Problem sets for the course Intermediate Microeconomics
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Topic 1: Games in normal form

Problem 1

Consider the following normal form game:

Player 2 / Player 1	Left	Center	Right
Top	2, 2	3, 2	5, x
Bottom	y, 3	1, 4	0, 1

As usual, the left number in each box represents the payoff of player 1 (the "row" player) while the right number represents the payoff of player 2 (the "column" player).

(a) For which values of x will Right be a dominated strategy for player 2? Explain your answer.

(b) Can you find a value of x for which player 2 will have a dominant strategy? Explain your answer.

(c) For which values of y will Player 1 have a dominant strategy? Explain your answer.

(d) Suppose that x = y = 1. Solve the game by iterative elimination of dominated strategies.

(e) Suppose that x = 3 and find the Nash equilibrium of the game. Explain why you do not need to know the value of y in order to find the Nash equilibrium.

Problem 2

The following question is based on the paper "Coordination Through Committees and Markets" by Farrell and Saloner (*Rand Journal of Economics*, 1988 pp. 235-252).

Two users choose simultaneously between two technologies, A and B. The two users differ in their preference for the two technologies: User 1 prefers technology A to B. His utility from adopting A is equal to $a > 0$, while his utility from adopting B is 0. User 2, on the other hand, prefers technology B to A. His utility when he adopts B is equal to $b > 0$, while his utility from adopting A is 0. However, because of positive network externalities, each user draws additional utility of $c > 0$ when both users adopt the same technology.

(a) Write out the game in a normal form.

(b) What are the conditions on a,b and c that ensure the existence of an equilibrium in dominant strategies? Describe this equilibrium.

(c) What are the conditions on a,b and c that ensure the existence of a Nash equilibrium in which both users adopt technology A? Is this equilibrium also an equilibrium in dominant strategies? Explain why.

(d) Suppose that $a = b < c$. Find all Nash equilibria in this case and compute the corresponding payoffs.

Topic 2: Oligopoly

Problem 1

Consider a market for a homogenous product with two identical firms that compete by setting quantities. The cost function of each firm is $C(q_i) = 4q_i+q_i^2/2$, where q_i is the output of firm i = 1,2. The inverse demand function for the product is $P = A-(q_1+q_2)$, where $A > 4$.

(a) Compute the symmetric Nash equilibrium in the market and the equilibrium profit of each firm.

(b) Show the best response functions and the equilibrium point in a figure that has q_1 on the horizontal axis and q_2 on the vertical axis.

(c) Compute the equilibrium under the assumption that firm 2 observes q_1 before it chooses q_2 and firm 1 knows that firm 2 will observes q_1 before it will choose q_2.

Problem 2

Consider an industry with quantity-setting firms that produce a homogeneous good. The inverse demand function for the good is $p = A - Q$, where $Q = q_1+q_2+...+q_n$ is the aggregate quantity. All firms have a cost function $C = F + q^2/2$. Initially there are many potential firms that can enter the industry: firms first decide whether or not to enter and then all firms that entered the market compete by simultaneously choosing quantities. If a firm decides to stay out, its payoff is 0.

(a) Suppose that n firm decided to enter. Compute the symmetric Nash equilibrium when all entrants choose their quantities simultaneously.

(b) Given your answer to (a), how many firms will decide to enter the market? (Hint: entry will take place until the equilibrium profit of each firm that enters the market is equal to 0)

(c) How does the number of entrants that you computed in (b) vary with the fixed cost F?

(d) Explain the intuition for your answer.

Problem 3

Consider two quantity-setting firms that produce a homogeneous good. The inverse demand function for the good is $p = A - (q_1+q_2)$. Each firm i has a cost function $C = q_i^2$.

(a) What is the Nash equilibrium if both firms choose their quantities simultaneously? What are the equilibrium profits of the two firms?

(b) Suppose that firm 1 can switch to a new technology under which its cost function becomes $C_1 = F + q^2/2$. The cost function of firm 2 remains $C = q^2$. What the largest value of F for which firm 1 will switch to the new technology if it assumes that both firms will continue to produce the same equilibrium quantities that you computed in (a)? (Hint: you should evaluate the profit of firm 1 after the switch at the equilibrium quantities that you computed in (a)).

(c) Are the equilibrium quantities that you computed in (a) still optimal for both firms after firm 1 adopts the new technology? Explain your answer and show why or why not this is true. (Hint: you should compute the best response of each firm after firm 1 makes the switch to the new technology and then check if its quantity in (a) is a best response against the rival's quantity in (a)).

