Corporate Finance: Payout policies

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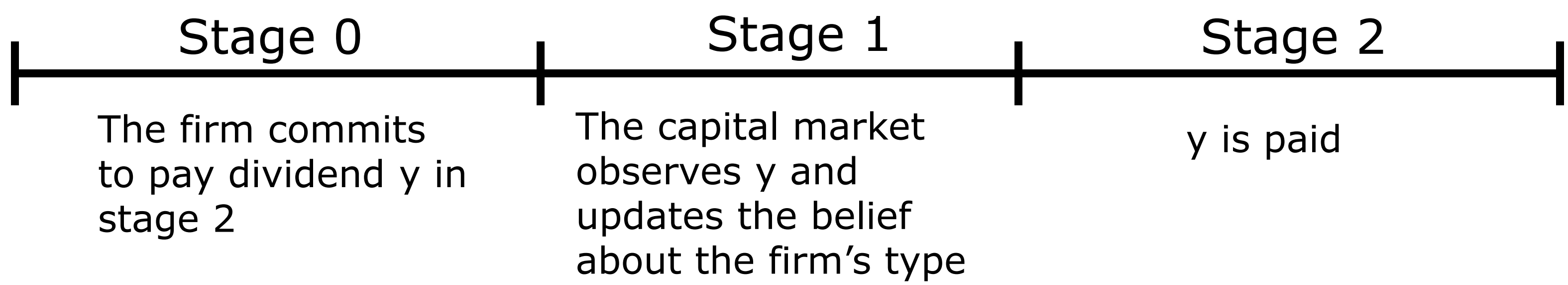
A two-period model

- The timing:



- Cash flow is distributed on $[X_0, X_1]$ according to $f(X)$
- The firm can always pay y
- There's a dividend tax t

The effect of dividend payments

- The value of the firm:

$$V(y) = \int_{X_0}^{X_1} [(X - y) + y(1 - t)]dF(X) = \hat{X} - ty$$

- If t = 0, then V(y) is independent of y

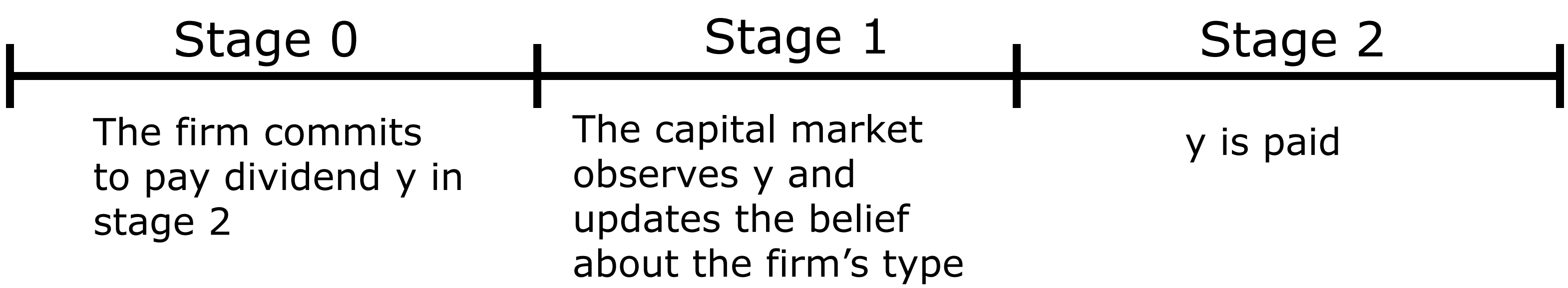
- If t > 0, the firm should pay no dividends

Bhattacharya, BJE 1979

“Imperfect Information, Dividend Policy, and "The Bird in the Hand" Fallacy”

The model

- The timing:



- Cash flow is $x \sim U[0,T]$, where $T \in \{L,H\}$, $L < H$
- T is private info. to the firm; the market believes that $T = H$ with prob. γ
- If $y > x$, the firm bears a loss of $b(y-x)$
- Dividend tax t
- The manager's utility: $U = \alpha V + (1-\alpha)V_T$

The full information case

- The value of the firm when T is common knowledge:

$$V = V_T = \left[\int_0^T \left[(x-y) + y(1-t) \right] dF(x) - b \int_0^y (y-x) dF(x) \right]$$

- With uniform dist.

