Market Liquidity and Performance Monitoring

Holmstrom and Tirole (*JPE*, 1993)

The main idea

A firm would like to issue shares in the capital market because once these shares are publicly traded, speculators will gather information on the firm's performance, so the stock price will (at least partly) reflect this information. The firm would then be able to condition the manager's compensation on the stock price, thereby providing him with sharper incentives to exert effort and enhance the firm's performance. This information however comes at a cost because the profits that the speculators make come at the expense of other stockholders. Realizing that they will lose money on trades with speculators, the stockholders pay less for the shares they buy when the firm issues them. At the optimum, the firm trades-off the beneficial impact of the increase in managerial effort against the lower price it gets for its shares when it issues them in the capital market.

The sequence of events:

The model evolves in 3 periods as follows:

Period 0:
- The firm is established and a fraction $1-\delta$ of the shares are sold to outside equityholders at a price p_0.
- The firm signs an incentive contract with the manager.

Period 1:
- The manager chooses his effort level in periods 1 and 2, e_1 and e_2.
- The first period earning, π_1, are realized.
- The manager gets paid and the rest of the earnings are paid as a dividend.
- Speculators observe (after paying a cost) a signal on the period 2 earnings.
- The firm's shares are traded in the capital market.

Period 2:
- The second period earning, π_2, are realized.
- The manager gets paid.
- The firm is liquidated.

Technology and information

The manager chooses his effort levels in periods 1 and 2 at the outset. These effort levels are denoted e_1 and e_2. Given e_1 and e_2, the earnings in periods 1 and 2 are as follows:

$$\pi_1 = e_1 + \varepsilon_1, \quad \textbf{(1)}$$

and

$$\pi_2 = e_2 + \theta + \varepsilon_2, \quad \textbf{(2)}$$

where ε_1, ε_2, and θ are normally distributed, zero mean, and independent random variables (in fact all random variables in this model are assumed to be independent). Their variances are σ_1^2, σ_2^2, σ_θ^2.

In period 1 the speculator observes a signal, s, on the liquidation value of the firm in period 2:

$$s = e_2 + \theta + \eta, \quad \textbf{(3)}$$

where η is a normally distributed, zero mean, random variables, with variance σ_η^2. The cost of the signal depends on its precision. In particular it is given by $g(1/\sigma_\eta^2)$, where g', $g'' > 0$, and $g(0) = 0$.

Before continuing it is worth mentioning that the variable θ although is seems superfluous actually plays a crucial role in the analysis. The reason is that in equilibrium, the effort level of the manager will be anticipated. Hence the unique information that speculators will have will be about θ. If $\sigma_\theta^2 = 0$ (i.e., θ is deterministic), speculators will simply observe the effort of the manager with a noise, but since the equilibrium action of the manager will be anticipated, their information will be useless.

The manager

Let I denote the manager's income. The manager is risk averse and his utility function is given by:

$$U(I,e_1,e_2) = E(I) - \frac{r}{2}\,var(I) - c(e_1,e_2). \quad \textbf{(4)}$$

The manager's income depends on the contract the firm gives him. This contract depends on π_1, π_2, and on the share price in period 1, p_1. The assumption is that the contract is linear:

$$I = W + B\pi_1 + Ap_1 + S(\pi_2 - Ap_1), \quad \textbf{(5)}$$

where W is a base salary paid out of π_1, S are shares transferred from insiders to the manager, and Ap_1 are stock appreciation rights paid out of π_2. In other words, the manager get $W+B\pi_1$ in period 1 plus $Ap_1+S(\pi_2-Ap_1)$ in period 2.

The rational expectations equilibrium in the stock market

As in Kyle 1985, trading evolves in two steps:

- Step 1: Speculators privately observe s (in Kyle the signal is perfect; here it is noisy) and choose a quantity, x(s), to be traded (speculators commit to x(s) even though they still do not know the price at which trade will take place). x(s) could be either positive or negative so speculators either buy or sell shares.
- Step 2: Market makers observe q = y+x(s), where y is the net demand of noise traders, update their belief about π_2, and set a fair price for the share, p_1. The fair price just reflects the belief of the market makers about the value of the firm in period 2 given the available information which is q.

