TOPIC 4: ASYMMETRIC INFORMATION MODELS OF CAPITAL STRUCTURE

1. Introduction

The recent economic literature has emphasized a lot the importance of asymmetric information to our understanding of a wide range of issues varying from taxation and regulation of firm to optimal nonlinear prices and insurance contracts. These types of models have also shown to be very powerful in the context of corporate finance since they can explain why firms rely on one type of financing rather than another and why firms may have preferences over different ways of paying out profits to shareholders. In this chapter we shall review some of the classic applications of this approach to the capital structure literature.

2. The lemons problem

One of the most important contributions to the literature on asymmetric information is Akerlof's paper "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism," (*Q.J.E.*, 1970). The main point in this paper is that the presence of asymmetric information creates an adverse selection problem: if consumers cannot tell the quality of a product and are willing to pay only an average price for it, then this price is more attractive for sellers who have bad products than to seller who have good products (hence the term adverse selection). Consequently, more bad products (i.e., lemons) will be offered than good products. Now, if consumers are rational, they should anticipate this adverse selection and expect that at any given price, a randomly chosen product is more likely to be a lemon than a good product. Of course, these expectations imply a lower willingness to pay for products and so the proportion of good products that is actually offered falls further. Eventually, this process may lead to a complete break down of the market.

This lemons idea is also important for corporate finance: if investors cannot observe the value of firms before they buy them then they would be willing to pay only an average price for the equity of firms. Given that the price is average, selling equity on the market is much more attractive to owners of bad firms than to owners of good firms so the average value of firms that are actually offered on the market should be below average. This implies that investors should be suspicious that if they are offered equity than it must mean that the firm's value is more likely to be below the average. Hence, investors will no longer be willing to pay an average price for the equity of firms once they are offered to buy this equity.[1]

To illustrate the main idea suppose that there is a large population of firms whose future cash flows are drawn from a uniform distribution on the interval [0, 100]. If investors are risk neutral and cannot observe the cash flow of a randomly selected firm, then the maximum amount that they will agree to pay for it is 50 which is the average cash flow of firms. But, given this price, owners of firms with cash flows that exceed 50 will not be willing to sell their firms realizing that they are better off keeping the firms rather than selling them at a loss. Hence, at a price of 50, only owners of firms with cash flows below 50 will offer their firms for sale. Now however, firms whose equity is offered on the market are no longer randomly selected from the population of firms. Investors will anticipate that the price 50 attracts only firms with cash flow of below 50 and will therefore revise their beliefs about the cash flows of firms on the market. Specifically, given the price 50, the distribution of firms that are offered for sale on the market is uniform on the interval [0, 50], and the average of this distribution is 25. Thus investors being risk neutral, will be willing to pay at most 25 for firms. Given this price, owners of firms whose cash flows exceed 25 will withdraw their firms from the market so the new distribution of firms on the market is uniform on the interval [0, 25] with an average on 12.5. Now consumers revise their beliefs

[1] In a sense then, the idea is similar to the famous saying attributed to Groucho Marx that he "wouldn't want to belong to any club that is would have him as a member." An investor should never be willing to buy equity and become a (part) owner of a firm that would offer him to buy its equity.

again and the process continues.

The conclusion from this story is that in equilibrium, only owners of firms whose value is 0 will offer their firms for sale and investors will be willing to pay for these firms 0. In other words, only absolutely the worst kind of firms will be traded on the market. Since we assumed that the cash flows of firms are drawn from a uniform distribution on the interval [0, 100] there is a measure 0 of firms with 0 cash flows so the market break down completely and no trade will take place. This is an extreme version of the Akerlof's lemons problem. In what follows we shall see models that examine the implications of the lemons problem and offer solutions to the problem.

3. The mispricing of equity and the pecking order of financing

The first paper that we are going to examine is "Corporate Financing and Investment Decisions When firms Have Information that Investors Do Not Have" by Myers and Majluf (*JFE*, 1984). This paper shows that in the presence of asymmetric information, the equity of high-profit firms will be underpriced and if this underpricing is sufficiently severe, these firms may forgo investments in project that have a net present value. This idea of this paper is very close to the lemons problem but as we shall see, the assumption here is that there is asymmetry of information with respect to only part of the future cash flow of the firm so under certain conditions there will not be a complete break down of the market.

Before we describe the model, let's examine the main idea is more detail. Suppose that a firm wants to raise money to invest in a project. If the firm issues equity, it invites outside investors to become partners in the firm and share not only returns from the project but also the current assets of the firm. Hence investors should rightfully suspect that if the firm wants them to become partners it is only because the assets of the firm are not worth much. This suspicion implies that outside investors who do not have access to all the relevant information about the firm, will not be willing to pay high prices for the equity they are offered fearing that this equity represents ownership of assets that are not very valuable. Hence, if contrary to what investors suspect, the firm has valuable assets after all, its equity will be underpriced.

3.1 A model with two firm's types

Having discussed the main idea of the paper let us review the model itself. We will first assume that the firm's type can take only two possible values: it can be either high or low. Later on we shall generalize this to the case where there is a continuum of firm's types. The model evolves as usual in two periods. In period 1, an entrepreneur establishes a firm and needs to raise I dollars from external sources to invest in the firm. The firm operates only once in period 2 and then it is liquidated and its cash flow in that period is X+R if the entrepreneur makes the investment and X if he does not. In what follows, we shall refer to X as the "intrinsic value" of the firm. In this section we will assume that X can take two values, X_L and X_H, with $X_H > X_L > 0$. To simplify matters, let's assume that there is no discounting (the intertemporal interest rate is equal to 0) and that the investment has a net positive value in the sense that the increase in the firm's cash flow, R, exceeds the cost of investment I. Clearly then, if the entrepreneur has enough money to finance the investment, he invest in the project and thereby increase his wealth by R-I without taking any risk.

