Corporate Finance: Modigliani and Miller

Yossi Spiegel
Recanati School of Business

The M&M 1958 setting

- A firm operates for infinitely many periods
- Each period, the firm generates a random cash flow, X, distributed over the interval [X_0, X_1] according to some distribution function
- Assumptions:
  - A1 - No transactions costs for buying and selling securities and no bid-ask spreads (i.e., buying and selling prices are the same).
  - A2 - The capital market is perfectly competitive
  - A3 - No bankruptcy costs
  - A4- No corporate or personal taxes
  - A5 - All agents (firms and investors) have the same information

The market value of an all-equity firm

- Since the market is perfectly competitive:

$$V_U = \frac{\hat{X}}{1+\rho} + \frac{\hat{X}}{(1+\rho)^2} + \frac{\hat{X}}{(1+\rho)^3} + \ldots = \frac{\hat{X}}{1+\rho}\underbrace{\left[1 + \frac{1}{1+\rho} + \frac{1}{(1+\rho)^2} + \ldots\right]}_{z}$$

- Now

$$z - \frac{z}{1+\rho} = 1 + \frac{1}{1+\rho} + \frac{1}{(1+\rho)^2} + \ldots - \frac{1}{1+\rho}\underbrace{\left[1 + \frac{1}{1+\rho} + \frac{1}{(1+\rho)^2} + \ldots\right]}_{z} = 1$$

- Hence $z = \dfrac{1+\rho}{\rho}$

The market value of an all-equity firm

- Since the market is perfectly competitive:

$$V_U = \frac{\hat{X}}{1+\rho} + \frac{\hat{X}}{(1+\rho)^2} + \frac{\hat{X}}{(1+\rho)^3} + \ldots$$

$$= \frac{\hat{X}}{1+\rho}\left[1 + \frac{1}{1+\rho} + \frac{1}{(1+\rho)^2} + \ldots\right]$$

$$= \frac{\hat{X}}{1+\rho} \times \frac{1+\rho}{\rho} = \frac{\hat{X}}{\rho}$$

The market value of an all-equity firm – another derivation

- Since the market is perfectly competitive:

$$V_U = \frac{\hat{X}}{1+\rho} + \frac{\hat{X}}{(1+\rho)^2} + \frac{\hat{X}}{(1+\rho)^3} + \ldots = \frac{\hat{X}}{1+\rho} + \frac{1}{1+\rho}\underbrace{\left[\frac{\hat{X}}{1+\rho} + \frac{\hat{X}}{(1+\rho)^2} + \ldots\right]}_{V_U}$$

- Hence

$$V_U = \frac{\hat{X}}{1+\rho} + \frac{V_U}{1+\rho} \quad \Rightarrow \quad V_U = \frac{\hat{X}}{\rho}$$

The market value of a leveraged firm

- The market value of debt:

$$B_L = \frac{rD}{1+r} + \frac{rD}{(1+r)^2} + \frac{rD}{(1+r)^3} + \ldots$$

$$= \frac{rD}{1+r}\left[1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots\right]$$

$$= \frac{rD}{1+r} \times \frac{1+r}{r} = D$$

- The market value of equity

$$E_L = \frac{\hat{X} - rD}{\rho_L}$$

The market value of a leveraged firm

- The total value of the firm:

$$V_L = B_L + E_L = D + \frac{\hat{X} - rD}{\rho_L}$$

The M&M propositions

M&M Proposition 1

- Given Assumptions A1-A5, the market values of U and L are equal: $V_U = V_L$

- The proof of M&M1 uses a no-arbitrage argument
  - Under Assumptions A1-A5, investors in U can replicate the cash flows of investors in L and vice versa so the prices of U and L must be same

Proof of M&M1 – part 1

- Suppose by way of negation that $V_U > V_L$
- Consider an investor who holds a fraction α of firm U
- The investor's payoff is αX and the market value of his portfolio is αV_U
- Consider the following investment strategy: "Sell the holdings in firm U and buy a fraction α of firm L's equity and debt"

Proof of M&M1 – part 1

- The investor's payoff

$$\alpha\left(X - rD\right) + \alpha rD = \alpha X$$

- The investor's wealth:

$$\alpha V_U - \alpha V_L > 0$$

- The strategy yields the same payoff but a higher wealth so all investors should follow it; but then, it's no longer true that $V_U > V_L$