The net demand of noise traders is assumed to be normally distributed random variable, y, with zero mean and a variance σ_y^2.

A rational expectations trading equilibrium is a pair (x, p_1) that satisfies the following conditions:

(i) The trading strategy of speculators maximizes their expected payoff given their information and their correct expectation about p_1: x(s) = Argmax_x R(x), where

$$\begin{aligned} R(x) &= x\left(E\left[\pi_2 - Ap_1 \mid s,x\right] - E\left[p_1 \mid x\right]\right) \\ &= x\left(E\left[\pi_2 \mid s\right] - (1+A)E\left[p_1 \mid x\right]\right). \end{aligned} \quad \textbf{(6)}$$

The expression in the square brackets is just the profit that speculators make for each share they buy. This profit comes about because the speculators pay in expectation the price p_1 (when they submit their net demand, they still do not know what p_1 will be) but expect to get a profit of π_2 when the firm is liquidated in period 2. Note that the expectation of π_2 is conditioned s while p_1 is conditional on x.

(ii) Market makers set a fair price for the share given q:

$$p_1 = E\left[\pi_2 - Ap_1 \mid y + x(s) = q\right]. \quad \textbf{(7)}$$

Attention is restricted to linear strategies of speculators so we will conjecture that

$$x(s) = \alpha + \beta s. \tag{8}$$

Of course we will have to check that this conjecture is consistent with the optimal behavior of the speculators.

Equilibrium characterization

First let us solve for the equilibrium share price. From (7), it follows that

$$p_1 = \frac{E\left[\pi_2 \mid y + x(s) = q\right]}{1 + A}. \tag{9}$$

Using a univariate projection (for details, see p. 223-226 in T. Sargent, *Macroeconomic Theory*, 2nd. edition) of π_2 on q:

$$E\left[\pi_2 \mid y + x(s) = q\right] = E(\pi_2) + \frac{cov(\pi_2, q)}{var(q)}(q - E(q)), \tag{10}$$

where $E(\pi_2) = e_2$. The univariate projection is nothing but a simple OLS estimator of π_2 that we get using q as the "explanatory variable" (note that the "noise term," y, is distributed normally). In other words, suppose we use observations of q to estimate π_2 using OLS. Then, we hypothesize that $\pi_2 = \alpha_0 + \alpha_1 q$ and we find the coefficients α_0 and α_1 by minimizing the expression

$$E\left(\pi_2 - \alpha_0 - \alpha_1 q\right)^2. \tag{11}$$

The first order conditions for minimization require that

$$\begin{aligned} -2E\left(\pi_2 - \alpha_0 - \alpha_1 q\right) &= 0, \\ -E\left(\pi_2 - \alpha_0 - \alpha_1 q\right)q &= 0. \end{aligned} \tag{12}$$

From these conditions we get:

$$\begin{aligned} E(\pi_2) &= \alpha_0 + \alpha_1 E(q), \\ E(\pi_2 q) &= \alpha_0 E(q) + \alpha_1 E(q^2). \end{aligned} \tag{13}$$

Solving the two equations for α_0 and α_1, yields:

$$\alpha_0 = E(\pi_2) - \alpha_1 E(q),$$

$$\alpha_1 = \frac{E(\pi_2)E(q) - E(\pi_2 q)}{E(q)^2 - E(q^2)} = \frac{E(\pi_2 - E(\pi_2))(q - E(q))}{E(q - E(q))^2} = \frac{cov(\pi_2, q)}{var(q)}. \quad \textbf{(14)}$$

Having "estimated" α_0 and α_1 using q, the estimated value of π_2 is

$$E\left[\pi_2 \mid y + x(s) = q\right] = \alpha_0 + \alpha_1 q$$
$$= \left(E(\pi_2) - \alpha_1 E(q)\right) + \alpha_1 q = E(\pi_2) + \alpha_1 (q - E(q)). \quad \textbf{(15)}$$

If we substitute for α_1 from (14) we get equation (10).