$$V = V_T = \frac{T}{2} - ty - \frac{by^2}{2T}$$

- The manager's utility:

$$U = \alpha V_T + (1-\alpha) V_T = \frac{T}{2} - ty - \frac{by^2}{2T}$$

- The manager will set y = 0

Asymmetric information

- y_T* is the dividend of type T
- B* is the prob. that the capital market assigns to the firm being of type T
- Perfect Bayesian Equilibrium (PBE), (y_H*,y_L*,B*):
  - y_H* and y_L* are optimal given B*
  - B* is consistent with the Bayes rule

Separating equilibria: $y_H^* \neq y_L^*$

- The belief function:

$$B^* = \begin{cases} 1 & y = y_H^* \\ 0 & y = y_L^* \\ ? & o/w \end{cases}$$

- In a separating equil., $y_L^* = 0$ because type L cannot boost V by paying divided

- $y_L^* = 0 \Rightarrow B^*(0) = 0 \Rightarrow V(0) = L/2$

Separating equilibrium

- The IC constraint of H:

$$\underbrace{\alpha V+(1-\alpha)\left(\frac{H}{2}-ty-\frac{by^2}{2H}\right)}_{\substack{\text{Payoff of H}\\ \text{when paying dividend y}}}\geq \underbrace{\alpha\frac{L}{2}+(1-\alpha)\frac{H}{2}}_{\substack{\text{Payoff of H}\\ \text{when y}=0}}$$

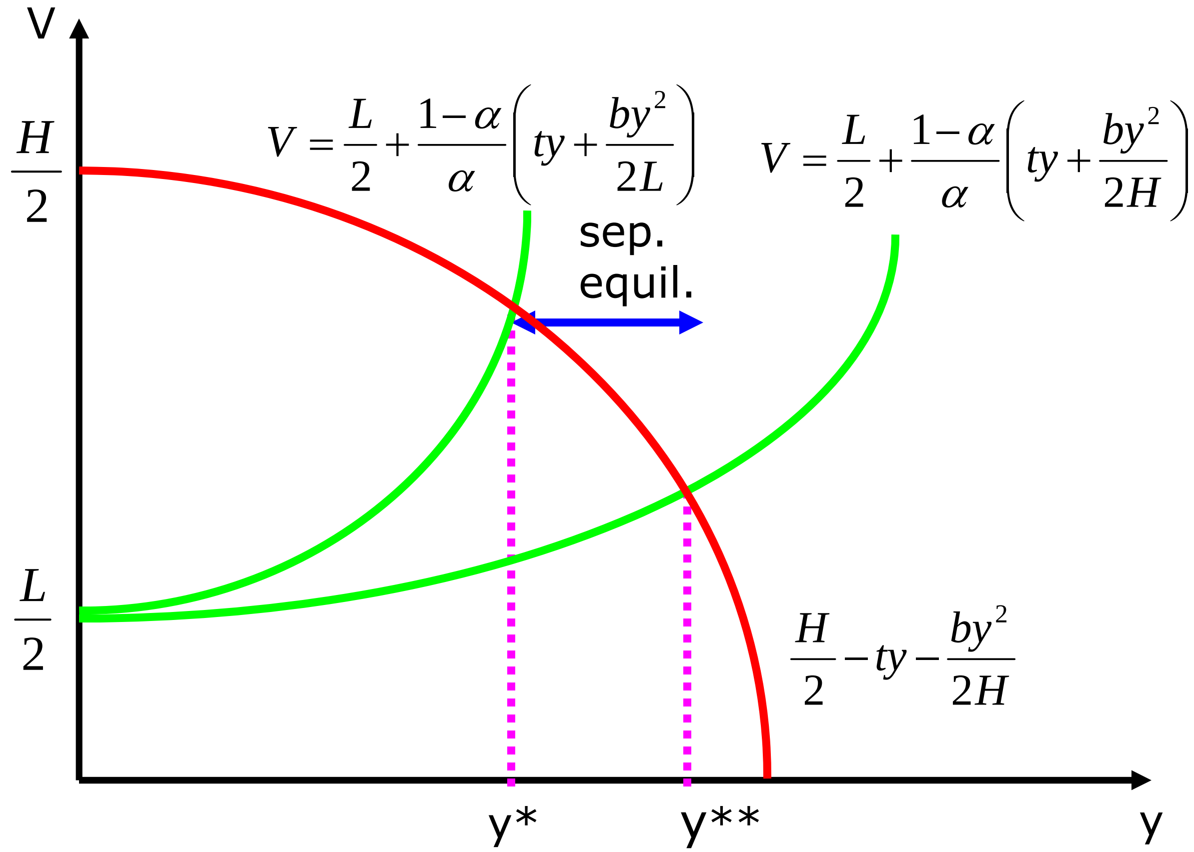
- The IC constraint of L:

$$\underbrace{\alpha V+(1-\alpha)\left(\frac{L}{2}-ty-\frac{by^2}{2L}\right)}_{\substack{\text{Payoff of L}\\ \text{when paying dividend y}}}\leq \underbrace{\alpha\frac{L}{2}+(1-\alpha)\frac{L}{2}}_{\substack{\text{Payoff of L}\\ \text{when y}=0}}$$