Now suppose that the entrepreneur does not have the required funds and he needs to raise I dollars from outsiders. One thing he can do is to issue debt with face value I. Since the cash flow of the firm in period 2 exceeds I, debt is completely riskless so investors will be willing to pay for it I dollars in period 1 (recall that there is no discounting). This enables the entrepreneur to cover the cost of investment. Note that this conclusion is true regardless of the intrinsic value of the firm, X. In other words, outsiders do not need to worry about the intrinsic value of the firm when they buy the firm's debt since whatever this value is, the increase in the firm's cash flow due to investment, R, is already sufficiently large to cover the debt payments. Hence, debt is not problematic in the Myers and Majluf model and so from now on we shall assume that for whatever reason the firm is restricted to use equity. As we shall see, the use of equity is problematic when investors do not know the intrinsic value of the firm for sure.

But before we consider the asymmetric information case let us consider the full information benchmark. Under full information, investors know precisely what is the intrinsic value of the firm, X. Hence when they pay the entrepreneur I dollars in period 1, they will require a sufficiently large fraction of the firm's equity to compensate them. Specifically, if they get a fraction α of the firm's equity, their wealth in period 2 is $\alpha(X+R)$. If the capital market is perfectly competitive, the following equation should hold:

$$I = \alpha(X + \boldsymbol{R}). \tag{1}$$

This equation says that the firm's equity is fairly priced in the sense that by buying the firm's equity, outside investors just break even. Solving this equation for α, we obtain the amount of equity that the entrepreneur has to issue to outsiders to induce to induce them to pay him \$I:

$$\alpha^* = \frac{I}{X + \boldsymbol{R}}. \tag{2}$$

Now, the personal wealth of the entrepreneur is given by:

$$W = (1 - \alpha^*)(X + \boldsymbol{R}) = X + \boldsymbol{R} - I. \tag{3}$$

Since $R > I$ the entrepreneur should issue equity and finance the investment, otherwise his wealth is simply equal to X. The fact that the capital market is perfectly competitive means that outsiders break even so they receive in period 2 exactly I, so the entire cash flow of the firm in excess of I goes to the entrepreneur.

Now suppose that only the entrepreneur knows whether the intrinsic value of the firm is X_L or X_H. Outside investors do not observe the true intrinsic value of the firm but they believe that $X = X_L$ with probability p and $X = X_H$ with probability 1-p. Let $\hat{X} = pX_L+(1-p)X_H$ be the expected intrinsic value of the firm according to the beliefs of outsiders.

If the entrepreneur does not raise I and does not invest in the project, then his personal wealth given his type t (t = L, H) is given by

$$W_t(NI) = \begin{cases} X_L & \text{if } t = L, \\ X_H & \text{if } t = H. \end{cases} \tag{4}$$

On the other hand, if the entrepreneur decides to raise money in a perfectly competitive capital market by issuing equity, then the following equation should hold:

$$I = \alpha(\hat{X} + R). \tag{5}$$

This equation is exactly like equation (1) except that $\hat{X}$ is replacing X because outside investors are not exactly sure what is the true intrinsic value of the firm so they need to break even only in expectation. Solving this equation for α, the equity fraction that outsiders will require in order to be willing to give the entrepreneur $I in period 1 is given by:

$$\alpha^* = \frac{I}{\hat{X} + R}. \tag{6}$$

Note that α* is increasing with p: the more likely is the firm to be an L type, the more equity outsiders will require because they anticipate that the firm is not that valuable and hence they require a large fraction of it to compensate them for their investment.

Given α*, the personal wealth of the entrepreneur if his firm is worth X_t is given by:

$$W_t(I) = (1 - \alpha^*)(X_t + R) = \left(\frac{\hat{X} + R - I}{\hat{X} + R}\right)(X_t + R). \tag{7}$$

In order to examine whether the entrepreneur will or will not issue equity and finance the investment we need to compare $W_t(I)$ with the entrepreneur's wealth if he does not invest, $W_t(NI)$. To this end let us examine the difference between the entrepreneur's wealth in the two situations:

$$W_t(\boldsymbol{I}) - W_t(NI) = \left(\frac{\hat{X} + \boldsymbol{R} - \boldsymbol{I}}{\hat{X} + \boldsymbol{R}}\right)(X_t + \boldsymbol{R}) - X_t = \frac{\boldsymbol{R}(\hat{X} + \boldsymbol{R}) - \boldsymbol{I}(X_t + \boldsymbol{R})}{\hat{X} + \boldsymbol{R}}$$
$$= \boldsymbol{R} - \boldsymbol{I}\left(\frac{X_t + \boldsymbol{R}}{\hat{X} + \boldsymbol{R}}\right). \tag{8}$$

The expression we got is the total increase in the wealth of the entrepreneur due to investment. In the full information case, this expression was equal to R-I. Now, if the firm has a low intrinsic value so that X = X_L, the coefficient of I is less than 1 (recall that $\hat{X} = pX_L+(1-p)X_H > X_L$), so the whole expression is larger than R-I. The reason for this is that the cost of investment from the entrepreneur's perspective becomes smaller since he needs to give away less equity to outsiders than in the full information case because outsiders believe that with a positive probability this equity is worth more than it actually does. Since investment increases the wealth of the entrepreneur of a low value firm by more than R-I and since R-I > 0, it is clear that the investment decisions of these firm will not be affected by the presence of asymmetric information.