Given our conjecture that $x(s) = \alpha + \beta s$ and since $E(y) = 0$,

$$E(q) = E(x(s)) + E(y) = \alpha + \beta e_2.$$

Recalling that all random variables are independent and have 0 means, it follows that

$$E(\theta\eta) = E(\theta y) = E(\eta y) = E(\theta\varepsilon_2) = E(\eta\varepsilon_2) = E(y\varepsilon_2) = 0.$$

Hence,

$$cov(\pi_2, q) = E\left[\left(\pi_2 - E(\pi_2)\right)\left(q - E(q)\right)\right]$$
$$= E\left[\left(\theta + \varepsilon_2\right)\left(\beta(\theta + \eta) + y\right)\right] \quad \textbf{(16)}$$
$$= \beta E\left(\theta^2\right) = \beta \sigma_\theta^2.$$

Similarly,

$$var(q) = E\left(q - E(q)\right)^2 = E\left[\left(\beta(\theta + \eta) + y\right)^2\right]$$
$$= \beta^2 E\left[(\theta + \eta)^2\right] + E(y^2) + 2\beta E\left[(\theta + \eta) y\right], \quad \textbf{(17)}$$
$$= \beta^2\left(\sigma_\theta^2 + \sigma_\eta^2\right) + \sigma_y^2.$$

Substituting the expressions for $cov(\pi_2,q)$ and $var(q)$ in (10) and rearranging terms,

$$E\left[\pi_2 \mid y + x(s) = q\right] = e_2 + \frac{\beta\,\sigma_\theta^2}{\beta^2(\sigma_\theta^2 + \sigma_\eta^2) + \sigma_y^2}\left(x + y - \alpha - \beta\, e_2\right) \tag{18}$$

$$= (1 - \mu\,\beta)\,e_2 + \mu\,(x + y - \alpha),$$

where

$$\mu \equiv \frac{\beta\,\sigma_\theta^2}{\beta^2(\sigma_\theta^2 + \sigma_\eta^2) + \sigma_y^2}. \tag{19}$$

Substituting from (18) into (9) yields:

$$p_1 = \frac{(1 - \mu\,\beta)\,e_2 + \mu\,(x + y - \alpha)}{1 + A}. \tag{20}$$

Recalling that E(y) = 0, it follows that

$$E\left[p_1 \mid x\right] = \frac{(1 - \mu\,\beta)\,e_2 + \mu\,(x - \alpha)}{1 + A}. \tag{21}$$

Next we can solve for the equilibrium trading strategy of the speculators. To this end, we first need to find out what $E[\pi_2|s]$ is. Using a univariate projection of π_2 on s yields:

$$E\left[\pi_2 \mid s\right] = E(\pi_2) + \frac{cov(\pi_2, s)}{var(s)}\,(s - E(s)), \tag{22}$$

where $E(\pi_2) = E(s) = e_2$. Since all random variables are independent, it follows that

$$E(\theta\eta) = E(\theta\varepsilon_2) = E(\eta\varepsilon_2) = 0.$$

Hence,

$$cov(\pi_2, s) = E\left[\left(\pi_2 - E(\pi_2)\right)(s - E(s))\right]$$
$$= E\left[\left(\theta + \varepsilon_2\right)(\theta + \eta)\right] = E\left(\theta^2\right) = \sigma_\theta^2. \tag{23}$$

Similarly,

$$var(s) = E(s - E(s))^2 = E\left[(\theta + \eta)^2\right] = \sigma_\theta^2 + \sigma_\eta^2. \tag{24}$$

Substituting from (24) and (23) into (22) and rearranging terms,

$$E\left[\pi_2 \mid s\right] = e_2 + \lambda\left(s - e_2\right); \qquad \lambda \equiv \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\eta^2}. \tag{25}$$

