

After the exam of 01.02.2015: Typical errors, comments etc.

QUESTION 1

ERRORS:

No attention to the problem of $\lambda = 0$: 7 points.

Wrong explanation why $\lambda \neq 0$: 5 points. (“By the theorem”; “because $h' > 0$ ”; just “ $\lambda \neq 0$ ” with no explanation; “ $\nabla g = \lambda \nabla f$ ” instead of “ $\nabla f = \lambda \nabla g$ ”; etc.)

Hint instead of explanation: 2 points (“since $h' > 0$ and P, Q, R are not on a line”).

QUESTION 2

FATAL ERRORS:¹ Implicit function theorem is applied when $r + c \neq n$; or, without checking that the relevant determinant is not zero.

ERRORS:

Item (b): local and global uniqueness confused: 7 points.

REMARK to Item (b): some students provide a counterexample to the global uniqueness, which is more than I really expected. These got 2 points above the 35 points. (But in some cases 35 is really 35+2-2.)

QUESTION 3

ERRORS:

Argument that holds only when $f, g \geq 0$ is applied to arbitrary f, g : 7 points. Example: “ $\sup(f^2) - \inf(f^2) \leq (\sup f)^2 - (\inf f)^2$ ” (think, what happens if $\sup f = 1$ and $\inf f = -1$); another example: “ $\inf(fg) \geq (\inf f)(\inf g)$ ”.

FATAL ERROR: Ridiculously, 3 students “proved” the equality $\int fg = \int fg$ by “deducing” the inequality

$$\frac{1}{2} \left(\int f^2 + \int g^2 \right) \leq \int fg \leq \int fg \leq \frac{1}{2} \left(\int f^2 + \int g^2 \right)$$

from $\frac{1}{2}(f^2 + g^2) - fg = \frac{1}{2}(f - g)^2 \geq 0$. Thus, they “prove” that, moreover, $\int fg = \frac{1}{2}(\int f^2 + \int g^2)$, that is, $\int (f - g)^2 = 0$ for all integrable f, g . Wow!

¹It means, no points for this question!

GRADES STATISTICS

| Total | Question 1 | Question 2 | Question 3 | Question 4 |
|-------|------------|------------|------------|------------|
| 107 | 35 | 37 | | 35 |
| 102 | 35 | 37 | 30 | |
| 102 | 35 | 37 | 30 | |
| 100 | 35 | 35 | 30 | |
| 100 | 35 | 35 | 30 | |
| 100 | 35 | 35 | 30 | |
| 100 | 35 | 35 | 30 | |
| 100 | 35 | 35 | | 30 |
| 98 | 33 | 35 | 30 | |
| 97 | 30 | 37 | 30 | |
| 97 | 30 | 37 | 30 | |
| 96 | 33 | | 28 | 35 |
| 95 | 35 | | 30 | 30 |
| 94 | 30 | 37 | 27 | |
| 93 | 35 | | 23 | 35 |
| 93 | 35 | 28 | 30 | |
| 93 | 35 | 35 | 23 | |
| 93 | 35 | 35 | 23 | |
| 93 | 35 | 35 | 23 | |
| 91 | 35 | 35 | 21 | |
| 89 | 26 | 28 | | 35 |
| 88 | 30 | 28 | 30 | |
| 85 | 35 | 20 | 30 | |
| 83 | 30 | | 23 | 30 |
| 81 | 30 | 28 | 23 | |
| 80 | 20 | 30 | 30 | 23 |
| 75 | 35 | 10 | 30 | |
| 75 | 35 | 10 | 30 | |
| 73 | 35 | 8 | 30 | |
| 69 | 26 | 20 | 23 | |
| 66 | 28 | 28 | | 10 |
| 60 | 30 | 0 | 30 | |

| Total | Question 1 | Question 2 | Question 3 | Question 4 |
|-------|------------|------------|------------|------------|
| 55 | 32 | 0 | 23 | |
| 51 | 30 | 0 | 21 | |
| 41 | 26 | 0 | 15 | |
| 40 | 30 | 0 | 10 | |
| 35 | 15 | 0 | 20 | |
| 30 | 30 | 0 | 0 | |
| 28 | 28 | 0 | 0 | |
| 25 | 25 | 0 | 0 | |
| 23 | 23 | 0 | 0 | |
| 23 | 23 | 0 | | 0 |
| 23 | 23 | 0 | 0 | |
| 0 | | | | |
| 0 | | | | |
| 0 | | | | |