- Indifference of type T between V and y:

$$\alpha V+(1-\alpha)\left(\frac{T}{2}-ty-\frac{by^2}{2T}\right)=\alpha\frac{L}{2}+(1-\alpha)\frac{T}{2}\quad\Rightarrow\quad V=\frac{L}{2}+\frac{1-\alpha}{\alpha}\left(ty+\frac{by^2}{2T}\right)$$

The set of separating equilibria



The dividend of type H under the DOM criterion

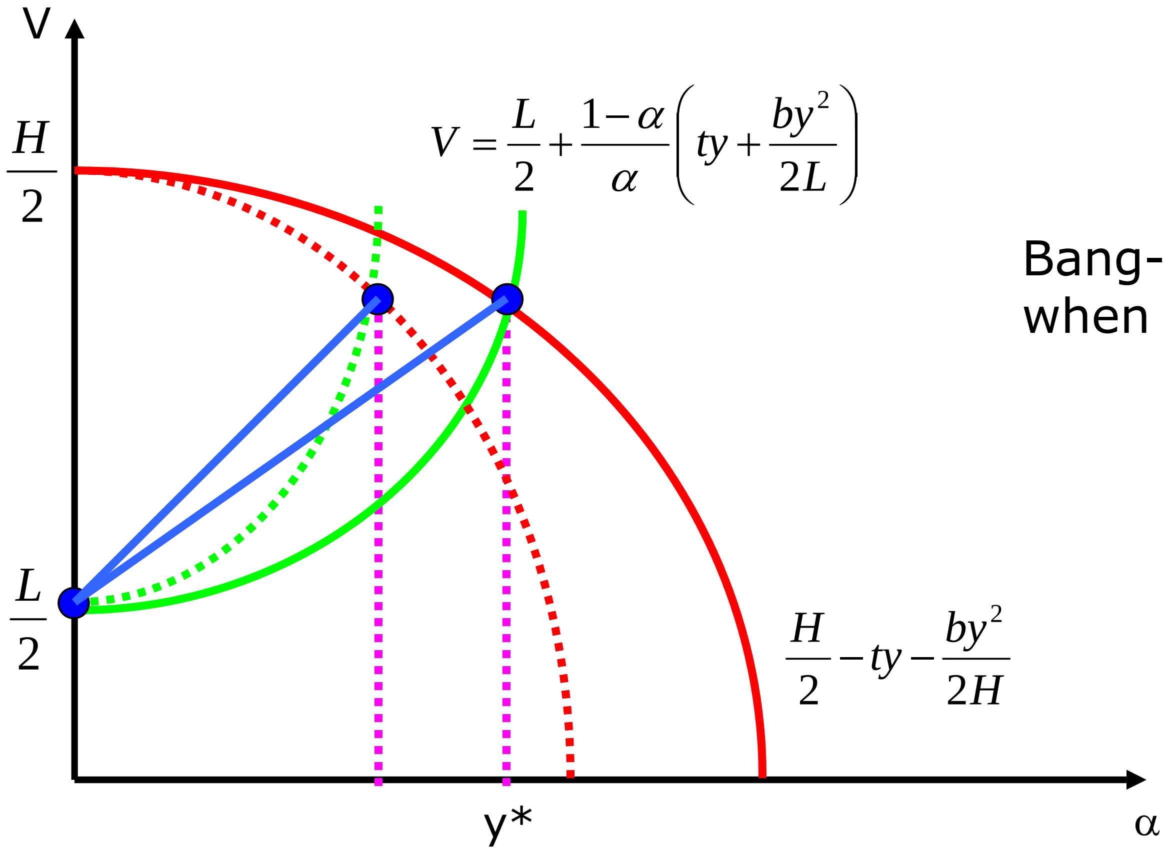
- The DOM criterion eliminates all separating equilibria except the Riley outcome:

$$\underbrace{\frac{H}{2} - ty - \frac{by^2}{2H}}_{\text{Value when the market believs that the firm's type is H}} = \underbrace{\frac{L}{2} + \frac{1-\alpha}{\alpha}\left(ty + \frac{by^2}{2L}\right)}_{\text{Indifference curve for firm L}}$$