In contrast, if the firm has a high intrinsic value, the coefficient of I exceeds 1 so the cost of investment from the entrepreneur's point of view is now larger than in the full information case for exactly the opposite reason that it is lower for a firm with a low intrinsic value: outsiders believe that with a positive probability the firm has a low intrinsic value so they require a larger equity participation in order to secure an expected payoff of I. This means that the entrepreneur needs to give up a larger fraction of the firm than he should have if he had a way to prove that the intrinsic value of his firm is in fact high. Hence we can conclude that the equity of firms with low intrinsic value is overpriced while the equity of firms with high intrinsic value is underpriced.

We already saw that the overpricing of the equity of firms with low intrinsic value does not distort their investment decisions in this model, but what is the impact of the underpricing of the equity of firms with high intrinsic values on their investment decisions? Well, if p is very close to 0 (outsiders believe that the firm is very likely to have a high intrinsic value), then $\hat{X}$ will be close to X_H so the underpricing effect will not be severe. Although the cost of investment from the point of view of the entrepreneur will exceed I it will not exceed it by much if p is sufficiently close to 0 so the investment decision of high intrinsic value firms will not change as well. But if p is sufficiently close to 1 (outsiders believe that with a high probability the firm has a low intrinsic value), then the coefficient of I will be very close to $(X_H+R)/(X_L+R)$, and if this number is sufficiently high the entrepreneur is better-off not investing at all. Again the reason for this is that since outsiders do not assign the full value to the firm they require a relatively high equity participation in order to be willing to give the entrepreneur the money for investment. This means that the entrepreneur gives away a large chunk of the firm and it might well be the case that this is more than he can gain by investing.

To illustrate this idea let us consider the following example. Suppose that $X_L = 0$, $X_H = 200$, $I = 30$ and $R = 50$. If the entrepreneur does not raise I and does not invest in the project, his personal wealth given his type t (t = L, H) is given by

$$W_t(NI) = \begin{cases} 0 & if\ t = L, \\ 200 & if\ t = H. \end{cases} \tag{9}$$

If the entrepreneur raises money, then using equation (6), the equity participation that outsiders will require in a competitive capital market is

$$\hat{\alpha}^* = \frac{30}{(1-p)200 + 50}. \tag{10}$$

Note that α^* is increasing with p: the more likely is the firm to be an L type, the more equity outsiders will require. The reason for this is that outside investors need to give the firm 30 but they receive only $\alpha 50$ if the firm is an L type. To compensate them for their investment they need an equity participation of 60%. If the firm is an H type, investors get $\alpha 250$, so an equity participation of 12% (30/250) is enough to compensate them for investing. Clearly then, as p increases, the firm is more likely to be an L type, so investors require an equity participation closer to 60% than to 12%.

Given α^*, the personal wealth of a type t entrepreneur (t = L, H), is given by

$$W_t(I) = \begin{cases} (1-\alpha^*)50 & \text{if } t = L, \\ (1-\alpha^*)250 & \text{if } t = H. \end{cases} \tag{11}$$

Since $W_L(NI) = 0$ while $W_L(I) > 0$, a type L entrepreneur will always raise money and invest. The reason is that investing can only increase his payoff. The interesting question is whether a type H entrepreneur will invest. To answer this question, lets substitute for α^* from equation (9) into the second line of equation (11). The personal wealth of the entrepreneur if his firm is worth 200 is given by:

$$W_H(I) = \left(\frac{(1-p)200 + 50 - 30}{(1-p)200 + 50}\right)(200 + 50) = \frac{100(11 - 10p)}{5 - 4p}. \tag{12}$$

Given this expression, the difference between the entrepreneur's wealth if he invests and if he does not is given by:

$$\begin{aligned} W_H(I) - W_H(NI) &= \frac{100(11 - 10p)}{5 - 4p} - 200 \\ &= \frac{100(1 - 2p)}{5 - 4p}. \end{aligned} \tag{13}$$

From this expression we can conclude that the entrepreneur will issue equity and invest only when $p < 1/2$. That is, a type H entrepreneur will invest only if the likelihood that the firm's type is L is sufficiently low. As we explained above, the reason for this is that outside investors, do not know the firm's type and hence they require a larger equity participation if they believe that the firm is more likely to be an L type. This means that an H type entrepreneur stands to lose more if he is thought to be an L type with a high probability since he needs to give up a larger fraction of his firm.

3.2 A model with a continuum of firm's types

We now generalize the two-types model that we saw in the previous section. To this end, let us assume that X is drawn from a continuous distribution on the interval $[X_0, X_1]$ and let $\hat{X}$ denote its expected value. As before the realization of X is private information of the entrepreneur. Outside investors only know the distribution of X. As we saw earlier, the value of the firm in a competitive equilibrium in the capital market will be based on the expected value of the firm. Consequently, the equity of firms with high intrinsic value will be underpriced. If this underpricing is severe, firms with high intrinsic value will not issue equity and will not finance the investment. Let $\bar{X}$ be the highest intrinsic value of the firm for which the entrepreneur decides to issue equity. Entrepreneurs who have firms with $X > \bar{X}$ will forgo the project. Notice that we do not preclude the possibility that $\bar{X} = X_1$: in equilibrium it could well be the case that all entrepreneur decide to finance the project by issuing equity.

