of bankruptcy and examine how it is affected by debt and investment. To this end, note that if $zg(I) \geq D$, the firm remains solvent: debtholders receive D in full and equityholders receive zg(I)-D. If $zg(I) < D$, the firm goes bankrupt. Let z* be the critical state of nature below which the firm goes bankrupt. Given D and I, z* is defined by

$$z^{*} = \frac{D}{g(I)}. \tag{62}$$

The probability of bankruptcy is given by F(z*). Note that

[2] Alternatively, it is possible to assume that the manager invests R-I in "pet projects" that either generate no return or generate a fixed return of say W, while the investment in the projects yields a return of W+zg(I). Then, one can think of zg(I) as the extra return from investment in the project. In any event, the money R-I is a pure waste from the firm's point of view since it can generate more money to shareholders if it is invested in the project rather than in the manager's "pet project".

$$\frac{\partial z^*}{\partial I} = -\frac{Dg'(I)}{g(I)} < 0, \qquad \frac{\partial z^*}{\partial D} = \frac{1}{g(I)} > 0. \tag{63}$$

Hence, bankruptcy becomes less likely if the firm invests more but more likely if the firm has more debt. As we shall see, the owners of the firm will exploit these properties as follows: the firm will issue debt thereby increasing the likelihood of bankruptcy in order to induce the manager to exert effort and make sure that the likelihood of bankruptcy is not too high.

To simplify the analysis, I shall assume that in case of bankruptcy the entire value of the firm is lost and moreover, the manager does not consume any perks and his payoff is 0; this is an extreme case of bankruptcy costs. Of course this assumption is unrealistic since if debtholders get nothing in case of bankruptcy, they have no reason to drag the firm into a bankruptcy court. We make this assumption for two reasons. First it simplifies the analysis greatly. Second, Grossman and Hart assume that debt is chosen by the manger of the firm who, as we shall see shortly, receives a zero payoff when the firm goes bankrupt. Hence, when the manager chooses D he ignores states of nature in which the firm goes bankrupt. Here I assume that D is chosen by the owners of the firm instead of the manager (this simplifies the analysis greatly). Assuming that the value of the firm is lost in the event of bankruptcy means that when the owners choose D they will also ignore states in which the firm goes bankrupt. Thus, the difference between the analysis in the original paper of Grossman and Hart and the analysis here is kept to a minimum.

Before we begin to analyze the model it should be noted that if the firm is all-equity, $z^* = 0$, so the manager always gets to consume his perks. Since the manager's perks are equal to R-I it is obvious that the manger is better-off not investing and keeping the entire resources of the firm to himself. If there is no way to force the manager to invest by say, signing a contract with him that stipulates that he should pay some penalty if he fails to invest, the value of an all-equity firm will be 0. Outside investors will anticipate this and will refuse to pay a positive price for the firm's securities. Hence the market value of the firm will be V(0) = 0. Thus, if there is no contractual solution to force the manager to invest, the owners of the firm face a severe moral hazard problem on the part of the manager. As we shall see, issuing debt is a way to discipline the manager and overcome the moral hazard problem.

The manager's investment decision

In period 0 after the firm raised \$R but before observing z, the manager needs the choose I in order to maximize his expected payoff which is given by:

$$U(I,D) = \int_{z^*}^{1} [R - I] dF(z) = (R - I)\left(1 - F(z^*)\right). \tag{64}$$

This expression is the value of perks, R-I, conditional on the firm staying solvent (i.e., conditional on the event that z ≥ z* ≡ D/g(I)). When the firm goes bankrupt, the manager receives nothing. Let I* be the optimal choice of investment for the manager. Assuming that 0 < I* < R, the first order condition for I* is:

$$\frac{\partial U(I,D)}{\partial I} = -\left[1 - F(z^*)\right] - (R - I) f(z^*) \frac{\partial z^*}{\partial I} = 0. \tag{65}$$

The first term on the right side of the equation is the marginal cost of investment from the manager's perspective. It is due to the fact that when the manager invests another dollar, there is less money left for him in the firm to consume as perks. The second term on the left side of the equation is the marginal benefit of investment from the manager's perspective, associated with the reduction in the likelihood of bankruptcy when the firm invests more. This is beneficial for the manager because it implies that he is more likely to get to consume perks.