- Solving:

$$y^* = \frac{-tHL + \sqrt{t^2H^2L^2 + 2\alpha b(H-L)HL(\alpha L + (1-\alpha)H)}}{b(\alpha L + (1-\alpha)H)}$$

Bang-for-the-buck (the effect of y on the firm's value)



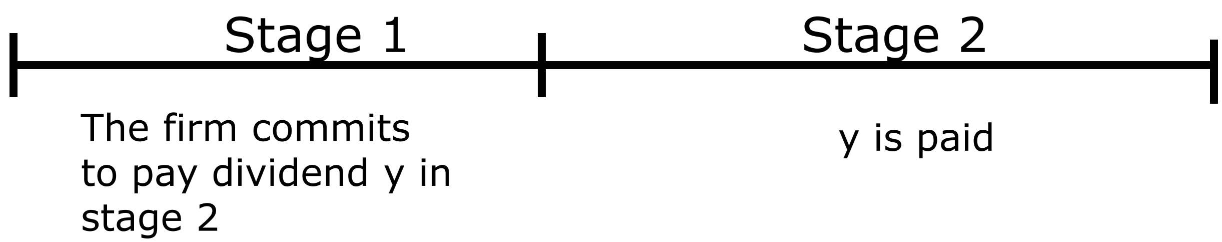
Bang-for-the-buck↑ when t↑

Bernheim and Wantz, AER 1995

"A tax-based test of the dividend signaling hypothesis"

An agency model of dividends - private info. about preferences

- The timing:



- Cash flow before dividends is Y
- The value of the firm is v(Y-y)
- Cum dividend value of the firm: v(Y-y)+(1-t)y,   (t is dividend tax)
- The manager gets private benefits w(Y-y)
- The manager's utility: U = v(Y-y) + (1-t)y + θw(Y-y), $\theta \sim [\theta_0, \theta_1]$

The manager's problem

- The manager chooses y to maximize:

$$U = \underbrace{v(Y-y)+(1-t)y}_{\text{Cum dividend value}} + \underbrace{\theta w(Y-y)}_{\text{Private benefits}}$$

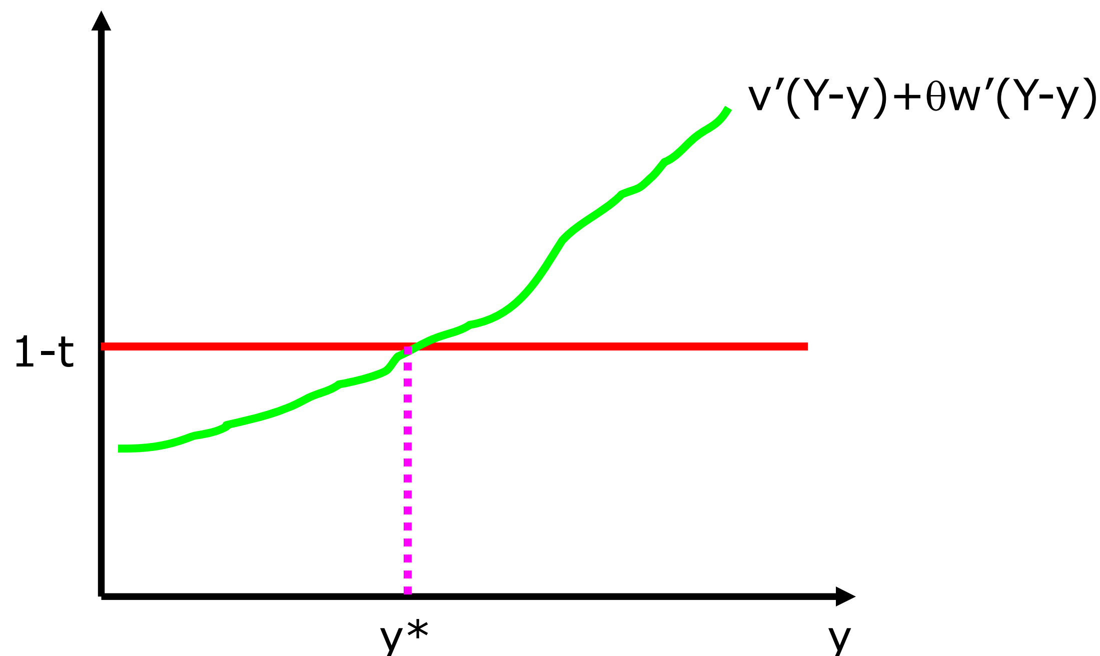
- F.O.C:

$$U' = \underbrace{1-t}_{\text{Benefit of dividends}} - \left(\underbrace{v'(Y-y)+\theta w'(Y-y)}_{\text{Cost of dividends}} \right) = 0$$