As usual, the equilibrium itself is common knowledge. Given $\bar{X}$, the expected value of the firm is given by

$$E(X \mid X \leq \bar{X}) = \int_{X_0}^{\bar{X}} X\, dF(X).$$

That is, it is the expected value conditional on X being less or equal to $\bar{X}$. Now, the condition for a competitive equilibrium in the capital market is given by

$$I = \alpha\left(E(X \mid X \leq \bar{X}) + R\right). \tag{15}$$

Solving this equation for α, we obtain the amount of equity that the entrepreneur has to issue to outsiders to induce to induce them to pay him \$I:

$$\alpha^* = \frac{\boldsymbol{I}}{E(X \mid X \leq \bar{X}) + \boldsymbol{R}}. \tag{16}$$

Using this expression, the personal wealth of the entrepreneur if he issues equity to finance the project is given by:

$$W_t(I) = (1 - \alpha^*)(X_t + \boldsymbol{R}) = \left(\frac{E(X \mid X \leq \bar{X}) + \boldsymbol{R} - I}{E(X \mid X \leq \bar{X}) + \boldsymbol{R}}\right)(X_t + \boldsymbol{R}). \tag{17}$$

The entrepreneur will issue equity and finance the investment only if this value exceeds the entrepreneur's wealth if he does not invest, W_t(NI):

$$\begin{aligned} W_t(I) - W_t(NI) &= \left(\frac{E(X \mid X \leq \bar{X}) + \boldsymbol{R} - \boldsymbol{I}}{E(X \mid X \leq \bar{X}) + \boldsymbol{R}}\right)(X_t + \boldsymbol{R}) - X_t \\ &= \boldsymbol{R} - \boldsymbol{I}\left(\frac{X_t + \boldsymbol{R}}{E(X \mid X \leq \bar{X}) + \boldsymbol{R}}\right). \end{aligned} \tag{18}$$

Notice that this expression decreasing with X_t so that indeed, entrepreneurs who know that the intrinsic value of their firms is high are less likely to issue equity in order to finance the investment. If this expression is positive for all values of X, and in particular for the largest value of X, i.e., X_1, then all entrepreneurs will issue equity. As a result we will have $\bar{X} = X_1$. Otherwise, $\bar{X}$ will be the highest value of X_t for which $W_t(I) = W_t(NI)$. That is, if at $X_t = X_1$, $W_t(I) < W_t(NI)$, then $\bar{X}$ is defined implicitly by the equation

$$W_t(I) = W_t(NI) \quad \Rightarrow \quad \boldsymbol{R} = \boldsymbol{I}\left(\frac{\bar{X} + \boldsymbol{R}}{E(X \mid X \leq \bar{X}) + \boldsymbol{R}}\right), \tag{19}$$

or

$$\frac{R}{I} = \frac{\bar{X} + R}{E(X \mid X \leq \bar{X}) + R}. \qquad (20)$$

That is, $\bar{X}$ is the value of X_t such that the ratio of the true value of the firm to its expected value (the right side of the equation) is just equal to the return on the investment (the left side of the equation). Note that since the investment is assumed to have a positive NPV, then R/I > 1. This implies in turn that $\bar{X}$ will exceed X_0 because if $\bar{X} = X_0$, then $E(X|X \leq \bar{X}) = X_0$ and the right hand side of the equation is equal to 1 which is impossible. Hence, in equilibrium there will be an interval of firms with intrinsic values above X_0 that will issue equity. Firms with high intrinsic values near X_1 may or may issue equity depending on whether the $R/I > (X_1+R)/(E(X|X \leq X_1)+R)$ or not.

As before, the extent of the underpricing of firms with high intrinsic values is more severe when there is a higher likelihood that the intrinsic value of a given firm is low. This is because in that case $E(X|X \leq \bar{X})$ is smaller for all $\bar{X}$.

4. The equity underpricing in IPOs

Stock issues can be divided to initial public offerings (IPOs) in which firms go to the capital market for the first time to raise funds, and seasoned offerings in which firms whose stocks are already traded in the market return to the capital market and issue new equity. The difference between the two types of offerings is that in IPOs there is a substantial information asymmetry between insiders and outsiders given that prior to the IPO the firm was private and its securities were not traded on the market. There is strong empirical evidence that IPOs give rise to three anomalies:

(i) Short-run equity underpricing: The closing price on the first day of trading is on average higher than the offering price. In fact, the average increase in the stock price of IPOs in the U.S. shortly after the IPO was 16.4% in the U.S. during the period 1960-1987, 31.9% in Japan during the period of 1979-1989, 79% in Korea during the period 1984-1990, and 149.3% in Malaysia during the period 1979-1984 (see Table 1 in Weiss Hanely and Ritter, "Going Public, *The New Palgrave Dictionary*).

(ii) Long-run equity overpricing: For several years following the IPO, the returns on IPOs are on average lower than those on comparable stocks. Ritter (*JF* 1991) documents that the average cumulative matching firm-adjusted return after 36 month after the IPO is -15.08%.

(iii) Oversubscription: IPOs typically attract a large number of investors and so the issue is typically oversubscribed with each investor receiving just a fraction of his order.

Probably the most convincing explanation for the short-run equity underpricing of IPOs is provided in Rock's paper "Why New Issues are Underpriced" (*JFE*, 1986). To see the main idea of Rock, let us consider a simplified version of his model. A firm plans to go to the capital market for the first time. The future cash flow of the firm is denoted X and it is drawn from a distribution function f(X) on the unit interval. The expected value of the firm is simply equal to its expected cash flow denoted $\hat{X}$.