Substituting from equations (21) and (25) into equation (6), yields:

$$\begin{aligned} R(x) &= x\left(e_2 + \lambda(s - e_2) - (1 - \mu\beta)e_2 - \mu(x - \alpha)\right) \\ &= x\left(\lambda s + (\mu\beta - \lambda)e_2 - \mu(x - \alpha)\right). \end{aligned} \tag{26}$$

Maximizing this expression with respect to x reveals that the equilibrium trading strategy of the speculators is given by:

$$x(s) = \frac{(\mu\beta - \lambda)e_2 + \alpha\mu}{2\mu} + \frac{\lambda}{2\mu}s. \tag{27}$$

Equation (27) shows that x(s) is indeed a linear function as we conjectured. Matching the coefficients in equation (27) with those in equation (8) and using the definitions of λ and μ it follows that

$$\beta = \frac{\sigma_y}{\sqrt{\sigma_\theta^2 + \sigma_\eta^2}}, \qquad \alpha = -\beta e_2. \tag{28}$$

Substituting for α and β from equation (28) into equation (8) and using equation (3), reveals that the equilibrium trading strategy of the speculators is:

$$x(s) = -\beta e_2 + \beta s = \beta(\theta + \eta). \tag{29}$$

Since θ and η are normally distributed, zero mean, random variables, the equilibrium amount of shares traded by the speculators is also a normally distributed, zero mean, random variable which is independent of managerial effort.

Now we need to determine the investment level of speculators in information gathering. Note that the precision of the signal s, $1/\sigma_\eta^2$, is chosen before the speculators know the actual realization of their trading profits. Consequently, the speculators will choose $1/\sigma_\eta^2$ on the basis of their expectations of these profits. Using ER to denote the expected profit of the speculators from trading, the speculators' maximization problem is given by:

$$\underset{\frac{1}{\sigma_\eta^2}}{Max} \; ER - g\left(\frac{1}{\sigma_\eta^2}\right). \tag{30}$$

Before we can solve this problem we first need to compute ER. Substituting for x(s) from (29) into (26) and using the definitions of μ and λ yields

$$\begin{aligned} \boldsymbol{R} \equiv \boldsymbol{R}(x(s)) &= x(s)\left(\lambda s + (\mu\beta - \lambda)e_2 - \mu(x(s) - \alpha)\right) \\ &= \beta(\theta + \eta)((\lambda - \mu\beta)(\theta + \eta)) \\ &= \frac{\sigma_\theta^2 \sigma_y}{2\left(\sigma_\theta^2 + \sigma_\eta^2\right)^{3/2}} (\theta + \eta)^2. \end{aligned} \tag{31}$$

Taking the expectation of this expression and recalling that θ and η are independent, the expected payoff of the speculators is:

$$ER = \frac{\sigma_\theta^2 \sigma_y}{2\left(\sigma_\theta^2 + \sigma_\eta^2\right)^{3/2}}\left(\sigma_\theta^2 + \sigma_\eta^2\right) = \frac{\sigma_\theta^2 \sigma_y}{2\sqrt{\sigma_\theta^2 + \sigma_\eta^2}}. \tag{32}$$

Substituting this expression into (30), the speculators' maximization problem becomes:

$$\underset{\sigma_\eta^2}{Max} \; \frac{\sigma_\theta^2 \sigma_y}{2\sqrt{\sigma_\theta^2 + \sigma_\eta^2}} - g\left(\frac{1}{\sigma_\eta^2}\right). \tag{33}$$

Before solving the problem it is worth noting that ER is increasing with σ_y. In other words, the more liquid the firm's stock is (recall that by assumption, as 1-δ increases, so does σ_y), the more profits speculators make. The reason for this is that as σ_y increases, speculators can better "hide" themselves and exploit their superior information better. Not surprisingly then, the higher is σ_y, the more information speculators collect and hence the stock price will be a better measure of the effort level that the manager exerts.