Proof of M&M1 – part 2

- Suppose by way of negation that $V_U < V_L$
- Consider an investor who holds a fraction β of firm L
- The investor's payoff is $\beta(X\text{-}rD)$ and the market value of his portfolio is βV_L
- Consider the following investment strategy: "Sell the holdings in firm L and buy a fraction β of firm U's equity and borrow βD at an interest rate r"

Proof of M&M1 – part 2

- The investor's payoff

$$\beta X - r\beta D = \beta\left(X - rD\right)$$

- The investor's wealth:

$$\beta E_L + \beta D - \beta V_U = \beta\underbrace{\left(E_L + D\right)}_{V_L} - \beta V_U > 0$$

- The strategy yields the same payoff, but a higher wealth, so all investors should follow it; but then, it's no longer true that $V_U < V_L$

M&M Proposition 2

- Given Assumptions A1-A5, the discount rate for a leveraged firm is

$$\rho_L = \rho + \frac{(\rho - r)D}{E_L}$$

- The expression $(\rho\text{-}r)D/E_L$ can be thought of as a financial risk premium

Financial risk premium

- Why is the equity of a leveraged firm "risky"?
- Consider an investment of 100 which yields either 100 or 300 with equal probabilities
- Under all equity financing, the net return is
  - prob. 50%: (100-100)/100 = 0%
  - prob. 50%: (300-100)/100 = 200%

Financial risk premium

- Suppose now the firm issued debt with face value 80
- If r = 0, investors pay 80 for the debt so the firm finance the remaining 20 with equity
- The net return is
  - prob. 50%: (100-80-20)/20 = 0%
  - prob. 50%: (300-80-20)/20 = 1000%
- The net return is much more volatile
- If investors do not like volatility they will require a higher return as compensation

Proof of M&M2

- From the market value of equity:

$$E_L = \frac{\hat{X} - rD}{\rho_L} \quad \Rightarrow \quad \rho_L = \frac{\hat{X} - rD}{E_L}$$

- By M&M1,

$$V_L = V_U = \frac{\hat{X}}{\rho} \quad \Rightarrow \quad \hat{X} = \rho\underbrace{\left(E_L + D\right)}_{V_L}$$

- Hence

$$\rho_L = \frac{\rho\left(E_L + D\right) - rD}{E_L} = \rho + \frac{\left(\rho - r\right)D}{E_L}$$

The implications of the M&M propositions

WACC

- In practice, investors often discount the cash flows from projects using a weighted average cost of capital:

$$WACC = r \times \frac{D}{V_L} + \rho_L \times \frac{E_L}{V_L}$$

- By M&M2,

$$WACC = r \times \frac{D}{V_L} + \underbrace{\left(\rho + \frac{(\rho - r)D}{E_L} \right)}_{\rho_L} \times \frac{E_L}{V_L} = \frac{\rho(E_L + D)}{V_L} = \rho$$

- WACC is independent of capital structure (otherwise it would have affected investment decisions)

The stock price reaction to recapitalization

- An all-equity firm with N outstanding shares priced at P_0 issues debt with market value D and uses the proceeds to repurchase M shares for a price P_r. What's the post-recapitalization price, P_1?
- $P_1 = P_r$, otherwise all shareholders tender or no shareholder does
- D must cover the cost of recapitalization:

$$D = M \times P_r = M \times P_1$$

- The value of the shares that are not tendered:

$$E_L = (N - M) \times P_1$$

- The post-recapitalization value of the firm:

$$V_L = D + E_L = M \times P_1 + (N - M) \times P_1 = N \times P_1$$

The stock price reaction to recapitalization

- By definition, the initial value of the firm is

$$V_U = N \times P_0$$

- By M&M1,

$$V_U = V_L = N \times P_1$$

- The last two equations imply that $P_1 = P_0$

- In an M&M world, a firm cannot boost its stock prices by repurchasing equity

- Why? Equityholders can create leverage by borrowing money personally, so they will not agree to pay more for the firm's equity just because the firm did something that they could have done themselves

The expected EPS response to recapitalization

- Expected EPS for an all-equity firm:

$$EPS_U = \frac{\hat{X}}{N}$$

- Post-recapitalization EPS:

$$EPS_L = \frac{\hat{X} - rD}{N - M}$$

- Since D must cover the cost of recapitalization and since $P_1 = P_0$:

$$EPS_L = \frac{\hat{X} - rMP_0}{N - M}$$

- Moreover:

$$V_U = \frac{\hat{X}}{\rho} = NP_0 \quad \Rightarrow \quad P_0 = \frac{\hat{X}}{\rho N}$$

