Corporate Finance: Asymmetric information and capital structure – signaling

Yossi Spiegel
Recanati School of Business

Ross, BJE 1977

"The Determination of Financial Structure: The Incentive-Signalling Approach"

The model

- The timing:

Stage 0	Stage 1	Stage 2
The firm issues debt with face value D	The capital market observes D and updates the belief about the firm's type	Cash flow is realized

- Cash flow is $x \sim U[0,T]$, where $T \in \{L,H\}$, $L < H$
- T is private info. to the firm
- The capital market believes that $T = H$ with prob. γ
- In case of bankruptcy, the manager bears a personal loss C
- The manager's utility: $U = V - C \times$ Prob. of bankruptcy

The full information case

- The value of the firm when T is common knowledge:

$$V_T = \left[\int_0^D x dF(x) + \int_D^T D dF(x) \right] + \int_D^T (x - D) dF(x) = \hat{x}_T$$

- With uniform dist.: $\hat{x}_T = T/2$

- The manager's utility:

$$U = \hat{x}_T - C \times F(D) = \hat{x}_T - C\frac{D}{T}$$

- The manager will not issue debt

Asymmetric information

- D_T* is the debt level of type T
- B* is the prob. that the capital market assigns to the firm being of type T
- Perfect Bayesian Equilibrium (PBE), (D_H*,D_L*,B*):
  - D_H* and D_L* are optimal given B*
  - B* is consistent with the Bayes rule

Bayes rule

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

- In our case, two levels of D are chosen. Suppose the probability that each type plays them is as follows:

	D_1	D_2
Type H (γ)	h_1	h_2
Type L $(1-\gamma)$	l_1	l_2

- Having observed D_1 the capital market believes that the firm is type H with prob.:

$$P(H \mid D_1) = \frac{P(D_1 \mid H)P(H)}{P(D_1)} = \frac{h_1\gamma}{h_1\gamma + l_1(1-\gamma)}$$

- In a sep. equil., $h_1 = 1$ and $l_1 = 0$. In a pooling equil., $h_1 = l_1 = 1$

Separating equilibria: $D_H^* \neq D_L^*$

- The belief function:

$$B^* = \begin{cases} 1 & D = D_H^* \\ 0 & D = D_L^* \\ \gamma' & o/w \end{cases}$$

- In a separating equil., $D_L^* = 0$ because type L cannot boost V by issuing debt

- $D_L^* = 0 \Rightarrow B^*(0) = 0 \Rightarrow V(0) = L/2$

Separating equilibrium

- The IC of H:

$$\underbrace{V - C \times \frac{D}{H}}_{\substack{\text{Payoff of H} \\ \text{when it issues D}}} \geq \underbrace{\frac{L}{2}}_{\substack{\text{Payoff of H} \\ \text{when D} = 0}}$$