It is also worth noting that ER is independent of the manager's effort and of the manager's contract. Therefore the problem of motivating the speculators to collect information is distinct from the problem of motivating the manager to exert effort. The latter problem of course will depend on the information content of the stock price (which reflects the speculator's information) but the relationship runs only in one way. This is very useful property because it means that we do not need to solve the two problem simultaneously: we can first solve the speculators' problem and only then solve the manager's problem.

The first order condition for the speculators' problem is

$$\frac{\sigma_\eta^4 \sigma_\theta^2 \sigma_y}{4\left(\sigma_\theta^2 + \sigma_\eta^2\right)^{3/2}} = g'\left(\frac{1}{\sigma_\eta^2}\right). \tag{34}$$

The left hand side of the equation is the marginal benefit from information gathering and it decreasing in $1/\sigma_\eta^2$. The right hand side of the equation is the marginal cost of information gathering, and since g'' > 0, it is increasing in $1/\sigma_\eta^2$. Hence, there is a unique σ_η^2 that solves equation (34). This implies in turn that β is also unique so the linear rational expectations trading equilibrium is also unique. In addition note that the larger σ_y, the higher is the marginal benefit from information gathering. This implies that speculators will gather more information (i.e., $1/\sigma_\eta^2$ will be higher) as there is more noise trading. Consequently, the more noise trade there is in the market, the more informative the share prices will be about the liquidation value of the firm. This result seems somewhat paradoxical, but it turns out that when σ_y increases, x(s) adjusts as well exactly so that var(p_1) remains a constant. To see this, let us substitute from (28) into (20) and use the definition of μ to obtain:

$$p_1 = \frac{e_2 + \dfrac{\sigma_\theta^2(\theta + \eta)}{2\left(\sigma_\theta^2 + \sigma_\eta^2\right)} + \dfrac{\sigma_\theta^2}{2\sqrt{\sigma_\theta^2 + \sigma_\eta^2}}\dfrac{y}{\sigma_y}}{1 + A}. \tag{35}$$

Since E(θ) = E(η) = E(μ) = E(y) = 0, it follows that

$$E(p_1) = e_2/(1+A).$$

Consequently,

$$var(p_1) = E\left(p_1 - E(p_1)\right)^2 = \frac{E\left(\dfrac{\sigma_\theta^2(\theta+\eta)}{2(\sigma_\theta^2+\sigma_\eta^2)} + \dfrac{\sigma_\theta^2}{2(\sigma_\theta^2+\sigma_\eta^2)^{\frac{1}{2}}}\dfrac{y}{\sigma_y}\right)^2}{(1+A)^2} \tag{36}$$

$$= \frac{\sigma_\theta^4}{4(1+A)^2\left(\sigma_\theta^2+\sigma_\eta^2\right)^2}\left[\sigma_\theta^2+\sigma_\eta^2+\frac{\sigma_y^2(\sigma_\theta^2+\sigma_\eta^2)}{\sigma_y^2}\right] = \frac{\sigma_\theta^4}{2(1+A)^2\left(\sigma_\theta^2+\sigma_\eta^2\right)}.$$

Equation (36) shows that var(p_1) is independent of σ_y. But since var(p_1) is increasing with $1/\sigma_\eta^2$ which is increasing with σ_y, p_1 becomes more volatile as σ_y increases. That is, the more liquid the firm's stock is, the more volatile the stock price is. This increased volatility reflects the speculators information.

Clearly, the firm may wish to condition the manager's compensation on p_1 only if p_1 is an informative signal on π_2. Using a univariate projection of π_2 on p_1:

$$E\left[\pi_2 \mid p_1\right] = E(\pi_2) + \frac{cov(\pi_2,p_1)}{var(p_1)}\left(p_1 - E(p_1)\right), \tag{37}$$

where $E(\pi_2) = e_2$. Using equation (35) and recalling that by independence,

$$E(\theta\eta) = E(\theta y) = E(\eta y) = E(\theta\varepsilon_2) = E(\eta\varepsilon_2) = E(y\varepsilon_2) = 0,$$

it follows that,

$$cov(\pi_2,p_1) = E\left[\left(\pi_2 - E(\pi_2)\right)\left(p_1 - E(p_1)\right)\right]$$