- S.O.C:

$$U'' = \underbrace{v''(Y-y)}_{<0 \text{ by assumption}} + \underbrace{\theta w''(Y-y)}_{<0 \text{ by assumption}} < 0$$

Illustrating the manager's problem



- $t\uparrow \Rightarrow y^*\downarrow$ (the benefits of y is smaller)
- $\theta\uparrow \Rightarrow y^*\downarrow$ (the cost of y is bigger)

Bang-for-the-buck

- The cum dividend value of the firm in equil.:

$$V^* = v(Y - y^*) + (1 - t)y^*$$

- Suppose that θ is private info. for the manager. The ex. value of the firm before y is paid:

$$E_\theta V^* = E_\theta v(Y - y^*) + (1 - t)E_\theta y^*$$

- Effect of y on firm value:

$$A \equiv V^* - E_\theta V^*$$

- Bang-for-the-buck:

$$\frac{\partial A}{\partial y} = \frac{\partial V^*}{\partial y} = -v'(Y - y^*) + (1 - t) \underbrace{=}_{\text{by F.O.C}} \theta w'(Y - y^*)$$

Effect of t on bang-for-the-buck

- The effect of t on ∂A/∂y:

$$\frac{\partial}{\partial t}\frac{\partial A}{\partial y} = -\theta \underbrace{w''(Y - y^*)}_{(-)} \underbrace{\frac{\partial y^*}{\partial t}}_{(-)} < 0$$

- t↑ ⇒ ∂A/∂y↓

- The effect of the opposite of what we had with signaling ⇒ we have a test to discriminate between the two explanations for dividend payments

An agency model of dividends – private info. about cash flows

- Consider the same model as before except that now Y = θ and the manager chooses y to maximize:

$$U = \underbrace{v(\theta - y) + (1-t)y}_{\text{Cum dividend value}} + \underbrace{w(\theta - y)}_{\text{Private benefits}}$$

- F.O.C:

$$U' = \underbrace{1-t}_{\text{Benefit of dividends}} - \left(\underbrace{v'(\theta - y) + w'(\theta - y)}_{\text{Cost of dividends}} \right) = 0$$

- F.O.C determines θ – y. Hence, θ – y = const. and

$$\frac{\partial y^*}{\partial \theta} = 1$$

Effect of dividend on the firm's value

- The cum dividend value of the firm in equil.:

$$V^* = v(\theta - y^*) + (1 - t)y^*$$

- Suppose that θ is private info. for the manager. The ex. value of the firm <u>before</u> y is paid:

$$E_\theta V^* = E_\theta v(\theta - y^*) + (1 - t)E_\theta y^*$$

- Effect of y on firm value:

$$A \equiv V^* - E_\theta V^*$$

Bang-for-the-buck

- Suppose $\theta\uparrow$; then A and y change. We can define bang-for-the-buck as:

$$B \equiv \frac{\text{Change in value}}{\text{Change in y}} = \frac{\frac{\partial A}{\partial \theta}}{\frac{\partial y^*}{\partial \theta}}$$

- The effect of θ on A:

$$\frac{\partial A}{\partial \theta} = \frac{\partial V^*}{\partial \theta} + \frac{\partial V^*}{\partial y}\frac{\partial y^*}{\partial \theta} \quad \Rightarrow \quad \underbrace{\frac{\frac{\partial A}{\partial \theta}}{\frac{\partial y^*}{\partial \theta}}}_{B} = \frac{\frac{\partial V^*}{\partial \theta}}{\frac{\partial y^*}{\partial \theta}} + \frac{\partial V^*}{\partial y}$$

Effect of t on bang-for-the-buck

- Computing bang-for-the-buck:

$$B = \frac{\partial V^*}{\partial y} + \frac{\frac{\partial V^*}{\partial \theta}}{\frac{\partial y^*}{\partial \theta}} = \underbrace{-v'(\theta - y^*) + (1-t)}_{\frac{\partial V^*}{\partial y}} + \underbrace{\frac{v'(\theta - y^*)}{1}}_{\frac{\partial V^*}{\partial \theta} \Big/ \frac{\partial y^*}{\partial \theta}} = 1 - t$$