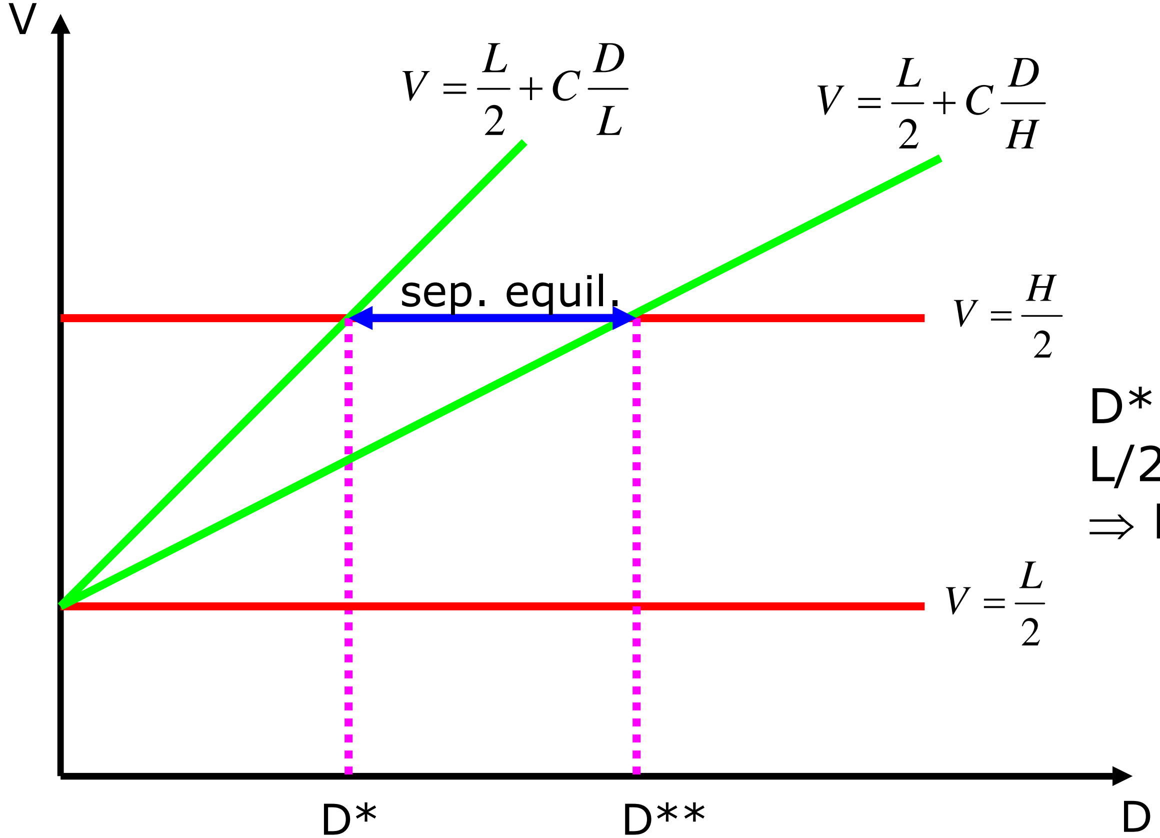
- The IC of L:

$$\underbrace{\frac{L}{2}}_{\substack{\text{Payoff of L} \\ \text{when D} = 0}} \geq \underbrace{V - C \times \frac{D}{L}}_{\substack{\text{Payoff of L} \\ \text{when it issues D}}}$$

- Indifference of type T between V and D:

$$\underbrace{\frac{L}{2}}_{\substack{\text{Payoff} \\ \text{if D=0}}} = \underbrace{V - C \times \frac{D}{T}}_{\substack{\text{Payoff when issuing D} \\ \text{if it induces value V}}} \Rightarrow V = \frac{L}{2} + C \times \frac{D}{T}$$