(d) Compute the Nash equilibrium after firm 1 adopts the new technology (note that this equilibrium is not symmetric)

(e) What is the largest value of F for which firm 1 will switch to the new technology? (Hint: you should compare the profits of firm 1 in (a) and in (d))

(f) Compare your answers to (b) and (e). Explain the intuition in detail; that is, why is/isn't there a difference between the two answers?

(g) Now suppose that instead of switching to a new technology, firm 1 can select q_1 before firm 2 selects q_2 (that firm 2 observes q_1 before choosing q_2 and both firms know that this will be the case). Solve for the equilibrium in this case.

(h) Compare the profits of firm 1 in the equilibrium in (d) assuming that F = 0 and in (g). What will firm 1 prefer: to be more efficient or to be a market leader?

Problem 4

Two firms engage in an R&D race to discover a new technology. If both firms discover the new technology, then each of them earns R in the market from selling the new technology. If only one firm succeeds, then it earns αR, where $\alpha > 1$, while the rival firm earns 0. If both firms fail, then each of them has 0 earnings. The two firms need to invest in R&D. If firm i invests $p_i^2/2$ dollars in R&D, then its probability of discovering the new technology is p_i, where $p_i < 1$. The probabilities of discovery are independent across firms. Hence, with probability p_1p_2 both firms succeed, with probability $p_1(1-p_2)$ firm 1 succeeds and firm 2 fails and with probability $p_2(1-p_1)$ firm 2 succeeds and firm 1 fails.

(a) What is the Nash equilibrium if both firms choose their investments in R&D simultaneously (that is firm 1 chooses p_1 and firm 2 chooses p_2)?

(b) Now suppose that the two firms form an R&D consortium and choose p_1 and p_2 jointly with the objective of maximizing their joint profits. Compute the values of p_1 and p_2 at the optimum.

(c) Are the investment levels higher in (a) or in (b)? Carefully explain the intuition for your answer.

(d) Are the expected profits of firms 1 and 2 higher in (a) or in (b)? Carefully explain the intuition for your answer.

Problem 5

Consider a market in which the inverse demand function is $p = 20 - Q$, where Q is the aggregate quantity. Initially there is only one firm, firm 1, in the market. The marginal cost of firm 1 is constant and equal to 4 per unit. Firm 2, whose constant marginal cost per unit is k contemplates entry into the market. If firm 2 enters, then firm 1 and 2 engage in Cournot competition.

(a) What is the Nash equilibrium of the game if firm 2 enters the market?

(b) What is the highest value of k for which firm 2 would wish to enter the market? Carefully explain your answer.

(c) Repeat your answer to parts (a) and (b), assuming that initially there are two firms in the market, firms 1 and 3, both of whom have a constant marginal cost of 4 per unit.

Problem 6

Two firms compete in an auction run by a local government for the construction of a new highway worth $60 million for the local government. According to the rules of the auction, each firm needs to place a bid in a sealed envelop. The firm that makes the lowest bid is then chosen to build the highway and gets a price equal to its own bid. If the two firms make the same bid, then firm 1 is chosen to build the highway (that is, firm 2 can win the auction only if its bid is strictly lower than that of firm 1).

(a) Suppose that the cost of building the highway for both firms is $10 million. Find the Nash equilibrium of the game.

(b) Now suppose that firm 2's cost of building the highway is $15 million while firm 1's cost is $10 million. Find the Nash equilibrium of the game.

Problem 7

Consider two price-setting firms that produce differentiated products. The demand for the product of firm 1 is $q_1 = 4 - 3p_1 + 2p_2$ and the demand for the product of firm 2 is $q_2 = 4 - 3p_2 + 2p_1$. For simplicity, suppose that the firms have no production costs. Compute the Nash equilibrium of the game.

Topic 3: Extensive form games

Problem 1

Consider the following extensive form game between a university professor and a student who missed the midterm exam. The professor can either "trust" the student and give him a take home exam provided that the student will not ask any of his friends for help, or "mistrust" the student and give him an exam in the professor's office. In the latter case, both the professor and the student get a payoff of 1. If the student gets a take home exam, he can either "honor" his commitment not to ask friends for help and then both he and the professor get a payoff of 2, or "cheat." If the student cheats, the professor can either "grade" the exam as is, in which case his payoff is 0 and the student's payoff is 3, or "investigate" whether the student has cheated, in which case both he and the student get a payoff of -1.

(a) Draw the game tree for the game.

(b) Compute the subgame perfect equilibrium of the game. Carefully explain your answer.

(c) Suppose now that the student believes that once he cheats there would be an automatic investigation (the student fails to notice that the professor can actually grade the exam as is without investigating). What is the subgame perfect equilibrium of this new game? carefully explain your answer.