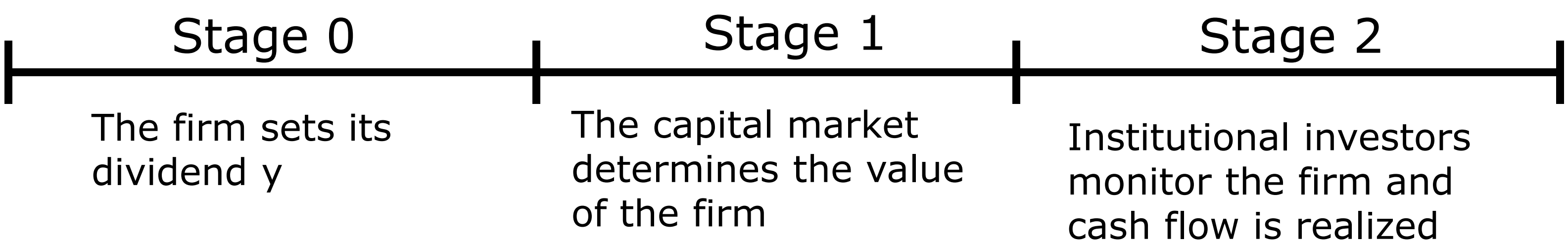
- B is decreasing with t

Allen, Bernardo, and Welch, JF 2000

"A Theory of Dividends Based on Tax Clienteles"

The model

- The timing:



- Cash flow is $V \sim N(\mu,\sigma^2)$
- There are institutional investors and retail investors
- Monitoring by institutional investors adds ln(M) to the cash flow
- The cost of monitoring is cM
- Dividend taxes: $t_R > t_I$

Payoffs

- Wealth of institutional and retail investors:

$$W_I = \underbrace{\theta_I}_{\text{Stake}}\left(V + \ln(M) - \underbrace{t_I y}_{\text{Div. tax}}\right) + \underbrace{w_I}_{\substack{\text{Initial}\\\text{wealth}}} - \underbrace{p\theta_I}_{\substack{\text{Payment}\\\text{for shares}}} - \underbrace{cM}_{\substack{\text{Cost of}\\\text{monitoring}}}$$

$$W_R = \underbrace{\theta_R}_{\text{Stake}}\left(V + \ln(M) - \underbrace{t_R y}_{\text{Div. tax}}\right) + \underbrace{w_R}_{\substack{\text{Initial}\\\text{wealth}}} - \underbrace{p\theta_R}_{\substack{\text{Payment}\\\text{for shares}}}$$

- The expected payoff of type t = I,R investors:

$$EU(W_t) = EW_t - \frac{\gamma_t}{2} \times Var(W_t)$$

- EW_t is equal to W_t except that μ replaces V

- $Var(W_t)$: $$Var(W_t) = E(W_t - EW_t)^2 = \theta_t{}^2\sigma^2$$

Institutional investors: monitoring and demand for equity

- Institutional investors choose M and θ_I to maximize:

$$EU(W_I) = \underbrace{\theta_I(\mu + \ln(M) - t_I y) + w_I - p\theta_I - cM}_{EW_I} - \frac{\gamma_I}{2} \times \underbrace{\theta_I{}^2\sigma^2}_{Var(W_I)}$$

- F.O.C for M and θ_I:

$$\frac{\theta_I}{M} - c = 0 \quad \Rightarrow \quad M^* = \frac{\theta_I}{c}$$

$$\mu + \ln(M) - t_I y - p - \gamma_I \theta_I \sigma^2 = 0$$

- Using F.O.C: $\theta_I \gamma_I \sigma^2 - \ln\left(\frac{\theta_I}{c}\right) = \mu - t_I y - p$

- Ambiguous price effect: $\frac{\partial p}{\partial \theta_I} = \frac{1}{\theta_I} - \gamma_I \sigma^2$

Retail investors: demand for equity

- Retail investors choose θ_R to maximize:

$$EU(W_R) = \underbrace{\theta_R\left(\mu + \ln\left(\frac{\theta_I}{c}\right) - t_R y\right) + w_R - p\theta_R}_{EW_R} - \frac{\gamma_R}{2} \times \underbrace{\theta_R{}^2 \sigma^2}_{Var(W_R)}$$