The set of separating equilibria



D* is defined by
L/2+CxD/L = H/2
$\Rightarrow$ D* = (H-L)L/2C

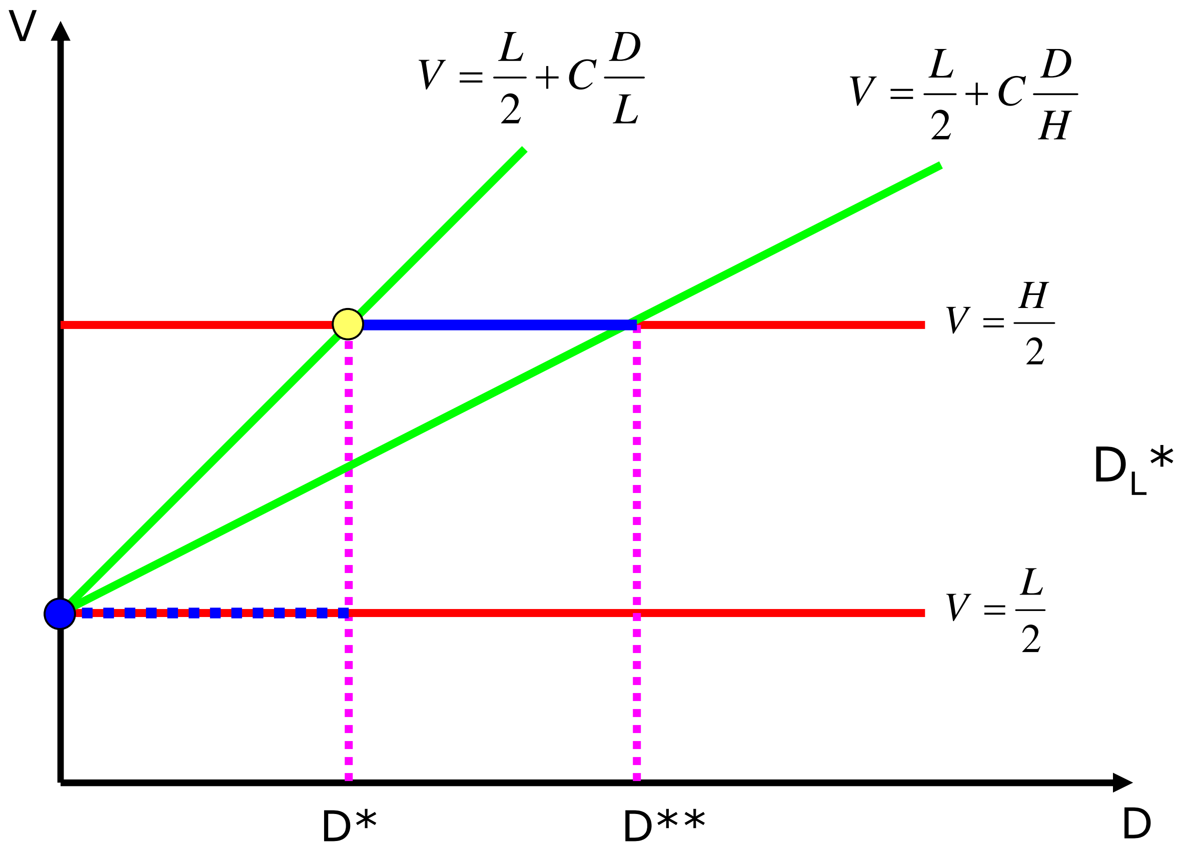
The DOM criterion

- $D \geq D^*$ is a dominated strategy for type L: $D = 0$ always guarantees L a higher payoff no matter what the capital market believes
- The DOM criterion:

$$B^* = \begin{cases} 1 & D = D_H{}^* \\ 0 & D = D_L{}^* \\ 1 & D \geq D^* \\ \gamma' & o/w \end{cases}$$

- The idea: type L will never play $D \geq D^*$ while type H might. Hence, if $D \geq D^*$, then the firm's type must be H
- The DOM criterion still does not determine the beliefs for $D < D^*$ and $D \neq 0$
- Under the DOM criterion: $D_L{}^* = 0$, $D_H{}^* = D^*$