Problem 2

Consider two price-setting firms. The demand for firm 1's product is $q_1 = 9-4p_1-p_2$ and the demand for firm 2's product is $q_2 = 9-4p_2-p_1$. For simplicity, suppose that the firms have no production costs.

(a) Are the products of the two firm substitutes or complements? Explain your answer.

(b) Are the strategies of the two firms strategic substitutes or strategic complements? Explain your answer.

(c) Suppose that firm 1 sets its price before firm 2 (firm 2 sets its price after it observes p_1). Compute the subgame perfect equilibrium of the game.

Topic 4: GE in exchange economies

Problem 1

Consider an exchange economy with 2 agents, A and B, and 2 goods, x and y. The two agents have identical utility functions given by

$$U(x_i, y_i) = \sqrt{x_i} + \sqrt{y_i}, \qquad i = A, B.$$

The initial endowments of agents A and B, respectively, are (4,0) and (0,4). That is, agent A has 4 units of x and 0 units of y while agent B has 0 units of x and 4 units of y.

(a) Write down the maximization problem that needs to be solved in order to characterize the set of Pareto efficient allocations, derive the first order conditions for this problem, and use them to express the condition that ensures Pareto efficiency. Explain in simple words what is the meaning of this condition.

(b) Using your answer in (a), write down the equation that describes the contract curve and show the contract curve graphically in an Edgeworth box.

(c) Given your answer in (b), derive the utility possibilities frontier (UPF) in the economy and show the UPF in a graph that has the utility of agent A on the horizontal axis and the utility of agent B on the vertical axis.

(d) Suppose that a social planner wants to allocate the resources in the economy with the objective of maximizing the following welfare function:

$$W(U^A, U^B) = 3U^A + 4U^B.$$

Given the UPF you derived in (c), what are the utility levels that agent A and agent B will get when the social planner maximizes $W(U^A,U^B)$?

(e) Find the allocation of x and y between the two agents such that each agent will get the utility level that you computed in (d).

Topic 5: GE in production economies

Problem 1

Consider a production economy with 10 units of capital, K, and 10 units of labor, L. Capital and labor can be used to produce the goods, x and y. The production functions of x and y are

$$x = Min\{K, L\}, \qquad y = K + L.$$

That is, K and L are perfect complements in the production of x (you need one unit of each to produce each unit of x) but are perfect substitutes in the production of y.

(a) Draw an Edgeworth box that corresponds to this economy, with units of K on the horizontal axis and units of L on the vertical axis. Put good x at the lower left corner of the box and good y at the top right corner. Show representative isoquants for x and y and derive the contract curve.

(b) Derive the production possibility frontier for this economy and illustrate it on a graph.

Problem 2

Consider an island economy with one agent. There are 9 coconut trees on the island that can be cut down and used to build a hut. If a tree is not cut down, it produces 2 coconuts. Suppose that the size of the hut in square feet is $h = 8T^{1/2}$, where T is the number of trees that the agent cuts down and uses for housing. The preferences of the agent over housing and coconuts are

$$U = Min\left\{h, \frac{c}{2}\right\}.$$

(a) Derive the Production Possibilities Frontier (PPF) for the island economy and show it on a graph that has coconuts on the horizontal axis and housing on the vertical axis.

(b) Given the PPF you derived in (a), write the conditions that define the Pareto efficient allocation and show them in the graph you drew in (a) (make sure you clearly explain what's going on in the graph). Note: to characterize the Pareto efficient allocation you need to solve a quadratic equation. It is enough to write this equation and explain how it determines the Pareto efficient allocation (although you do not have to solve it, the solution of this equation is straightforward).

(c) Suppose that the agent establishes a profit maximizing firm that buys trees from the agent at a price of w per tree and sells housing on the market at a price of p per square feet. Solve the firm's problem and derive the firm's demand for trees and the firm's supply of housing as functions of p and w.

(d) The agent's income comes from (i) selling the 9 trees at a price of w per tree (the trees are sold either to the firm who uses them to build the hut or to the agent who uses them to produce coconuts), and (ii) the firm's profits. Compute the agent's income as a function of p and w and write down the agent's budget constraint.

(e) Given your answer in (d), derive the agent's demand function for housing and for trees (remember: the agent cares for trees because of the 2 coconuts that each tree produces).

(f) Write down the market clearing conditions in the market for trees and the market for housing. Normalize w to 1 and derive the (quadratic) equation that determines p (again, the solution of this equation is straightforward but you do not have to solve it).