- F.O.C for θ_R:

$$\mu + \ln\left(\frac{\theta_I}{c}\right) - t_R y - p = \gamma_R \theta_R \sigma^2$$

- Using F.O.C:

$$\theta_R \gamma_R \sigma^2 - \ln\left(\frac{\theta_I}{c}\right) = \mu - t_R y - p$$

- Downward sloping demand

Equilibrium

- Demands:

$$\theta_I \gamma_I \sigma^2 - \ln\left(\frac{\theta_I}{c}\right) = \mu - t_I y - p$$

$$\theta_R \gamma_R \sigma^2 - \ln\left(\frac{\theta_I}{c}\right) = \mu - t_R y - p$$

- Market clearing: $\underbrace{\theta_I + \theta_R}_{\text{demand}} = \underbrace{1}_{\text{supply}}$

- The equil. is given by the sol'n to the system of 3 equations (demands + market clearing) in 3 unknowns: θ_I, θ_R, p, given y

- The owners choose y to maximize p

Characterizing the equilibrium

- Rewriting the demand equations:

$$\theta_I \gamma_I \sigma^2 + t_I y = \ln\left(\frac{\theta_I}{c}\right) + \mu - p$$

$$\theta_R \gamma_R \sigma^2 + t_R y = \ln\left(\frac{\theta_I}{c}\right) + \mu - p$$

- From the 2 demand equations:

$$\theta_I \gamma_I \sigma^2 + t_I y = \underbrace{(1-\theta_I)}_{\theta_R} \gamma_R \sigma^2 + t_R y \quad \Rightarrow \quad \theta_I * = \frac{\gamma_R \sigma^2 + (t_R - t_I) y}{(\gamma_R + \gamma_I)\sigma^2}$$

- In equilibrium, $\theta_I = \theta^*_I$ and $\theta^*_R = 1-\theta^*_I$ (using market clearing)
- We found θ^*_I and θ^*_R as functions of y. We now must determine y
- The owners of the firm will set y* to maximize p

The effect of y on p:

- p is determined by the equil. system:

$$\ln\left(\frac{\theta_I}{c}\right) - \theta_I \gamma_I \sigma^2 + \mu - p = t_I y$$

$$\ln\left(\frac{\theta_I}{c}\right) - \theta_R \gamma_R \sigma^2 + \mu - p = t_R y$$

$$\theta_I + \theta_R = 1$$

- To find the effect of y on p we must differentiate the equil. system w.r.t. y

$$\left(\frac{1}{\theta_I} - \gamma_I \sigma^2\right) \times \partial\theta_I + 0 \times \partial\theta_R - 1 \times \partial p = t_I \times \partial y$$

$$\frac{1}{\theta_I} \times \partial\theta_I - \gamma_R \sigma^2 \times \partial\theta_R - 1 \times \partial p = t_R \times \partial y$$

$$1 \times \partial\theta_I + 1 \times \partial\theta_R + 0 \times \partial p = 0 \times \partial y$$

The comparative static matrix:

- Writing the system in matrix form:

$$\underbrace{\begin{pmatrix} \frac{1}{\theta_I} - \gamma_I \sigma^2 & 0 & -1 \\ \frac{1}{\theta_I} & -\gamma_R \sigma^2 & -1 \\ 1 & 1 & 0 \end{pmatrix}}_{\text{Comparative statics matrix}} \times \begin{pmatrix} \partial \theta_I \\ \partial \theta_R \\ \partial p \end{pmatrix} = \begin{pmatrix} t_I \\ t_R \\ 0 \end{pmatrix} \times \partial y$$

- Using Cramer's rule:

$$\frac{\partial p}{\partial y} = \frac{\frac{t_R - t_I}{\theta_I} - \left(\gamma_R t_I + \gamma_I t_R\right)\sigma^2}{\left(\gamma_R + \gamma_I\right)\sigma^2}$$