Separating equilibria under the DOM criterion



$D_L^* = 0$, $D_H^* = D^*$

Pooling equilibria: $D_H^* = D_L^* = D_p^*$

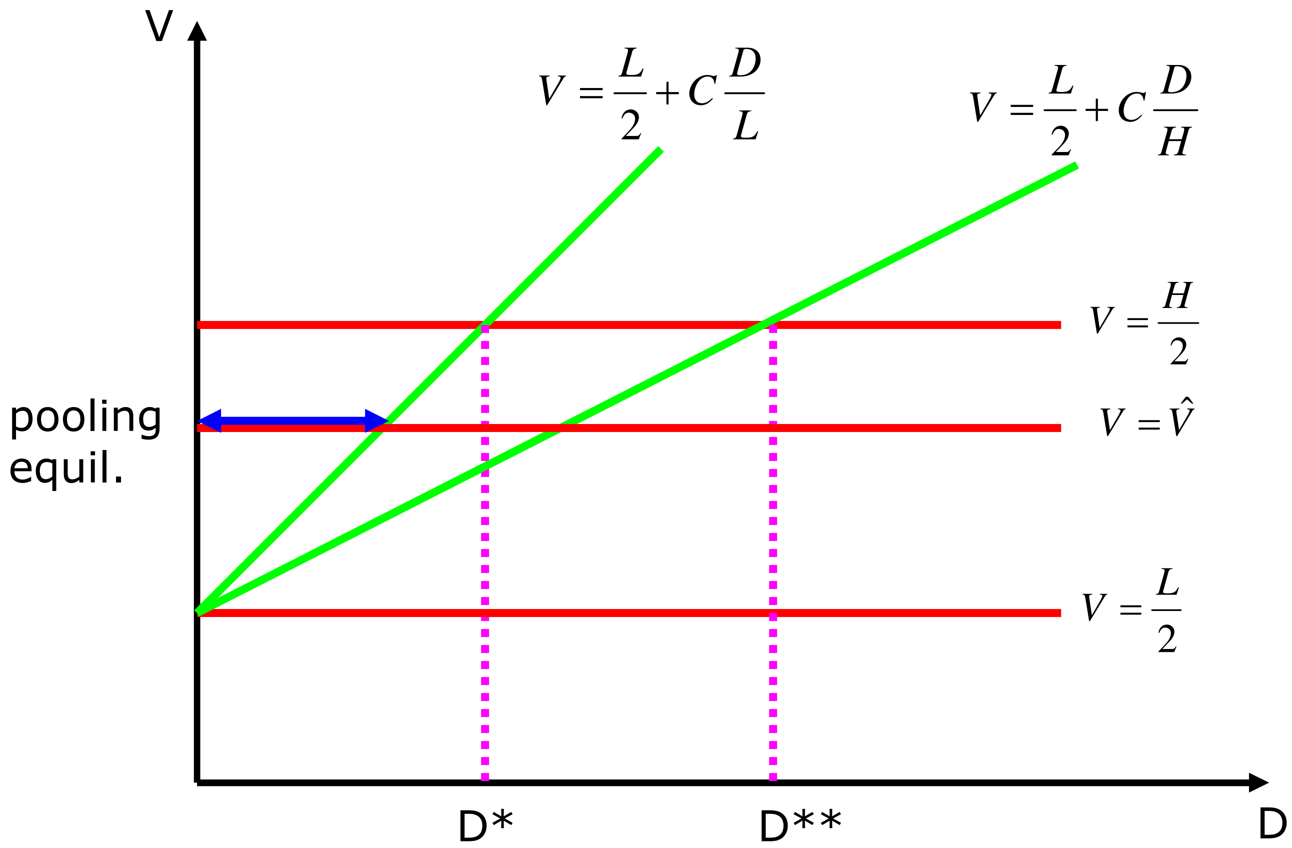
- The belief function:

$$B^* = \begin{cases} \gamma & D = D_p^* \\ \gamma' & o/w \end{cases}$$

- In a pooling equil., the choice of D_p^* is uninformative; hence

$$\hat{V} = \gamma \times \frac{H}{2} + (1-\gamma) \times \frac{L}{2}$$

The set of pooling equilibria



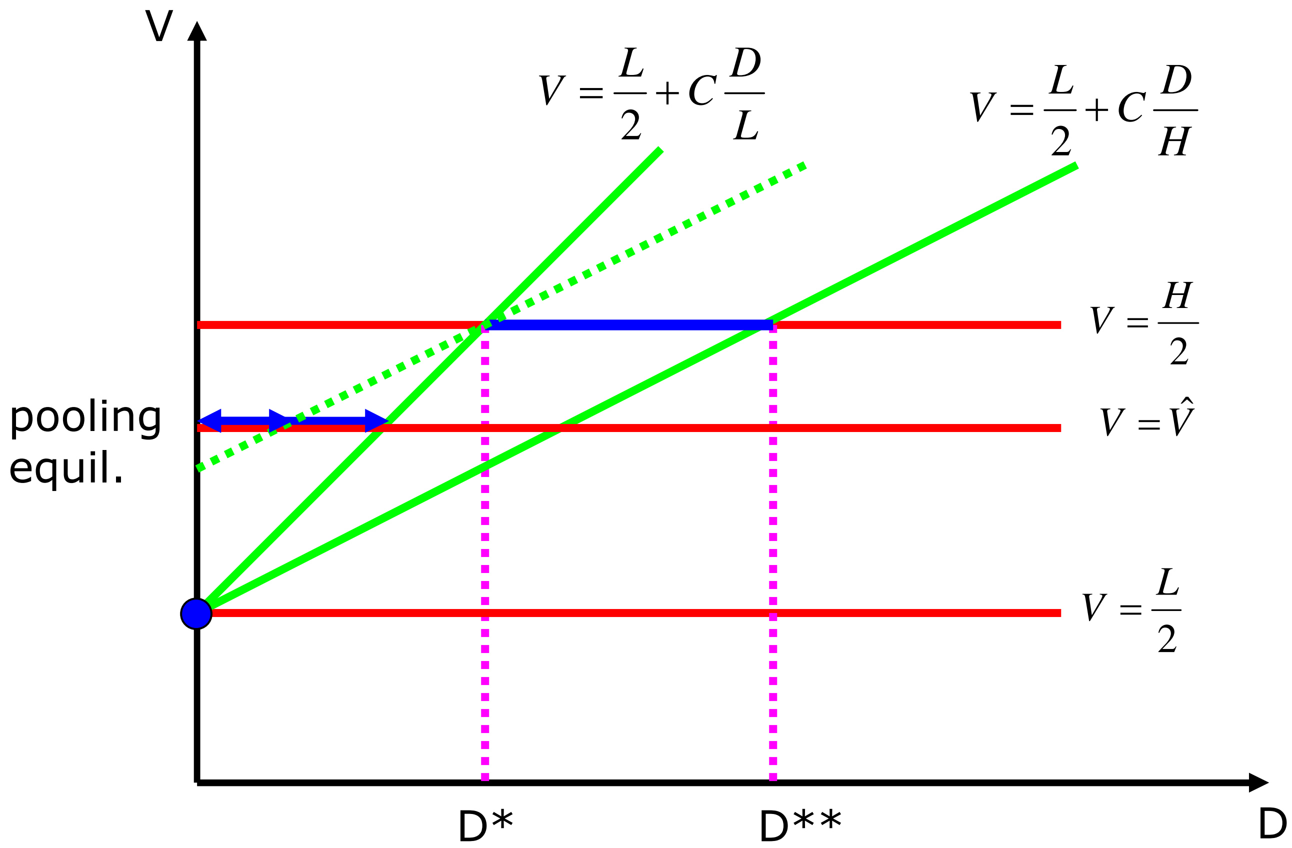
The DOM criterion

- The DOM criterion:

$$B^* = \begin{cases} \gamma & D = D_p^* \\ 1 & D \geq D^* \\ \gamma' & o/w \end{cases}$$

