310-2 - Yossi Spiegel

Solution to Problem set 4

Problem 1

(b) To compute the subgame perfect equilibrium of the game, let's start with the last subgame that starts after the professor Trusts and the student Cheats (this of course does not mean that the professor will trust the students and that students will cheat; we just solve for what happens IF we get to that point in the game). Then since grading is best for the professor, he'll grade the student's exam thus avoiding a costly investigation. Realizing that his exam will be graded anyway, the student will cheat as his payoff is going to be 3 rather than 2. But since the professor's payoff from trust will eventually be 0 (given that the student cheats and the professor grades), it better for the professor not to trust the student in the first place. Hence the SPE is ((Mistrust, Grade), (Cheat)). Notice that we specify the professors actions even for subgames that are not arrived at on the equilibrium path; that is, given that the professor does not trust the student there is no chance for him to grade. But what we are after is a characterization of the SPE. The reason why the student cheats is that he realizes that the professor will grade and this is precisely why the professor does not trust the student in the first place.

(c) If the student believes that once he cheats there would be an automatic investigation, then he'll honor the professor's trust to avoid the -1 from being investigated and this in turn will induce the professor to trust the student. The SPE now is ((Trust, Investigate), (Honor)).

Problem 2

If firm 1 chooses its price before firm 2, then the best-response function of firm 2 is $BR_2(p_1) = (4+2p_1)/6$. Firm 1 takes $BR_2(p_1)$ as given and maximizes its profit by choosing p_1. The resulting price is $p_1^* = 8/7$. Firm 2's price is $p_2^* = (4+2p_1^*)/6 = 44/42$.