The optimal y

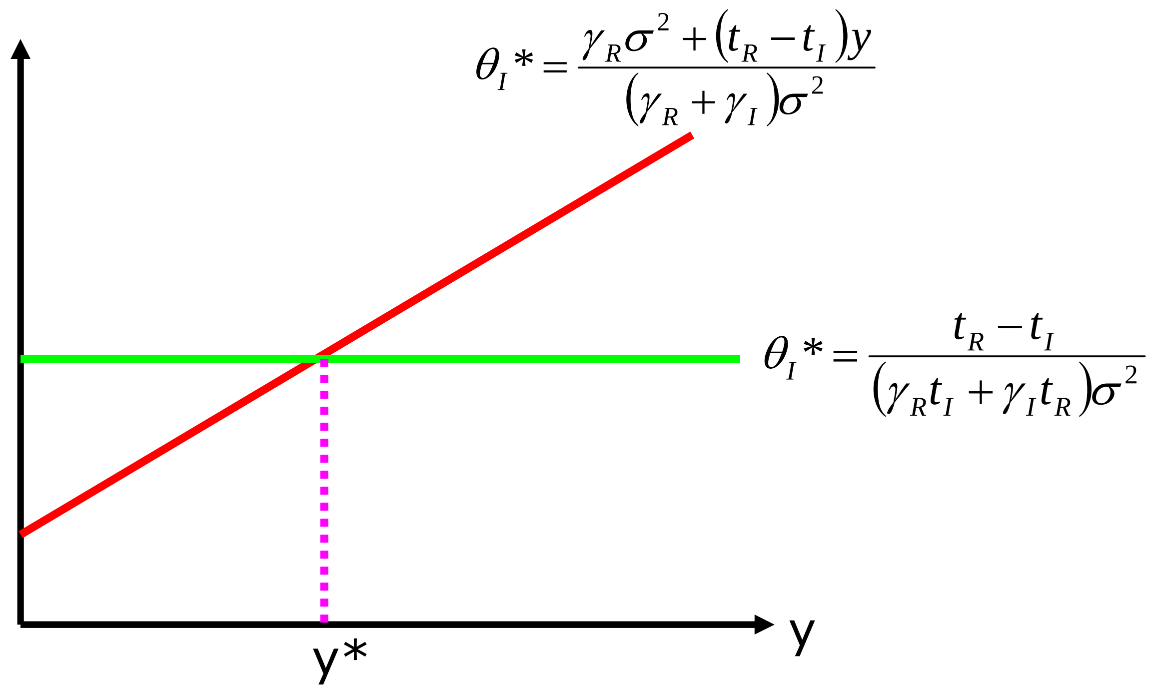
- The owners of the firm set y to maximize p:

$$\frac{\partial p}{\partial y} = \frac{\frac{t_R - t_I}{\theta_I} - (\gamma_R t_I + \gamma_I t_R)\sigma^2}{(\gamma_R + \gamma_I)\sigma^2} = 0 \quad \Rightarrow \quad \theta_I * = \frac{t_R - t_I}{(\gamma_R t_I + \gamma_I t_R)\sigma^2}$$

- y* is chosen so that $\theta_I = \theta^*_I$

- We found earlier that in equil. : $\theta_I * = \frac{\gamma_R \sigma^2 + (t_R - t_I) y}{(\gamma_R + \gamma_I)\sigma^2}$

Solving for y*



- The optimal dividend: $y^* = \frac{\gamma_R + \gamma_I}{(\gamma_R t_R + \gamma_I t_I)} - \frac{\gamma_R \sigma^2}{t_R - t_I}$

$\Rightarrow$ y*↑ when σ^2↓ and $t_R - t_I$↑

Bang-for-the-buck

- The value of the firm is:

$$V^* = V + \ln\left(\frac{\theta_I^*}{c}\right) \quad \Rightarrow \quad \frac{\partial V^*}{\partial y} = \frac{c}{\theta_I^*} \times \frac{\partial \theta_I^*}{\partial y}$$

- From before we know that:

$$\theta_I^* = \frac{\gamma_R \sigma^2 + (t_R - t_I) y}{(\gamma_R + \gamma_I)\sigma^2} \quad \Rightarrow \quad \frac{\partial \theta_I^*}{\partial y} = \frac{t_R - t_I}{(\gamma_R + \gamma_I)\sigma^2} > 0$$

$$\Rightarrow \quad \frac{\partial V^*}{\partial y} = \frac{c}{\theta_I^*} \times \frac{t_R - t_I}{(\gamma_R + \gamma_I)\sigma^2} = \frac{c}{\frac{\gamma_R \sigma^2 + (t_R - t_I) y^*}{(\gamma_R + \gamma_I)\sigma^2}} \times \frac{t_R - t_I}{(\gamma_R + \gamma_I)\sigma^2}$$

$$= \frac{c(t_R - t_I)}{\gamma_R \sigma^2 + (t_R - t_I) y^*} = \frac{c}{\frac{\gamma_R \sigma^2}{t_R - t_I} + y^*}$$