- The DOM criterion eliminates some pooling equilibria but not all

The set of pooling equilibria



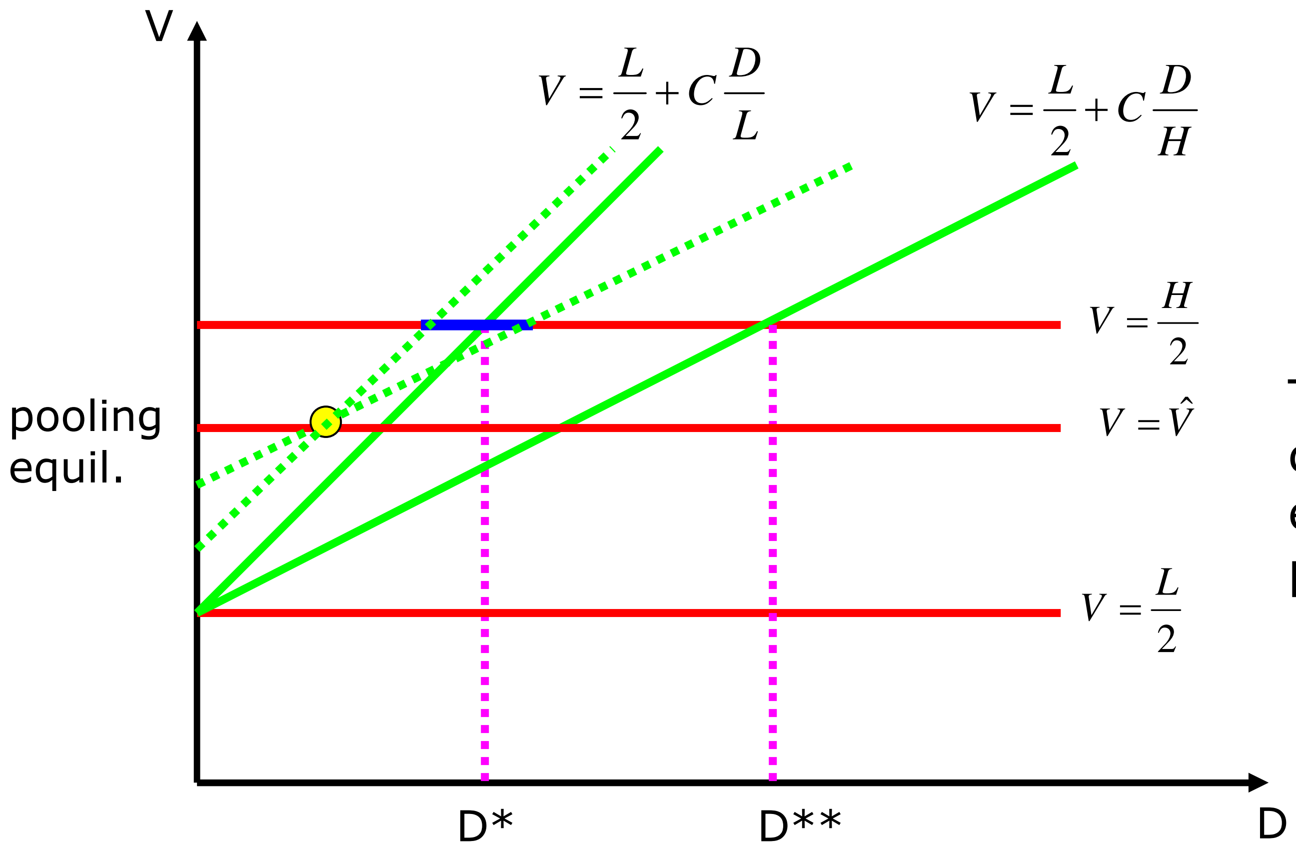
The intuitive criterion

- Due to Cho and Kreps, QJE 1987
- Fix a equilibrium (D_L*, D_H*) and consider a deviation from this equilibrium to D'. If the deviation never benefits type x (even if it induces the most favorable beliefs by the capital market) but can benefit type y, then the deviation was played by type y
- The intuitive criterion:

$$B^* = \begin{cases} \gamma & D = D_p^* \\ 1 & D \text{ is dominated by } D_p^* \\ \gamma' & o/w \end{cases}$$

- The intuitive criterion still does not determine the beliefs everywhere

The set of pooling equilibria under the intuitive criterion



The intuitive criterion eliminates all pooling equil.

Conclusions

- The only equilibrium which survives the intuitive criterion is $D_L^* = 0$ and $D_H^* = D^*$
- This equilibrium is the Pareto undominated separating equilibrium and it is called the "Riley outcome"
- Debt can be used as a signal of high cash flow
- The debt of type H:

$$\frac{L}{2} + C\frac{D}{L} = \frac{H}{2} \quad \Rightarrow \quad D^* = \frac{(H-L)L}{2C}$$

- $D^*\uparrow$ when $L\uparrow$, $H-L\uparrow$, and $C\downarrow$, D^* is independent of γ!