The expected EPS response to recapitalization

- Altogether then:

$$EPS_L = \frac{\hat{X} - rMP_0}{N - M} = \frac{\hat{X} - rM\dfrac{\hat{X}}{\rho N}}{N - M} = \frac{\hat{X}}{N}\left[\frac{\rho N - rM}{\rho N - \rho M}\right]$$

- Using the EPS for an all-equity firm:

$$EPS_L = EPS_U \underbrace{\left[\frac{\rho N - rM}{\rho N - \rho M}\right]}_{>1}$$

- Hence, $EPS_L > EPS_U$

The expected EPS response to recapitalization

- To see why $EPS_L > EPS_U$ note that by definition

$$p_0 = \frac{EPS_U}{\rho} \qquad p_1 = \frac{EPS_L}{\rho_L}$$

- Since $P_1 = P_0$:

$$\frac{EPS_U}{\rho} = \frac{EPS_L}{\rho_L}$$

- By M&M2, $\rho_L > \rho$, so $EPS_L > EPS_U$

- Equityholders must be compensated for the added risk they take, by having higher EPS

M&M and EPS-price ratios

- Practitioners often compare earnings-price ratios across firms in the same industry and use the rule: "buy firms with higher EPS-price ratios"
- The logic: stocks with higher EPS-price ratios yield a higher returns and must be better
- Since firms are in the same industry, their risks are presumably the same
- What is ignored is financial risk: leverage raises EPS but not prices so it raises EPS-price ratios even of values are the same

M&M 1963: corporate taxes

M&M with corporate taxes

- Instead of Assumption A4, assume now that there's a corporate tax rate t_c

- The market value of an all-equity firm:

$$V_U = \frac{\hat{X}(1-t_c)}{1+\rho} + \frac{\hat{X}(1-t_c)}{(1+\rho)^2} + \frac{\hat{X}(1-t_c)}{(1+\rho)^3} + \ldots = \frac{\hat{X}(1-t_c)}{\rho}$$

M&M Proposition 1 with corporate taxes

- Proposition: $V_L = V_U + t_c D$

- To prove the proposition, consider an-all-equity firm and a leveraged firm which are identical, save for their capital structures

- The market values are V_U and $V_L = E_L + D$

Proof of M&M1 – part 1

- Consider two investments:
  - Buy a fraction α of firm U
  - Buy a fraction α of firm L's equity and a fraction $\alpha(1\text{-}t_c)$ of firm L's debt
- The annual payoff from investment 1:

$$\alpha X(1-t_C)$$

- The annual payoff from investment 2:

$$\alpha(X-rD)(1-t_C)+\alpha rD(1-t_C)=\alpha X(1-t_C)$$

Proof of M&M1 – part 1

- Since the two investments yield the same payoff, they must have the same value:

$$\alpha V_U = \alpha E_L + \alpha D(1 - t_C) \quad \Rightarrow \quad V_U = \underbrace{E_L + D}_{V_L} - t_C D$$

- Intuition: with leverage, the firm saves on corporate taxes, so its value increases by the present value of the tax shield on future interest payments

M&M Proposition 2

- Proposition:

$$\rho_L = \rho + \frac{(\rho - r)D(1 - t_C)}{E_L}$$

- By definition:

$$\rho = \frac{\hat{X}(1 - t_C)}{V_U} \qquad \rho_L = \frac{(\hat{X} - rD)(1 - t_C)}{E_L}$$

- Combining the two:

$$\rho_L = \frac{\left(\frac{\rho V_U}{1 - t_C} - rD\right)(1 - t_C)}{E_L} = \frac{\rho V_U - rD(1 - t_C)}{E_L}$$

M&M Proposition 2

- By M&M1: $V_L = V_U + tC_D$

$$\rho_L = \frac{\rho V_U - rD(1-t_C)}{E_L} = \frac{\rho\left(\overbrace{E_L + D}^{V_L} - t_C D\right) - rD(1-t_C)}{E_L}$$

$$= \rho + \frac{\rho D(1-t_C) - rD(1-t_C)}{E_L}$$

$$= \rho + \frac{(\rho - r)D(1-t_C)}{E_L}$$

Interpretation of M&M Proposition 2

- We can rewrite ρ_L as follows:

$$\rho_L = \rho + \frac{(\rho - r)D(1 - t_C)}{E_L} = \frac{\rho(E_L - t_C D) + (\rho - r)D + t_C rD}{E_L}$$

- $\rho(E_L - t_C D)$ – a return ρ on invested equity
- $(\rho - r)D$ – financial risk premium
- $t_C rD$ – per-period tax saving