In the capital market, there are N+1 risk neutral investors, each of whom is willing to invest up to K dollars in the firm's equity, where $K < \hat{X} < NK$. The last assumption means that individual investors do not have enough money to buy the whole firm on their own but collectively even N investors have enough money to buy the firm. A key element of the model is that some investors are informed about the realization of X while others only know the distribution of X but not the actual realization. In the next subsection we shall assume that all investors are uninformed and establish the symmetric information benchmark. Then we shall assume that one investor is informed about X while the other N investors are uninformed and show that in this situation, the firm's equity is underpriced. To simplify matters we shall assume that the entire equity of the firm is up for sale and there is no discounting. The assumption that the firm sells its entire equity is made in order to avoid the Myers and Majluf problem.

All investors are uninformed

Suppose that all N+1 investors are uninformed about X. However, since the investors know the distribution of X, they do know that the expected cash flow of the firm is $\hat{X}$. Hence, an investor who buys a fraction α of the firm's equity expects a payoff of $\alpha\hat{X}$. Since there is no discounting, this will also be the maximum amount that each investor will be willing to pay for a fraction α of the firm's equity. Consequently, the market value of the firm will be $E = \hat{X}$. Given that every investor has K dollars to invest, each will demand a fraction $\alpha = K/\hat{X}$ of the firm's equity. In total, investors will demand a fraction $(N+1)K/\hat{X}$ of the firm's equity. Since by assumption, $NK > \hat{X}$, the total demand for equity exceeds the total supply which is equal to 1. Hence, the IPO will be oversubscribed and each investor will receive only part of his order. Assuming equal rationing, each investor will receive only a fraction $1/((N+1)K/\hat{X}) = \hat{X}/(N+1)K$ of his order and will therefore end up with equity participation of $\hat{X}/(N+1)K$ x $K/\hat{X} = 1/(N+1)$. In other words, each investor will get an equity participation of $1/(N+1)$.

One informed investor

Next, suppose that there is exactly one informed investor in the capital market. Since the informed investor knows the true realization of X he will participate in the IPO only if the market value of the firm, E, is less or equal to X because if he buys a fraction α of the firm's equity, the price he pays is αE while his return is αX. If E > X, the informed investor is better-off staying out of the IPO since the price that he needs to pay exceeds his return. Thus the investment strategy of the informed investor given his information on X is

$$S^{I}(X) = \begin{cases} \textit{Particiapte in the IPO} & \textit{if } E \leq X, \\ \textit{stay out of the IPO} & \textit{if } E > X. \end{cases} \tag{21}$$

Uninformed investors cannot observe whether the informed investor participates in the IPO or not. This assumption implies that the market is anonymous. If this was not the case then the uninformed were able to mimic the informed investor so there would not be any asymmetry of information in the model. But, although the uninformed investors do not know whether the informed investor actually participates in the IPO or not they do anticipate his strategy, $S^I(X)$. Consequently, they anticipate that if E > X, they alone will participate in the IPO and will receive a fraction 1/N of the firm's equity each, and if E ≤ X, the informed investor will participate in the IPO so each uninformed investor will receive a fraction 1/(N+1) of the firm's equity. Therefore, given $S^I(X)$, the equity participation of each uninformed investor as a function of the cash flow of the firm, X, is given by

$$\alpha^{U}(X) = \begin{cases} \dfrac{1}{N+1} & \textit{if } E \leq X, \\ \dfrac{1}{N} & \textit{if } E > X. \end{cases} \tag{22}$$

The expected payoff to an uninformed investor is:

$$Y^U(E) = \int_0^E \alpha^U(X)(X-E)\,dF(X) + \int_E^1 \alpha^U(X)(X-E)\,dF(X)$$
$$= \int_0^E \frac{X-E}{N}\,dF(X) + \int_E^1 \frac{X-E}{N+1}\,dF(X). \tag{23}$$

The first term in $Y^U(E)$ is the expected payoff of an uninformed investor in states of nature in which X < E, in which case the informed investor does not participate in the IPO and each uninformed ends up with 1/N of the firm's equity. The second term in $Y^U(E)$ is the expected payoff of an uninformed investor in states of nature in which X ≥ E, in which case the informed investor participates in the IPO and each uninformed gets a fraction 1/(N+1) of the firm's equity.

In a competitive capital market, $Y^U(E)$ must be equal to 0, for if $Y^U(E) > 0$, each investor will be willing to pay a higher price for the firm's equity and get a larger fraction of equity (recall that each investor receive only a fraction of his order), whereas if $Y^U(E) < 0$, each uninformed investor will wish to stay away from the IPO. Thus, in a competitive equilibrium, the market value of the firm, E*, is given implicitly by the equation $Y^U(E) = 0$. To characterize E*, let's rewrite $Y^U(E)$ as follows:

$$Y^U(E) = \int_0^E \frac{X-E}{N}\,dF(X) + \int_E^1 \frac{X-E}{N}\,dF(X) - \int_E^1 \frac{X-E}{N}\,dF(X) + \int_E^1 \frac{X-E}{N+1}\,dF(X)$$
$$= \frac{\hat{X}-E}{N} + \int_E^1 \left[\frac{X-E}{N+1} - \frac{X-E}{N}\right] dF(X) \tag{24}$$
$$= \frac{\hat{X}-E}{N} - \int_E^1 \frac{X-E}{N(N+1)}\,dF(X).$$

Since in a competitive equilibrium this expression should be equal to 0, it follows that

$$E^* = \hat{X} - \int_{E^*}^{1} \frac{X - E^*}{N + 1} dF(X) < \hat{X}, \qquad (25)$$

where the inequality follows because over the interval [E*, 1], X > E*. Equation (25) shows that in equilibrium, the market value of the firm will be less than its expected cash flow. The reason for this result is simple: uninformed investors get the entire equity of the firm when the price is too high but only N/(N+1) of the equity when the price is too low. Hence when the uninformed end up losing money on their investment (i.e., E* > X), they bear the entire loss, but when they end up making a profit (i.e., E* < X) they have to share this profit with the informed investor. In other words, the share of each uninformed investor in losses is larger than his share in profits because the informed investor participates in the IPO only when there are profits to be made but never when the IPO is overpriced. Since the uninformed investors must break even in expectation in order to participate in the IPO, it must be the case that their expected profits are larger than their expected losses. To ensure that this is the case, the value of the firm's equity has to be discounted below its expected value so the expected profits of the uninformed investors exceed their expected losses. Thus, E* is the highest price that the firm can ask for its equity in the IPO such that the uninformed investors are willing to participate.