Topic 6: Externalities and public goods

Problem 1

Consider an economy with two agents, A and B. Each agent has an endowment of 24 units of good y that he can consume. Apart from good y, the two agents can spend time surfing the web. Since the server that the two agents use to surf the web can get congested, the utility of each agent from surfing the web is negatively affected by the amount of time that the other agent spends on surfing the web. Specifically, using h to denote the number of hours spent on surfing the web, the utility functions of the two agents are given by

$$U^A(h_A, h_B, y_A) = h_A(12 - h_A - h_B) + y_A, \qquad U^B(h_A, h_B, y_B) = h_B(12 - h_A - h_B) + y_B$$

(a) Write down the maximization problem that needs to be solved in order to characterize the set of Pareto efficient allocations, and solve it for the Pareto efficient allocations (hint: note that in total there are 48 units of good y).

(b) Suppose that the agents play a game in which each agent chooses how many hours to spend on surfing the web (note that the payoff of each agent is his utility and that surfing the web is costless). Solve for the Nash equilibrium of the game (hint: note that the equilibrium is symmetric).

(c) Compare the amount of time that each agent will spend on surfing the web in (a) and in (b). Explain the intuition for this result.

(d) Now suppose that the government gives agent B the property rights over the use of the server that enables the agents to surf the web. In particularly, agent B can now prevent agent A from surfing the web. In addition, suppose that agent B approaches agent A and makes him a take-it-or-leave it offer according to which agent B will allow agent A to surf the web for h_A hours if in return agent A will give agent B T units of good y. If the offer is rejected, agent A cannot surf the web and each agent consumes his initial endowment of good y. Compute the optimal offer that agent B will make agent A and compare the resulting allocation with the one you got in (b). Explain the intuition for your answer.

(e) Now suppose that the government decides to charge a price p for each hour spent on surfing the web. Compute the price p needed in order to restore Pareto efficiency when the two agents play the game described in (b).

Problem 2

Consider an economy with 2 agents, A and B, each of whom has a utility function defined over a public good, G, and a private good y. The utility functions of the two agents are given by

$$U(G, y_i) = 2\ln(G) + y_i, \qquad i = A, B$$

The initial endowment of agent A is 5 units of good y and the initial endowment of agent B is 3 units of good y. The cost of producing the public good is $G^2/2$ units of good y.

(a) Write down the Production Possibilities Frontier (PPF) for the economy and show it on a graph that has the public good on the horizontal axis and the private good on the vertical axis.

(b) Given the PPF you derived in (a), write down the maximization problem that needs to be solved in order to characterize the set of Pareto efficient allocations and derive the Samuelson condition. Explain in words the meaning of this condition. How many units of the public good are produced at a Pareto efficient allocation?

(c) Now, suppose that the public good is produced by a profit maximizing firm that is jointly owned by the two agent (each agent is entitled to 50% of the firm's profit). Normalize the price of good y to 1 and let p be the price of the public good and assume that agents A and B, respectively, buy g_A and g_B units of the public good from the firm and contribute these units to a common pool, so the size of the public good is $G = g_A + g_B$. Solve for the Walrasian equilibrium in this economy.

(d) Prove that the Walrasian equilibrium you computed in (c) there is an underprovision of the public good relative to the Pareto efficient allocation that you computed in (b). Explain the intuition for this result.

(e) Solve for the system of subsidies that is needed to induce the two agents to provide a Pareto efficient level of the public good in a Walrasian equilibrium.

Problem 3

Consider an economy with two agents, A and B, each of whom has a utility function defined over a public good, G, and a private good y. The utility functions of the two agents are given by

$$U^A = 5G - \frac{G^2}{2} + y_A, \qquad U^B = 5G - \frac{G^2}{2} + y_B.$$

The initial endowment of agent A is 10 units of good y and the initial endowment of agent B is 8 units of good y. The technology is such that one unit of y can be converted to one unit of G (i.e., the cost of producing G units of the public good is y units of the private good).

(a) Write down the Production Possibilities Frontier (PPF) for the economy and show it on a graph that has G on the horizontal axis and y on the vertical axis.

(b) Given the PPF you derived in (a), write down the maximization problem that needs to be solved in order to characterize the set of Pareto efficient allocations and derive the

Samuelson condition. Explain in words the meaning of this condition. How many units of the public good are produced at a Pareto efficient allocation?

(c) Now consider a voluntary contribution game in which agent i, i = A,B, contributes z_i units of his private good to a common pool, such that the amount of the public good is $G = z_A+z_B$. Solve for the symmetric Nash equilibrium of this game.

(d) Is the equilibrium amount of G that you computed in (c) Pareto efficient? Explain the intuition for your answer.