$$= \frac{E\left[\left(\theta+\varepsilon_2\right)\left(\dfrac{\sigma_\theta^2(\theta+\eta)}{2\left(\sigma_\theta^2+\sigma_\eta^2\right)} + \dfrac{\sigma_\theta^2}{2\sqrt{\sigma_\theta^2+\sigma_\eta^2}}\dfrac{y}{\sigma_y}\right)\right]}{1+A} \tag{38}$$

$$= \frac{\sigma_\theta^4}{2(1+A)\left(\sigma_\theta^2+\sigma_\eta^2\right)}.$$

Hence, $cov(\pi_2,p_1) = (1+A)var(p_1)$. Therefore, equation (37) becomes

$$E(\pi_2 \mid p_1) = e_2 + (1 + A)\left(p_1 - E(p_1)\right)$$
$$= e_2 + \frac{\sigma_\theta^2(\theta + \eta)}{2\left(\sigma_\theta^2 + \sigma_\eta^2\right)} + \frac{\sigma_\theta^2}{2\sqrt{\sigma_\theta^2 + \sigma_\eta^2}}\frac{y}{\sigma_y}. \quad \textbf{(39)}$$

The speculators' information is $\theta+\eta$. Equation (39) therefore shows that p_1 becomes more sensitive to this information as $1/\sigma_\eta^2$ increases so the more information is gathered, the more informative p_1 becomes on π_2.

Managerial contract

Having solved for the equilibrium in the capital market and for the speculators' equilibrium level of investment in information gathering, we next solve for the equilibrium contract that insiders will give the manager. Insiders choose this contract to maximize their payoffs, net of the profits earned by the speculators, subject to the incentive compatibility and the individual rationality constraints of the manager. The profits of the speculators have to be subtracted from the insiders' payoff because when the firm issues shares, the buyers are noise traders who expect to make a loss on their trades with speculators. Hence, they will take these losses into account when deciding how much to pay for the firm's shares. This implies in turn that the speculators' profits are ultimately borne by insiders.

Hence, the insiders' problem is:

$$\underset{B,W,S,A}{Max}\; E(\pi_1 + \pi_2) - ER - EI$$
$$s.t.\quad (e_1,e_2) \in \underset{e_1,e_2}{Argmax}\; EI - \frac{r}{2}var(I) - c(e_1,e_2), \quad \textbf{(40)}$$
$$EI - \frac{r}{2}var(I) - c(e_1,e_2) \geq 0.$$

At the optimum, the individual rationality constraint must be binding otherwise the firm can always lower the manager's base salary, W, slightly and make more money. Substituting from the individual rationality into the objective function, the insiders' problem becomes:

$$\underset{B,W,S,A}{Max}\ E(\pi_1 + \pi_2) - ER - \frac{r}{2}var(I) - c(e_1,e_2)$$
$$s.t.\quad (e_1,e_2) \in \underset{e_1,e_2}{Argmax}\ EI - \frac{r}{2}\,var(I) - c(e_1,e_2). \qquad \textbf{(41)}$$

Equation (41) shows that insiders would like to minimize the variance of the manager's compensation. This is because the manager is averse to risk so the more risk he bares, the more money the firm will have to pay him in order to compensate him for this risk. The firm then needs to find an optimal compensation scheme that motivates the managet to exert effort while minimizing the risk he bears.

Instead of solving this problem, let's make the following transformations:

$$b \equiv \frac{A(1-S)\nu}{1+A}, \qquad z \equiv \frac{(1+A)p_1 - (1-\nu)e_2}{\nu}, \qquad d \equiv W + \frac{b(1-\nu)e_2}{\nu}, \qquad \textbf{(42)}$$

where ν is a positive constant. Using these definitions, we can write the manager's compensation as