Note that the size of the discount, given by the integral term in equation (25), gets smaller as N gets bigger. The reason for this is that when N is large, the proportion of informed investors in the market, 1/(N+1) falls so there is relatively less information asymmetry in the market. Another way to interpret this is as follows: the presence of an informed investor in the market imposes a negative externality on the uninformed investors by lowering their share in the firm equity when the firm happens to be underpriced. But as N gets larger, the externality is divided among a large number of investors so each one of them bears a small part of it.

Now let's consider what happens to the price of the share following the IPO. First immediately after the IPO ends, the market learns how many investors participated. In our model they learn whether

N or N+1 investors participated in the IPO by simply looking at the aggregate demand for shares. If the aggregate demand was NK/$\hat{X}$, then this means that N investors participated and if the aggregate demand is (N+1)K > $\hat{X}$, then N+1 investor participated. In the first case, only uninformed investors participated in the IPO so it must be the case that X < E*, otherwise the informed investor would have participated as well. In the second case where N+1 investors participated, the informed investor must have participated so X ≥ E*.

Now consider the case that only N investors participated. Then, the uninformed investors can conclude that X < E*, so the cash flow of the firm is distributed over the interval [0, E*]. The distribution function of X, conditional on the information that the informed investor did not participate is f(X)/F(E*), where the denominator ensures that the total area under f(X) from 0 to E* sums up to 1 (otherwise this is not a distribution function). Hence the value of the firm upon learning that the informed investor decided to stay out becomes

$$E^{/} = \int_{0}^{E^*} \frac{X}{F(E^*)} dF(X). \tag{26}$$

To compare this value with E* note that

$$\begin{aligned} E^{/} - E^* &= \int_{0}^{E^*} \frac{X}{F(E^*)} dF(X) - \frac{E^* F(E^*)}{F(E^*)} \\ &= \int_{0}^{E^*} \frac{X - E^*}{F(E^*)} dF(X) < 0. \end{aligned} \tag{27}$$

Hence, if the market learns that the informed investor did not participate (i.e., the IPO was only moderately oversubscribed), then this is bad news for the market and the value of the firm drops.

If N+1 investors participated in the IPO, then the uninformed investors can conclude that X ≥ E*, so the cash flow of the firm is distributed over the interval [E*, 1]. The distribution function of X, conditional on the information that the informed investor participated is f(X)/(1-F(E*)), where the denominator ensures that the total area under f(X) from E* to 1 sums up to 1. Hence the value of the firm upon learning that the informed investor decided to participate becomes

$$E' = \int_{E^*}^{1} \frac{X}{1 - F(E^*)} dF(X). \tag{28}$$

To compare this value with E* note that

$$\begin{aligned} E' - E^* &= \int_{0}^{E^*} \frac{X}{1 - F(E^*)} dF(X) - \frac{E^*\left(1 - F(E^*)\right)}{1 - F(E^*)} \\ &= \int_{E^*}^{1} \frac{X - E^*}{1 - F(E^*)} dF(X) > 0. \end{aligned} \tag{29}$$

Hence, if the market learns that the informed investor participated in the IPO (i.e., there was only a high oversubscription for shares), then this is good news for the market because its means that the firm is worth more than the market has previously thought.

5. Signalling with equity

So far we saw that in the presence of asymmetric information the owners of firms face a problem in that the market cannot tell whether the firm is worth a lot or not and so the equity of good firms is underpriced. As Myers and Majluf show, this underpricing may in fact be so large that the firm may forgo profitable investment projects. The question then is whether good firms can somehow alleviate this problem and communicate top the market that they are in fact good. One of the early contributions to the literature that deals with the impact of asymmetric information on capital structure is Leland and Pyle’s "Information Asymmetries, Financial Structure, and Financial Intermediation," (*JF*, 1977). the basic idea is that owners of firms can signal to the capital market that the firms they own are good by retaining a large fraction of the firm's equity in their hands. This signal convinces the capital market that the firm is indeed good because the capital market realizes that the owners are risk averse and hence they are better-off selling the whole firm and thereby replace the risky cash flows of the firm with certain returns. But if the owners are willing to retain equity despite the fact that it is risky then this means that the average returns are high enough to compensate the owners for having to bear risk.

To illustrate the main ideas in Leland and Pyle (1977), let us consider an entrepreneur who establishes a firm in period 0. The firm operates only once in period 1 and is then liquidated. The cash flow of the firm in period 1 is a random variable, $\tilde{X}$, whose mean is either X_L or X_H, with $X_H > X_L$, and its variance is σ^2. Therefore, there are two types of firms, L and H, that differ only with respect to the mean of the distribution of the cash flow in period 1. In what follows we shall use $\tilde{X}_t$ to denote cash flows that are taken from a distribution whose mean is X_t, where $t = L,H$. When the entrepreneur establishes the firm, he knows whether the firm is of the L type or the H type but he does not know the actual realization of $\tilde{X}$. Assuming that the entrepreneur is risk averse while investors in the capital market are risk neutral, the entrepreneur may wish to sell a fraction of the firm's equity to outside investors to reduce the amount of risk he bears. If the entrepreneur sells a fraction $1-\alpha$ of the firm's equity and retains a fraction α, his wealth in period 2 is given by

$$W_t = \alpha \tilde{X}_t + (1-\alpha) V \qquad t = L,H, \tag{30}$$

where V is the market value of the firm in the capital market. Note that V is not indexed by t since the market value of the firm depends on the beliefs of the capital market about t rather than on the true type of the firm.