To verify that $0 < I^* < R$, note that when I = 0, the firm goes bankrupt for sure so z* = 1. Hence, the marginal cost of debt vanishes so the manager would find it beneficial to invest at least one dollar in the project. On the other hand, if I = R, the marginal benefit of debt vanishes so the manager will never invest up to that point. Since the marginal benefit of investment increases with R, it is clear from equation (65) that when R is larger the manager will invest more. The intuition is clear: the reason why the manager invests is to lower the chances that the firm goes bankrupt in which case he loses his perks. When R is larger, the manager's perks are larger so it is more important for the manager to invest.

Since $\partial z^*/\partial I = -Dg'(I)/g(I)^2 = -z^*g'(I)/g(I)$ (the latter equality follows from the definition of z*), we can rewrite equation (65) as follows:

$$\frac{g(I)}{(R-I)g'(I)} = z^* \frac{f(z^*)}{1-F(z^*)} \equiv z^* H(z^*), \tag{66}$$

where H(z*) is the "hazard rate" of the distribution function f(z*). The interpretation of H(z*) is as follows: when the firm issues another dollar of debt the probability of bankruptcy increases. H(z*) is the marginal increase in the probability of bankruptcy, conditional on the firm staying solvent up to that point (of course, if the firm already went bankrupt when D was low, increasing D by a dollar has no impact on the bankruptcy). Using this equation, we can study the impact of debt on the manager's incentive to invest. To this end, let's differentiate equation (66) with respect to I and D. After rearranging terms we get:

$$\frac{\partial I}{\partial D} = \frac{\left[H(z^*) + z^* H'(z^*)\right] \dfrac{\partial z^*}{\partial D}}{\dfrac{(R-I)g'(I)^2 - \left[(R-I)g''(I) - g'(I)\right]g(I)}{\left[(R-I)g'(I)\right]^2} - \left[H(z^*) + z^* H'(z^*)\right]\dfrac{\partial z^*}{\partial I}}. \tag{67}$$

Assuming that H'(.) ≥ 0 as is the case in many commonly used distribution functions (including normal, log-normal, uniform, and exponential), this derivative is positive. Hence the more debt the firm issues, the more willing is the manager to invest in order to lower the probability that the firm goes bankrupt and his perks are lost.

The owners' problem

At the beginning of period 0, the owners of the firm need to decide how much debt to issue. Debt has a positive effect on the incentive of the manager to invest but it also increases the likelihood that the firm will go bankrupt and its value will be lost. The owners of the firm maximize the total value of the firm which is the sum of the value of equity and debt. The market value of equity is given by:

$$E(D) = \int_{z^*}^{1} \left[z g(I) - D\right] dF(z), \tag{68}$$

and the market value of debt is given by:

$$B(D) = \int_{z^*}^{1} D\, dF(z). \tag{69}$$

The total market value of the firm is

$$V(D) \;=\; E(D) + B(D) \;=\; \int_{z^*}^{1} z g(I) \, dF(z). \tag{70}$$

This expression is simply the expected value of the project over all states of nature in which the firm remains solvent.

Let D* be the optimal value of debt. Assuming that D* > 0, the first order condition for D* is

$$V'(D) \;=\; -z^* g(I) f(z^*) \frac{\partial z^*}{\partial D} \;+\; \left[-z^* g(I) f(z^*) \frac{\partial z^*}{\partial I} + \int_{z^*}^{1} z g'(I) \, dF(z^*) \right] \frac{\partial I}{\partial D} \;=\; 0. \tag{71}$$

The first term on the right side of equation (71) is the marginal cost of debt from the owners perspective. It represents the marginal increase in the probability of bankruptcy in which case the entire value of the firm, zg(I), is lost. The second term on the right side of equation (71) is the marginal benefit of debt associated with the increased incentive of the manager of the firm to invest. More investment in turn has two positive effects on the firm: first it lowers the likelihood that the firm will go bankrupt and second it enhances the value of the firm in states of nature in which the firm remains solvent.

To verify that D* > 0, note that when D = 0, the firm never goes bankrupt so z* = 0. Then the marginal cost of debt is 0. Since the second term inside the square brackets is positive even when D = 0, it follows that the owners of the firm will benefit from at least the first dollar of debt so the firm will have a positive level of debt. Given that the firm issue a positive debt level, the firm goes bankrupt with positive probability so debt leads to a distortion. Yet the firm is willing to bear this distortion as it allows it to provide the manager with incentives to invest.