$$I = B\pi_1 + S\pi_2 + bz + d. \qquad \textbf{(43)}$$

For future reference it is worth noting that since $E(p_1) = e_2/(1+A)$, then

$$E(z) = \frac{(1+A)p_1 - (1-\nu)e_2}{\nu} = e_2. \qquad \textbf{(44)}$$

The expected compensation of the manager is

$$E(I) = Be_1 + (S+b)e_2 + d, \qquad \textbf{(45)}$$

and its variance is

$$Var(I) = E\big(B\pi_1 + S\pi_2 + bz + d - (Be_1 + (S+b)e_2 + d)\big)^2$$
$$= B^2\sigma_1^2 + S^2(\sigma_2^2 + \sigma_\theta^2) + 2b^2(\sigma_\theta^2 + \sigma_\eta^2) + 2Sb\sigma_\theta^2. \qquad \textbf{(46)}$$

Noting that $E(\pi_1+\pi_2) = e_1 + e_2$, the insider's problem can be written as

$$\underset{B,S,b,d}{Max}\ e_1 + e_2 - ER - \frac{r}{2} var(I) - c(e_1,e_2)$$
$$s.t.\quad (e_1,e_2) \in \underset{e_1,e_2}{Argmax}\ Be_1 + (S+b)e_2 + d - \frac{r}{2} var(I) - c(e_1,e_2). \tag{47}$$

Now, the incentive compatibility constraint implies that the manager's effort is given by the following first order conditions:

$$B = c_1, \qquad S + b = c_2, \tag{48}$$

where c_1 and c_2 are the partial derivatives of $c(e_1,e_2)$.

Therefore, we can now simplify the insider's problem further and express it as:

$$\underset{B,S,b}{Max}\ e_1 + e_2 - ER - \frac{r}{2} var(I) - c(e_1,e_2)$$
$$s.t.\quad B = c_1,$$
$$S + b = c_2. \tag{49}$$

The second constraint shows clearly that what matters for incentives is only the sum of S and b. Hence insiders will choose the combination of S and b that mximizes their payoff. But since S and b only affect var(I), it is clear that they will be selected to minimize var(I). Now let

$$S + b = K.$$

Then, insiders need to solve the following problem:

$$\underset{S}{Min}\ Var(I) = B^2\sigma_1^2 + S^2(\sigma_2^2 + \sigma_\theta^2) + 2(K-S)^2(\sigma_\theta^2 + \sigma_\eta^2) + 2S(K-S)\sigma_\theta^2. \tag{50}$$

The first order condition for this problem is:

$$2S(\sigma_2^2 + \sigma_\theta^2) - 4(K-S)(\sigma_\theta^2 + \sigma_\eta^2) + 2(K-2S)\sigma_\theta^2 = 0. \tag{51}$$

Using the definition of K and rearranging terms we get:

$$\frac{b}{S} = \frac{\sigma_2^2}{\sigma_\theta^2 + 2\sigma_\eta^2}. \tag{52}$$

Equations (52) has several important implications. First, it shows that it must be the case that $S > 0$ (otherwise the left side of the inequality approaches ∞). Hence, any optimal compensation scheme will include shares. Second, unless $\varepsilon_2 = 0$, the optimal compensation scheme will be such that $b > 0$. Using the definition of b and noting that $S < 1$ (the manager cannot own more than 100% of the firm's shares), this means that $A > 0$. That is, the manager's compensation will include stock appreciation rights. Third, the ratio of b to S increases with ε_2. When ε_2 increases, π_2 becomes less attractive for compensation purposes from the firm's view point since it is becomes more risky and hence more expensive for the firm (reacll that the firm needs to compensate the manager for the risk he bears). Hence, the firm lowers S and increases b and thereby lowers the weight that π_2 has in the manager's overall compensation.