We shall now adopt the convenient assumption that the entrepreneur's expected utility function depends only on the mean of his wealth, EW_t, and the variance of his wealth, $Var(W_t)$. Specifically we shall assume that the entrepreneur's expected utility function is given by:[2]

$$U\left(W_t\right) = EW_t - \frac{b}{2}Var(W_t), \qquad t = L,H, \tag{31}$$

That is, the entrepreneur's expected utility is increasing with the mean of W_t and decreasing with variance of W_t. The latter property reflects the risk aversion of the entrepreneur: holding the mean of his wealth constant, an increase in the variance of his wealth makes the entrepreneur worse-off. The coefficient b then measures how risk averse the entrepreneur is, with larger values of b being associated with a larger degree of risk aversion. Now, since the market value of the firm in the capital market is deterministic, it follows from equation (30) that the expected wealth of the entrepreneur is

$$EW_t = \alpha X_t + (1-\alpha)V_t, \qquad t = L,H. \tag{32}$$

Using this expression, the variance of the entrepreneur's wealth is given by:

$$\begin{aligned} Var(W_t) &= E\left(\alpha\tilde{X}_t + (1-\alpha)V_t - EW_t\right)^2 \\ &= E\left(\alpha\left(\tilde{X}_t - X_t\right)\right)^2 = \alpha^2\sigma^2. \end{aligned} \tag{33}$$

Note that this expression is independent of the firm's type. This reflects the fact that the variance of the cash flow is the same regardless of what the mean cash flows are. Substituting from equations (32) and (33) into equation (31), the expected utility of a type t entrepreneur, as a function of the equity fraction that he retains, α, and the market value of the firm, V, can be written as

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[2] This expected utility function can be derived by assuming that the cash flows are distributed normally with mean X_i and variance σ^2 and the utility function of the entrepreneur over cash flows is given by $U = e^{-b\tilde{X}}$. For details, see Ch. 7.2 in Sargent T., *Macroeconomic Theory*, 2nd Ed., 1987, Academic Press.

$$EU_t(\alpha, V) = \alpha X_t + (1-\alpha)V - \frac{b}{2}\alpha^2\sigma^2, \qquad t = L,H, \tag{34}$$

Thus, the expected payoff of the entrepreneur is a weighted average of the mean cash flow of the firm and its market value, minus a term that depends on the variance of the cash flow of the firm and reflects the fact that the entrepreneur is risk averse. This term becomes larger the more risk averse the entrepreneur is, i.e., as b increases.

The full information benchmark

Under full information, the market knows whether the mean cash flow of the firm in period 1 is X_L or X_H. Since investors are assumed to be risk neutral, the market value of the firm is $V_t = X_t$, t = L,H. Substituting this expression into equation (34), the entrepreneur's expected utility becomes

$$U(\alpha, V) = X_t - \frac{\rho}{2}\alpha^2\sigma^2, \qquad t = L,H. \tag{35}$$

Clearly, the entrepreneur is better-off the smaller α is. The intuition is straightforward: the capital market is indifferent to risk and hence the market value of the firm is equal to its mean value. Since the entrepreneur is risk averse, he is better-off replacing the cash flows of the firm which are random with their certain mean. Hence under full information, the entrepreneur does not retain any fraction of the firm's equity.

The asymmetric information case

Now suppose that investors in the capital market only know the mean of the firm's cash flow can be X_L or X_H but cannot tell which one is it. If the entrepreneur will try to sell the firm on the capital market, investors will be willing to pay him the average between X_L and X_H. For instance, if investors believe that the mean cash flow of the firm is X_L with probability q and X_H with probability 1-q, they will be willing to pay for the firm's equity, $qX_L+(1-q)X_H$. Since this is less than X_H, an entrepreneur who knows that the cash flow of his firm is X_H will face an equity underpricing. The question then is whether the entrepreneur can do something about this underpricing. Of course, if the entrepreneur would just declare that the mean cash flow of his firm is X_H, the market will not believe him because they realize that even if the mean cash flow was X_L, the entrepreneur would still have an incentive to declare that it is X_H. Thus, the entrepreneur must somehow make his declaration credible and prove that he is telling the truth. One way to make the declaration credible is to retain some of the equity of the firm. By doing so, the entrepreneur demonstrates that he is willing to bear the risk only associated with holding the firm's equity because the cash flow of the firm is on average high enough to compensate for this risk.

Now we shall look for a separating equilibrium (if it exists) in which the two types of entrepreneurs act differently such that the capital market can tell them apart by observing their actions. In a separating equilibrium then, the identity of each entrepreneur is fully revealed to the capital market so that the market value of the firm is equal to its mean cash flows. Obviously, the firm's identity is not revealed all by itself: the H type entrepreneur must separate himself from the L type entrepreneur by retaining a sufficiently large fraction of the firm's equity to deter the L type entrepreneur from mimicking him. The reason why an L type entrepreneur will not find it attractive to also retain the same fraction of equity and thereby fool the market into believing that he is an H type is that the mean return on the equity of an L type firm is smaller than that of an H type firm so the return on the retained equity will be small and hence, mimicking an H type will be simply too costly for an L type entrepreneur.