Leland and Pyle, JF 1977

“Informational asymmetries, financial structure, and financial intermediation”

The model

- The timing:

Stage 0	Stage 1	Stage 2
An entrepreneur sell a fraction 1-α of the firm to outside equityholders	The capital market observes 1-α and updates the belief about the firm's type	Cash flow is realized

- Cash flow is $x \sim [0,\infty)$, with $Ex = x_T$ and $Var(x)=\sigma^2$, where $T \in \{L,H\}$, $x_L < x_H$
- T is private info. to the firm
- The capital market believes that T = H with prob. γ
- The entrepreneur's expected utility:

$$EU(W_T) = EW_T - \frac{b}{2} \times Var(W_T) \qquad W_T = \alpha x + (1-\alpha)V$$

The full information case

- The variance of W_T:

$$Var(W_T) = E(\alpha x + (1-\alpha)V - EW_T)^2$$
$$= E(\alpha(x - x_T))^2 = \alpha^2\sigma^2$$

- The entrepreneur's expected utility:

$$EU(W_T) = \underbrace{\alpha x_T + (1-\alpha)V}_{EW_T} - \frac{b}{2} \times \underbrace{\alpha^2\sigma^2}_{Var(x)}$$

- Under full info., $V = x_T$:

$$EU(W_T) = x_T - \frac{b}{2} \times \alpha^2\sigma^2$$

⇒ $\alpha^* = 0$ ⇒ The entrepreneur will sell the entire firm

- Why? Because the entrepreneur is risk-averse and the capital market is risk-neutral

Asymmetric information

- α_T* is the equity participation of type T

- B* is the prob. that the capital market assigns to the firm being of type T

- Perfect Bayesian Equilibrium (PBE), (α_H*, α_L*,B*):
  - α_H* and α_L* are optimal given B*
  - B* is consistent with the Bayes rule

Separating equilibria: $\alpha_H^* \neq \alpha_L^*$

- The belief function:

$$B^* = \begin{cases} 1 & \alpha = \alpha_H^* \\ 0 & \alpha = \alpha_L^* \\ \gamma' & o/w \end{cases}$$

- In a separating equil., $\alpha_L^* = 0$ because type L cannot boost V by keeping equity

- $\alpha_L^* = 0 \Rightarrow B^*(0) = 0 \Rightarrow V(0) = x_L$

Separating equilibrium

- The IC of H:

$$\underbrace{\alpha x_H + (1-\alpha)V - \frac{b}{2}\alpha^2\sigma^2}_{\text{Payoff of H when he keeps a fraction } \alpha \text{ of the firm}} \geq \underbrace{x_L}_{\substack{\text{Payoff of H} \\ \text{when } \alpha = 0}}$$

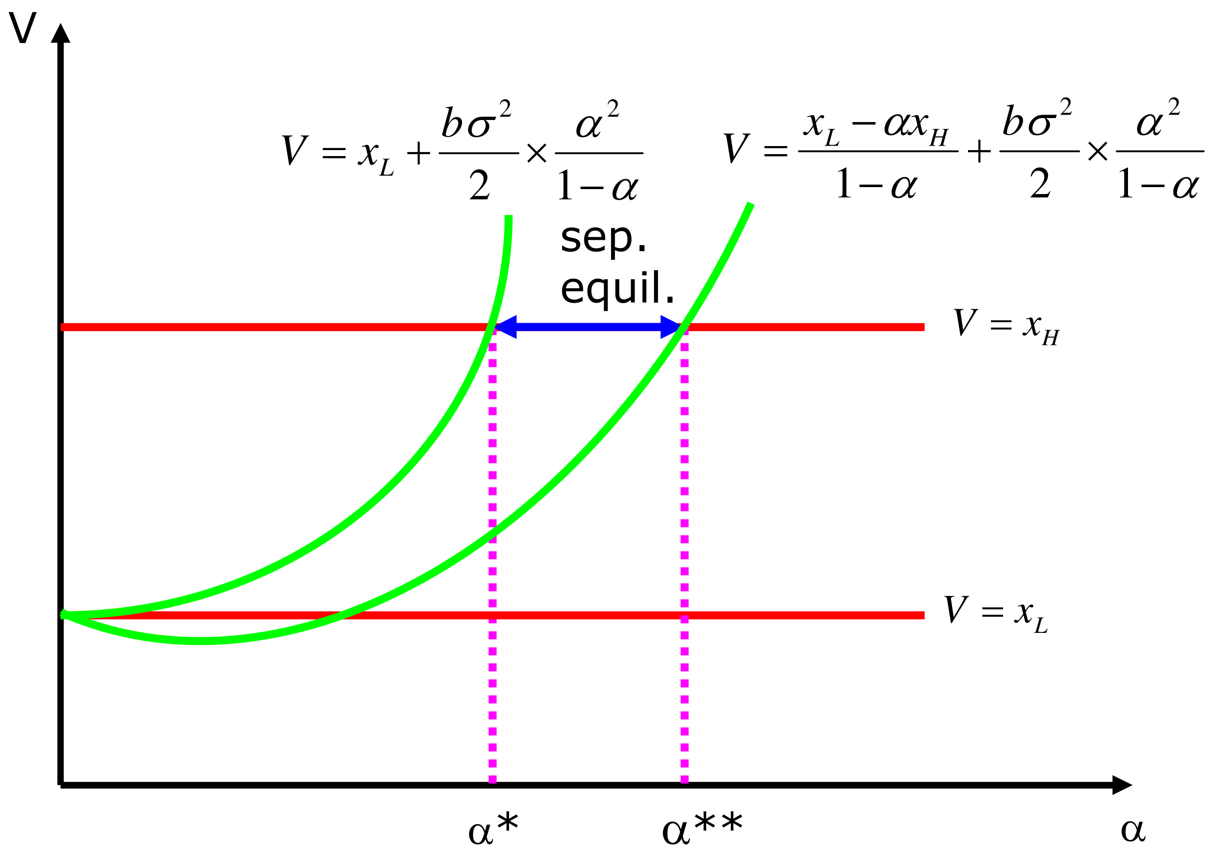
- The IC of L:

$$\underbrace{x_L}_{\substack{\text{Payoff of L} \\ \text{when } \alpha = 0}} \geq \underbrace{\alpha x_L + (1-\alpha)V - \frac{b}{2}\alpha^2\sigma^2}_{\text{Payoff of L when he keeps a fraction } \alpha \text{ of the firm}}$$

- Indifference of type T between V and α:

$$\underbrace{x_L}_{\substack{\text{Payoff} \\ \text{if } \alpha=0}} = \underbrace{\alpha x_T + (1-\alpha)V - \frac{b}{2}\alpha^2\sigma^2}_{\substack{\text{Payoff when keeping a fraction } \alpha \\ \text{of the firm if it induces a value V}}} \Rightarrow V = \frac{x_L - \alpha x_T}{1-\alpha} + \frac{b\sigma^2}{2} \times \frac{\alpha^2}{1-\alpha}$$

The set of separating equilibria



$$\alpha^* = -Z + \sqrt{Z(Z+2)} \qquad Z \equiv \frac{x_H - x_L}{b\sigma^2}$$

The Riley outcome

- Using L's IC, the ownership share of type H is:

$$\underbrace{x_L}_{\substack{\text{Payoff of L}\\ \text{when } \alpha=0}} = \underbrace{\alpha x_L + (1-\alpha)x_H - \frac{b}{2}\alpha^2\sigma^2}_{\substack{\text{Payoff of L when he keeps a fraction } \alpha \text{ of the firm}\\ \text{and the market believes that his type is H}}} \Rightarrow \alpha^* = -\underbrace{Z}_{\frac{x_H - x_L}{b\sigma^2}} + \sqrt{Z(Z+2)}$$

- α^* increases with Z which
  - increases with the extent of asymmetric info., x_H-x_L
  - decreases with risk aversion, b
  - decreases with the variance of cash flows, σ^2

- Using H's IC:

$$\underbrace{x_L}_{\substack{\text{Payoff of H}\\ \text{when } \alpha=0}} = \underbrace{\alpha x_H + (1-\alpha)x_H - \frac{b}{2}\alpha^2\sigma^2}_{\substack{\text{Payoff of H when he keeps a fraction } \alpha \text{ of the firm}\\ \text{and the market believes that his type is H}}} \Rightarrow \alpha^{**} = \sqrt{\frac{2(x_H - x_L)}{b\sigma^2}}$$

- $\alpha^{**} < 1$ iff $2(x_H - x_L) < b\sigma^2$; otherwise, type H prefers every $\alpha > 0$ over $\alpha = 0$ provided that he convinces outsiders that the firm's type is H