310-2 - Yossi Spiegel

Solution to Problem set 1

Problem 1

(a) If $x < 3$, then Left strictly dominates Right for Player 2 since no matter what Player 1 plays, Player 2's payoff under Left is higher. Using similar logic, it follows that if $x < 2$, then Center strictly dominates Right. Hence right is dominated as soon as $x < 2$ (recall that to show that a strategy is dominated it is enough to point out one other strategy that dominates it).

(b) To find a dominant strategy for Player 2 we need to find a strategy that gives Player 2 a higher payoff than any other strategy no matter what Player 1 does. Clearly Left is better than Center if 1 plays Top but Center is better than Left if 1 plays Bottom. And both Left and Center are better than Right if 1 plays Bottom. Hence, Player 2 cannot have a dominant strategy.

(c) If $y < 2$ than Top dominates Bottom for Player 1. Since Player 1 has only 2 strategies, Top is dominant.

(d) If $x = y = 1$, then Bottom is dominated for 1 and Right is dominated for 2 (by either Left or Center). Deleting them we are left with a game where 1 plays Top and 2 plays either Left or Center. But now Left dominates Center for 2 (remember that 1 now always plays Top) so the remaining outcome is (Top, Left).

(e) Note that (Bottom, Left) cannot be a NE because given Bottom, Player 2 will prefer to play Center. (Bottom, Center) cannot be a NE because Player 1 will deviate to Top. (Top, Center) is not a NE because when Player 1 plays Top, Player 2 will play Right. The only NE is (Top, Right). It is easy to check that no other outcome consists of a pair of mutual best-responses.

Problem 2

(a)

User 2 User 1	A	B
A	a+c, c	a, b
B	0, 0	c, b+c

(b) If a,b > c than user 1 always prefers A to B no matter what 2 does and 2 always prefers B no matter what 1 does. Hence, (A,B) is an equilibrium in dominant strategies.

(c) If c > b, then (A, A) is a NE since no user will deviate unilaterally from the equilibrium. This equilibrium is not in dominating strategies since User 2 prefers B is user 1 also chooses B. In other words, it is not true that user 2 prefers A no matter what user 1 does.

(d) If a = b < c, then (A, A) and (B, B) are two NE in pure strategies (there is also an equilibrium in mixed strategies). The payoffs in (A, A) are a+c and c and in (B, B) they are c and b+c.