Since the identities of each type of entrepreneur are fully revealed in a separating equilibrium, an L type entrepreneur has no reason to retain equity since he cannot fool the market anyway into believing that he is an H type. Hence in every separating equilibrium, it must be the case that $\alpha_L^* = 0$. Moreover, since the identity of an L type firm is fully revealed, the market value of a type L firm is X_L. Consequently, the utility of an L type entrepreneur in a separating equilibrium is given by,

$$EU_L(0, X_L) = X_L. \tag{36}$$

Having determined the strategy and the resulting utility of an L type entrepreneur, the next task is to determine the actions of an H type entrepreneur. To this end we first need to find the equity fraction that an H type entrepreneur must retain in order to deter an L type entrepreneur from mimicking him. In other words, we need to find the values of α that satisfy the following incentive constraint:

$$EU_L(\alpha, X_H) \leq X_L. \tag{37}$$

When this constraint holds, an L type entrepreneur is better-off when he retains no equity and the market believes that he is an L type than when he fools the market into believing that he is an H type by retaining a fraction α of the firm's equity. In other words, all values of α that satisfy inequality (37) make it unprofitable for an L type entrepreneur to pretend that he is an H type even if this boosts the market value of his firm from X_L to X_H. Now, using equation (34), this incentive constraint can be written as follows:

$$\alpha X_L + (1-\alpha) X_H - \frac{b}{2}\alpha^2 \sigma^2 \leq X_L. \tag{38}$$

Now let $\Delta_X \equiv X_H$-X_L be the difference between the mean cash flows of an H and L type firms. Δ_X is also a measure of the extent of information asymmetry: when Δ_X is small there is relatively little asymmetry of information because it does not make a lot of difference whether the firm is an H or an L type. On the other hand, as Δ_X increases it becomes more and more important to know the firm's type since the two types differ substantially from one another. Given the definition of Δ_X, we can solve inequality (38) and find out that the smallest α that satisfies the inequality is given by

$$\hat{\alpha}_L = \frac{\sqrt{\Delta_X(\Delta_X + 2\beta\sigma^2)} + \Delta_X}{b\,\sigma^2}. \tag{39}$$

That is, any $\alpha \geq \hat{\alpha}_L$ is large enough to deter an L type entrepreneur from mimicking an H type entrepreneur. It now remains to check whether an H type entrepreneur will find it profitable to retain such a fraction of equity. To this end, note that if an H type entrepreneur sells his entire firm to outside investors, he would be erroneously believed to be an L type and hence he will receive for the firm the amount X_L. On the other hand, if he retains a fraction α of the firm's equity and his true identity is revealed, his payoff is

$$EU_H(\alpha, X_H) = X_H - \frac{b}{2}\alpha^2\sigma^2. \tag{40}$$

Hence an H type entrepreneur will agree to retain an equity fraction α so long as $EU_H(\alpha,X_H) \geq X_L$. Using equation (40), this inequality can be expressed as

$$\Delta_X \geq \frac{b}{2}\alpha^2\sigma^2, \quad \Leftrightarrow \quad \alpha \geq \hat{\alpha}_H \equiv \sqrt{\frac{2\Delta_X}{b\sigma^2}}, \tag{41}$$

where $\hat{\alpha}_H$ is largest fraction of equity that an L type entrepreneur will agree to retain in order to separate himself. It can be checked that $\hat{\alpha}_H > \hat{\alpha}_L$. Hence every $\alpha \in [\hat{\alpha}_L, \hat{\alpha}_H]$ can be supported as the equity fraction that an H type entrepreneur will retain in a separating equilibrium. But, since the entrepreneur is better-off retaining as little equity as possible we shall focus on the Pareto undominated separating equilibrium in which an H type entrepreneur retains the fraction $\hat{\alpha}_L$ of the firm's equity which is the minimal amount the ensures separation.

To sum up we have concluded that in a separating equilibrium, $\alpha_L^* = 0$ and $\alpha_H^* = \hat{\alpha}_L$. We can now derive some empirical predictions from the model:

1. Firms that are run by managers who also own a large fraction of the firm's equity are more profitable than otherwise comparable firms that are run by managers who own a small fraction or do not own any fraction of the firm's equity.
2. When a manger of a firm sells his holdings in the firm, the market will interpret this sale as bad news and consequently there will be a negative price reaction in the market.
3. Comparative statics on $\hat{\alpha}_L$: $\hat{\alpha}_L$ increases with Δ_X and decreases with σ^2 and with b. Intuitively,

when Δ_X increases, there is more asymmetry of information in the market and hence it is more tempting for an L type entrepreneur to pretend that he is an H type (his gain from mimicking an H type is larger). Hence, a higher fraction of equity will have to be retained by an H type entrepreneur to induce separation. On the other hand, when either σ^2 or b are larger, the disutility from risk associated with retaining equity increases so a smaller fraction of equity will be sufficient to deter an L type entrepreneur from mimicking an H type. This implies that owners in industries where the extent of asymmetry of information is relatively large (e.g., fast growing industries or industries in which there is a lot of R&D such as high-tech) will retain a larger fraction of their firm's equity than managers in industries where there is little asymmetry of information (e.g., mature industries such as steel or food processing). Conversely, owners in industries where the variance of cash flows is relatively large (again examples include fast growing industries or industries that are sensitive to business cycles such as the construction industry) will retain a relatively small fraction of their firm's equity than managers in industries where the variance of cash flows is small (e.g., firms in regulated industries such as electricity, gas, and telecommunications).