= 0, the optimal compensation scheme will be such that $b > 0$. Using

Since b is the coefficient of z which and

θ are normally distributed, zero mean, and independent random variables (in fact all

$$\begin{aligned} \boldsymbol{R}(x) &= x(s)\left\{\lambda s + (\mu - \lambda)\, e_2 - \mu\, \frac{x(s) - \alpha}{\beta}\right\} \\ &= \frac{\beta}{4\mu}\left\{\lambda(e_2 + \theta + \eta) + (\mu - \lambda)e_2 + \frac{\alpha\,\mu}{\beta}\right\}^2 \qquad \textbf{(53)} \\ &= \frac{\beta}{4\mu}\left\{\mu\left(e_2 + \frac{\alpha}{\beta}\right) + \lambda\,(\theta + \eta)\right\}^2. \end{aligned}$$

Taking the expectation of this expression, using the definition of λ, and recalling that θ and η are independent, the expected payoff of the speculators is given by:

$$\begin{aligned} E(\boldsymbol{R}(x)) &= \frac{\beta}{4\mu}\left\{\mu^2\left(e_2 + \frac{\alpha}{\beta}\right)^2 + \left(\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\eta^2}\right)^2 (\sigma_\theta^2 + \sigma_\eta^2)\right\} \qquad \textbf{(54)} \\ &= \frac{\beta}{4\mu}\left\{\mu^2\left(e_2 + \frac{\alpha}{\beta}\right)^2 + \frac{\sigma_\theta^4}{\sigma_\theta^2 + \sigma_\eta^2}\right\}. \end{aligned}$$

Using this expression, the first order condition for the speculators' problem is

$$\frac{\beta}{4\mu} \times \frac{2\,\sigma_\theta^4\,\sigma_\eta}{\left(\sigma_\theta^2 + \sigma_\eta^2\right)^2} = \boldsymbol{g}'\left(\frac{1}{\sigma_\eta^2}\right) \times \frac{2}{\sigma_\eta^3}. \qquad \textbf{(55)}$$

Using the definitions of β and μ, we can write this first order condition as follows:

$$\frac{\sigma_\eta^4\,\sigma_\theta^2\,\sigma_y}{2\left(\sigma_\theta^2 + \sigma_\eta^2\right)^{3/2}} = \boldsymbol{g}'\left(\frac{1}{\sigma_\eta^2}\right). \qquad \textbf{(56)}$$

The left hand side of the equation is the marginal benefit from information gathering and it decreasing in σ_η. The right hand side of the equation is the marginal cost of information gathering, and since g'' > 0, it is increasing in σ_η. Hence, there is a unique σ_η that solves equation (28). This implies in turn that β is also unique so the linear rational expectations trading equilibrium is also unique. In addition note that the larger σ_y, the higher is the marginal benefit from information gathering. This implies that speculators will gather more information as there is more noise trading. Consequently, the more noise there is in the market, the more informative the share prices will be about the liquidation value of the firm.

Note that

$$E\left[\pi_2 \mid y + x(s) = q\right] = e_2 + \frac{\beta\,\sigma_\theta^2}{\beta^2(\sigma_\theta^2 + \sigma_\eta^2) + \sigma_y^2}\left(x + y - \alpha - \beta e_2\right)$$

$$= e_2 + \frac{\beta^2\,\sigma_\theta^2}{\beta^2(\sigma_\theta^2 + \sigma_\eta^2) + \sigma_y^2}\left(\frac{x + y - \alpha}{\beta} - e_2\right) \qquad \textbf{(57)}$$

$$= (1 - \mu)e_2 + \mu\,\frac{x + y - \alpha}{\beta}; \qquad \mu = \frac{\beta^2\,\sigma_\theta^2}{\beta^2(\sigma_\theta^2 + \sigma_\eta^2) + \sigma_y^2}.$$

$$E\left[\pi_2 \mid y + x(s) = q\right] = e_2 + \frac{\beta\,\sigma_\theta^2}{\beta^2(\sigma_\theta^2 + \sigma_\eta^2) + \sigma_y^2}\left(x + y - \alpha - \beta e_2\right)$$

$$= e_2 + \frac{\beta^2\,\sigma_\theta^2}{\beta^2(\sigma_\theta^2 + \sigma_\eta^2) + \sigma_y^2}\left(\frac{x + y - \alpha}{\beta} - e_2\right). \qquad \textbf{(58)}$$